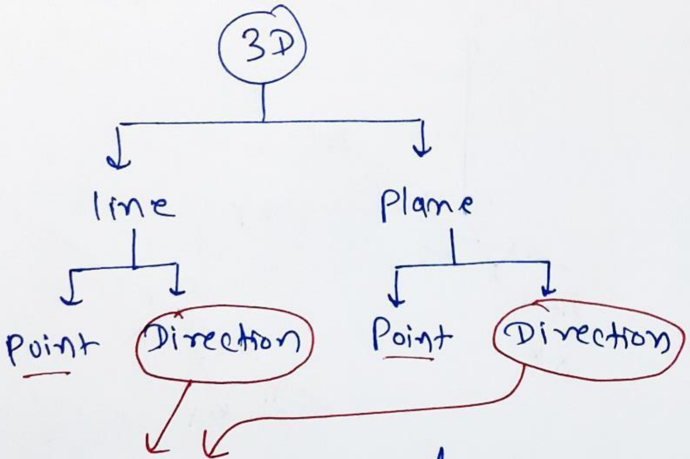
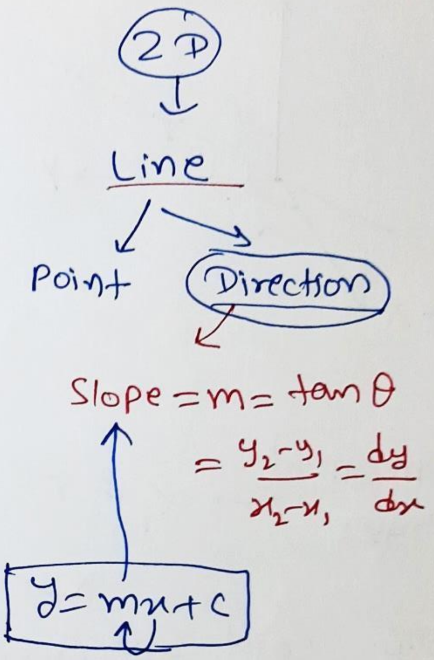
[redacted]	Class 12 Maths	Three Dimensional Geometry

Direction Cosines (D.C.) & Direction Ratios (D.R.)
(दिक् कोज्या और दिक् अनुपात)





Slope = m = tan θ

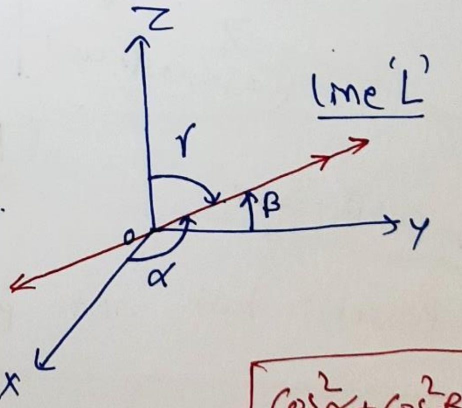
$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$

Direction Cosines ($\overset{l}{\cos\alpha}, \overset{m}{\cos\beta}, \overset{n}{\cos\gamma}$)

Direction Ratios ($\underset{\substack{\downarrow \\ a}}{\lambda l}, \underset{\substack{\downarrow \\ b}}{\lambda m}, \underset{\substack{\downarrow \\ c}}{\lambda n}$)

Direction Cosines (D.C.) & Direction Ratios (D.R.)

α, β, γ → Direction Angles between Line 'L' and $\underset{\substack{\downarrow \\ (+X)}}{OX}, \underset{\substack{\downarrow \\ (+OY)}}{OY}, \underset{\substack{\downarrow \\ (+Z)}}{OZ}$ respectively.



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$$\left(\underset{\substack{\downarrow \\ l}}{\cos\alpha}, \underset{\substack{\downarrow \\ m}}{\cos\beta}, \underset{\substack{\downarrow \\ n}}{\cos\gamma} \right) \rightarrow \text{Direction Cosines}$$

$$\boxed{\begin{array}{c} \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \\ \hline l^2 + m^2 + n^2 = 1 \end{array}}$$

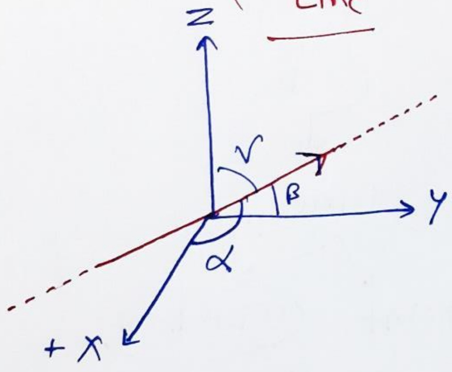
$$\left(\begin{array}{c} \lambda\cos\alpha, \lambda\cos\beta, \lambda\cos\gamma \\ \lambda l, \lambda m, \lambda n \\ a, b, c \end{array} \right) \rightarrow \text{Direction Ratios}$$

(Proportional to the D.C.)

$\lambda \in R - \{0\}$

Note: There are two possibilities of Direction Cosines (D.C.) for the same line.

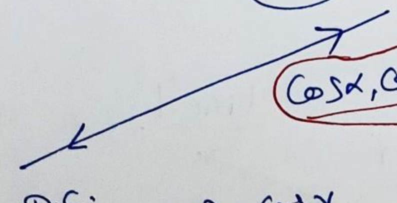
(Given Directed Line)



Direction Angles } → α, β, γ ✓

D.C. $\cos\alpha, \cos\beta, \cos\gamma$

Given



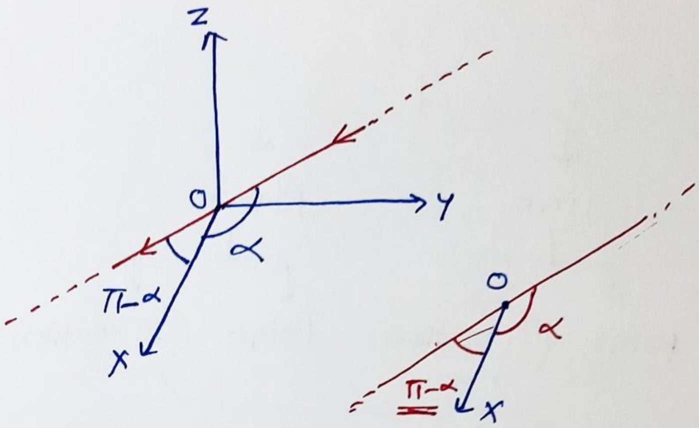
$\cos\alpha, \cos\beta, \cos\gamma$ ★ → Preferred

D.C.

$-\cos\alpha, -\cos\beta, -\cos\gamma$

possible but not preffored

(Reversed Line)



Direction Angles } → $\pi-\alpha$, $\pi-\beta$, $\pi-\gamma$

D.C. → $\cos(\pi-\alpha), \cos(\pi-\beta), \cos(\pi-\gamma)$

↳ $-\cos\alpha, -\cos\beta, -\cos\gamma$

$\boxed{\text{Conversion}}$

$\boxed{\text{D.C.}} \rightsquigarrow \boxed{\text{D.R.}}$

$l, m, n \longrightarrow \lambda l, \lambda m, \lambda n$

$\cos\alpha, \cos\beta, \cos\gamma \rightarrow \lambda\cos\alpha, \lambda\cos\beta, \lambda\cos\gamma$

$\lambda \in R - \{0\}$

$$\left\{\begin{array}{ll} & \text{DR.} \\ \lambda = 1, & l, m, n \\ \lambda = 2, & 2l, 2m, 2n \\ \lambda = 3, & 3l, 3m, 3n \\ \vdots & \vdots \end{array}\right\}$$

$\boxed{\text{D.R.}} \rightsquigarrow \boxed{\text{D.C.}}$

$$a, b, c \longrightarrow \frac{\pm a}{\sqrt{a^2+b^2+c^2}}, \frac{\pm b}{\sqrt{a^2+b^2+c^2}}, \frac{\pm c}{\sqrt{a^2+b^2+c^2}}$$

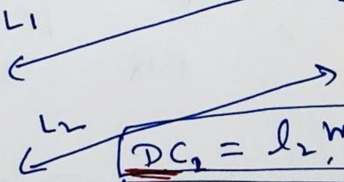
Note: A line Always has

- unique D.C.
- Infinite number of DR.

Note: Parallel lines:

$L_1 \parallel L_2$

$\boxed{DC_1 = l_1, m_1, n_1}$ → $\boxed{DR_1 = a_1, b_1, c_1}$



$\boxed{DC_2 = l_2, m_2, n_2}$

$\boxed{DR_2 = a_2, b_2, c_2}$

① $DC_1 = DC_2$ (Equal)

$l_1 = l_2$	$m_1 = m_2$	$n_1 = n_2$

② $DR_1 \propto DR_2$

(Proportion)

↓

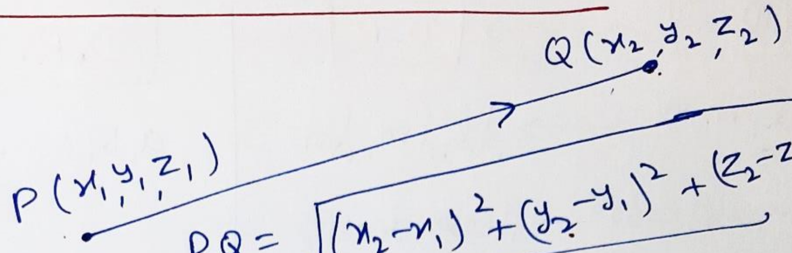
(Ratio → equal)

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

or

$$a_1 : b_1 : c_1 = a_2 : b_2 : c_2$$

D.R. & D.C. of a line joining two Points.



$$PQ = \underbrace{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

D.R. →
$$\begin{cases} x_2-x_1,\ y_2-y_1,\ z_2-z_1 \\ \text{or} \\ K(x_2-x_1),\ K(y_2-y_1),\ K(z_2-z_1) \end{cases}$$

D.C.
$$\begin{cases} \dfrac{(x_2-x_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}},\ \dfrac{(y_2-y_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}},\ \dfrac{(z_2-z_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}} \\ \text{or} \\ \dfrac{x_2-x_1}{PQ},\ \dfrac{y_2-y_1}{PQ},\ \dfrac{z_2-z_1}{PQ} \end{cases}$$

e.g. If a line makes $\underleftrightarrow{90^\circ, 60^\circ, 30^\circ}$ with the positive direction of x, y & z-axis respectively, find its Direction Cosines.

$$\begin{matrix} \alpha, & \beta, & \gamma \\ \downarrow & \downarrow & \downarrow \\ 90^\circ & 60^\circ & 30^\circ \end{matrix}$$

$$\text{D.C.} = \cos\alpha, \cos\beta, \cos\gamma$$

$$= \cos 90^\circ, \cos 60^\circ, \cos 30^\circ$$

$$= 0, \frac{1}{2}, \frac{\sqrt{3}}{2}$$

e.g. If a line has direction ratios 2, −1, −2, determine its Direction Cosines.

$$\boxed{\text{D.R.} = 2, -1, -2}$$

$$\text{D.C.} = \frac{2}{\sqrt{(2)^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}}$$

$$\text{D.C.} = \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

e.g. Find the direction cosines of the line passing through the two points $\underset{P\ ①}{(-2,4,-5)}$ & $\underset{Q\ ②}{(1,2,3)}$

$$PQ = \sqrt{3^2 + (-2)^2 + 8^2}$$

$$= \sqrt{9+4+64}$$

$$= \sqrt{77}$$

$$\text{D.R.} = x_2 - x_1, y_2 - y_1, z_2 - z_1$$

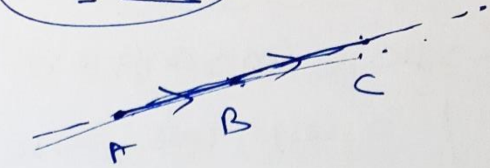
$$= 3, -2, 8$$

$$\text{D.C.} = \frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

e.g. Show that the points A(2, 3, −4), B(1, −2, 3) & C(3, 8, −11) are Collinear.

→ in a line

DC & DR



$D.R._{AB} = ?$

$DR._{BC} = ?$

Proportion (Ratio equal)

AB + BC = CA

$DR_{AB} = 1-2, -2-3, 3-(-4) = -1, -5, 7$

(2, 3, −4) (1, −2, 3)

~~−1 : −5 : 7~~

1 : 5 : −7

$DR_{BC} = 3-1, 8-(-2), -11-3 = 2, 10, -14$

(1, −2, 3) (3, 8, −11)

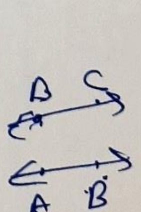
2 : 10 : −14

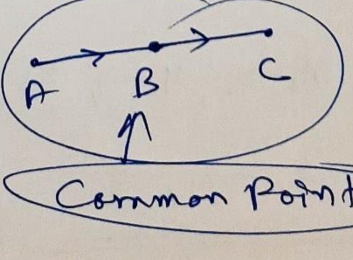
1 : 5 : −7

$$\frac{-1}{2} = \frac{-5}{10} = \frac{7}{-14}$$

$$\frac{-1}{2} = \frac{-1}{2} = \frac{-1}{2}$$

AB || BC





Common Point

Collinear

Exercise - 11.1

D.C. ☆
D. R.

Q.1 If a line makes angles 90°, 135°, 45° with the x, y and z-axes respectively, find its direction cosines.

Direction Angles

α, β, γ

$\downarrow \quad \downarrow \quad \downarrow$

$90°, 135°, 45°$

Direction Cosines

$\cos\alpha, \cos\beta, \cos\gamma$

$= \cos(90°), \cos(135°), \cos(45°)$

$= \boxed{0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}}$

Q.2 Find the direction cosines of a line which makes equal angles with the coordinate axes.

?

$\alpha = \beta = \gamma$ $\quad$ D.C. $\boxed{\cos\alpha, \cos\beta, \cos\gamma} \rightsquigarrow \cos\alpha, \cos\alpha, \cos\alpha$

Property.

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\rightarrow 3\cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{1}{3}$

$\Rightarrow \boxed{\cos\alpha = \pm\frac{1}{\sqrt{3}}}$

$$\boxed{\text{D.C.}\ \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}}$$

Q.3 If a line has the direction ratios −18, 12, −4, then what are its direction cosines?

D.R → −18, 12, −4

$$\text{D.C.} \rightarrow \frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{324+144+16}}, \frac{-4}{\sqrt{484}}$$

$$= \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22} = \boxed{\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}}$$

Points on a line

Q. 4 Show that the (2,3,4), (-1,-2,1), (5,8,7) are Collinear.
A B C

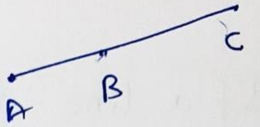
DR (Direction Ratios)

$DR_{AB} = -1-2, -2-3, 1-4$

$= -3, -5, -3$

$DR_{BC} = 5-(-1), 8-(-2), 7-1$

$= 6, 10, 6$



method - I

$AB + BC = \boxed{AC}$

method - II

$ar(\Delta ABC) = 0$

method - III

DR, DC

DR_{AB}	DR_{BC}
-3, -5, -3	6, 10, 6
-3 : -5 : -3	6 : 10 : 6
3 : 5 : 3	3 : 5 : 3

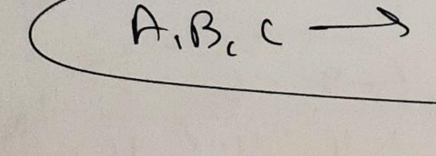
$$\frac{-3}{6} = \frac{-5}{10} = \frac{-3}{6}$$

$$-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$$

Ratio → equal.

DR → Proportional → Parallel

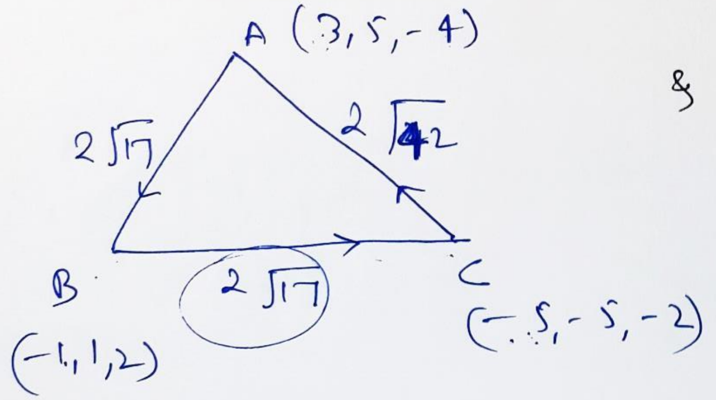
AB ∥ BC



Common

A, B, C → Collinear

Q.5 Find the direction cosines of the sides of the triangle whose vertices are $(3,5,-4) \rightarrow A$, $(-1,1,2) \rightarrow B$ & $(-5,-5,-2) \rightarrow C$



$$AB = \sqrt{4^2 + 4^2 + (-6)^2}$$

$$= \sqrt{68}$$

$$= 2\sqrt{17}$$

$$BC = \sqrt{(-4)^2 + (-6)^2 + (-4)^2}$$

$$= 2\sqrt{17}$$

~~AC~~

$$CA = \sqrt{8^2 + 10^2 + (-2)^2}$$

$$= \sqrt{64+100+4}$$

$$= \sqrt{168}$$

$$= 2\sqrt{42}$$

$$DC_{(AB)} = \frac{-1-3}{2\sqrt{17}}, \frac{1-5}{2\sqrt{17}}, \frac{2+4}{2\sqrt{17}}$$

$$= \frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}$$

$$= \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}} \checkmark$$

$$DC_{BC} = \frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}$$

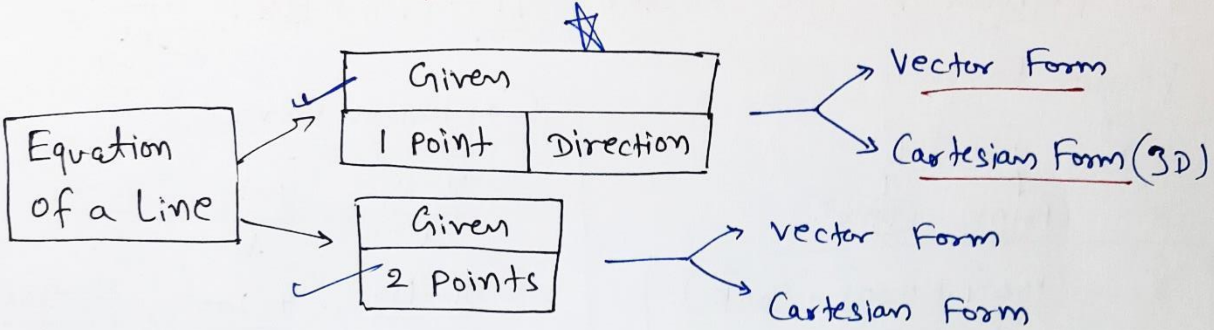
$$= \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$$

$$DC_{CA} = \frac{8}{2\sqrt{42}}, \frac{10}{2\sqrt{42}}, \frac{-2}{2\sqrt{42}}$$

$$= \frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$$

[redacted] | Class 12 Maths | Three Dimensional Geometry

Line (in 3D)



Equation of a Line (when 1 point & Direction are given)

Let the line passes through the point A & is parallel to $\vec{b}$

Coordinates of A (x_1, y_1, z_1)
Position Vector $(\vec{a})$ of A $= x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ — Point

$\boxed{\vec{b} = a\hat{i}+b\hat{j}+c\hat{k}}$ ← Direction

Equation of Line (L) ⇒

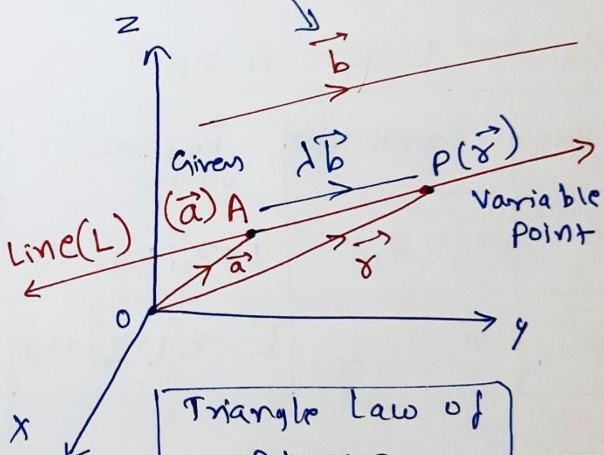
Vector Form

$$\boxed{\vec{r} = \underset{\substack{\uparrow \\ \text{Point}}}{\vec{a}} + \lambda \underset{\substack{\uparrow \\ \text{Direction}}}{\vec{b}}}$$

Cartesian Form

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$

Direction Ratios



Triangle Law of Addition

$$\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$$

variable ↓ $(x\hat{i}+y\hat{j}+z\hat{k})$

Point ↓ $(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})$

Direction ↓ $a\hat{i}+b\hat{j}+c\hat{k}$

Comparison

Solve

e.g. Find the vector & Cartesian form of the line passing through the point (100, 200, 300) and which is parallel to the vector $7\hat{i} - 17\hat{j} + 13\hat{k}$.

vector form,

$$\vec{r} = \underset{\text{Point}}{\vec{a}} + \lambda \underset{\text{Dire}^n}{\vec{b}}$$

$$\boxed{\vec{r} = (100\hat{i} + 200\hat{j} + 300\hat{k}) + \lambda(7\hat{i} - 17\hat{j} + 13\hat{k})}$$

Cartesian form,

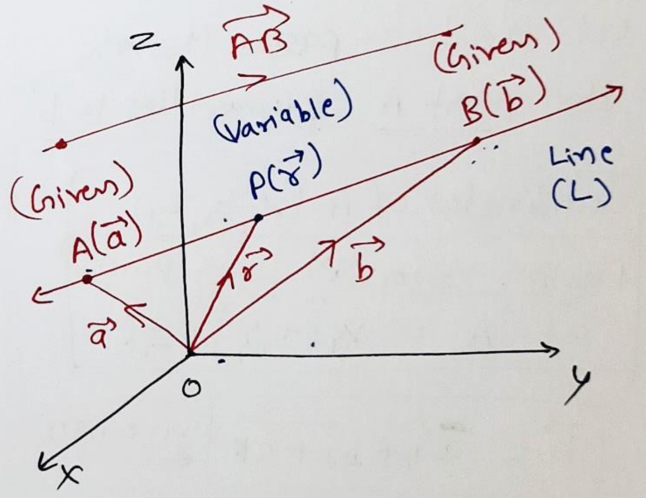
$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\Rightarrow \frac{x - 100}{7} = \frac{y - 200}{-17} = \frac{z - 300}{13}$$

Equation of a line (when two points on the line are given) →

Given Points A & B

Point	Coordinates	Position vector
A	(x_1, y_1, z_1)	$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$
B	(x_2, y_2, z_2)	$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$



Direction of line.

$$\vec{AB} = \vec{b} - \vec{a} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Equation of line (L)

Vector Form

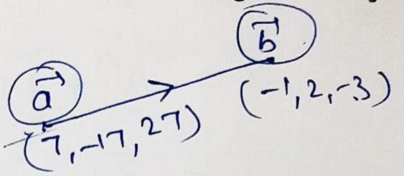
$$\boxed{\vec{r} = \underset{\text{Point}}{\vec{a}} + \lambda \underset{\text{Direction}}{(\vec{b} - \vec{a})}}$$

Cartesian Form

$$\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$$

Direction.

e.g. Find the vector equation for the line passing through the points (7, −17, 27) & (−1, 2, −3).



Vector form.

$$\vec{r} = \underset{\text{Point}}{\vec{a}} + \lambda(\underset{\text{Direction}}{\vec{b} - \vec{a}})$$

$$\boxed{\vec{r} = 7\hat{i} - 17\hat{j} + 27\hat{k} + \lambda(-8\hat{i} + 19\hat{j} - 30\hat{k})}$$

Cartesian form.

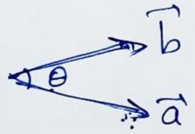
$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \boxed{\frac{x - 7}{-8} = \frac{y + 17}{19} = \frac{z - 27}{-30}}$$

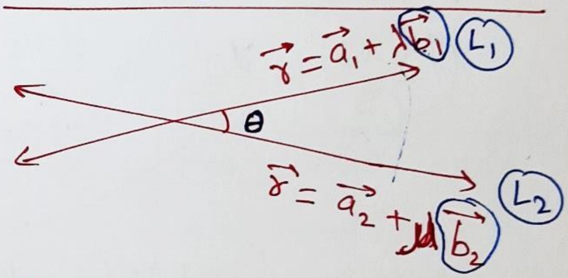
Angle between two lines :

Dot Product

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$



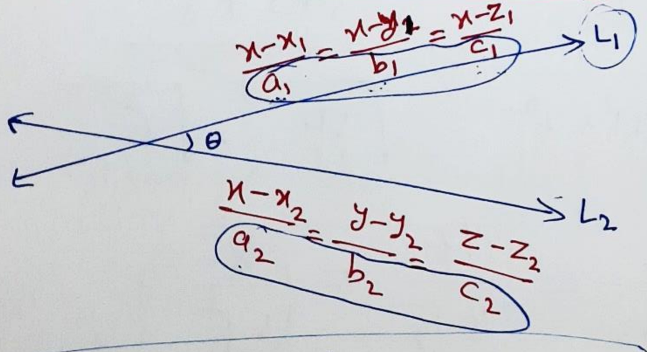
Understand in Vector Form



Directly by Dot product.

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

understand in Cartesian Form



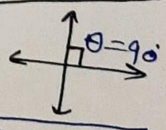
$$\boxed{\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}}$$

$\cos 90 = 0$

Conditions

Case-I perpendicular lines

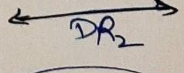
$\theta = 90°$



$$\boxed{a_1 a_2 + b_1 b_2 + c_1 c_2 = 0}$$

Case-II Parallel lines

$\theta = 0°$



$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

DR₁ ∝ DR₂

e.g. Find the angle between the pair of lines given by

$$\vec{r} = 3\hat{i} + \cancel{2\hat{j}} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) \quad \vec{b_1} \rightarrow \text{Directions}$$

$$\vec{r} = 5\hat{i} \cancel{-} 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k}) \quad \vec{b_2}$$

$$\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|}$$

$$\cos\theta = \frac{(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{1^2+2^2+2^2}\,\sqrt{3^2+2^2+6^2}} = \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

e.g. Find the angle between the pair of lines

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4} \quad \& \quad \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}.$$

Direction

$$\cos\theta = \frac{(3\times1)+(5\times1)+(4\times2)}{\sqrt{3^2+5^2+4^2}\cdot\sqrt{1^2+1^2+2^2}} = \frac{3+5+8}{\sqrt{50}\cdot\sqrt{6}}$$

$$\cos\theta = \frac{16}{\sqrt{\cancel{150}\,300}} = \frac{16}{\sqrt{100\times3}} = \frac{16}{10\sqrt{3}} = \frac{8}{5\sqrt{3}}$$

~~150 = 25×6~~

$$\boxed{\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)}$$

Distance between Two Skew Lines

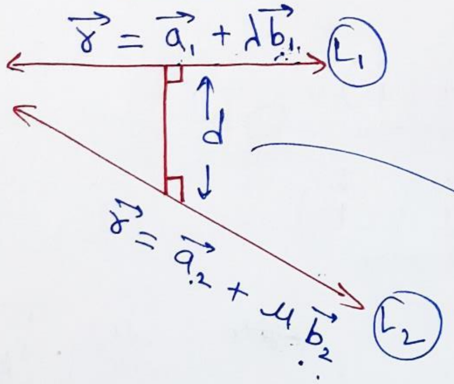
Not Parallel

Never meet

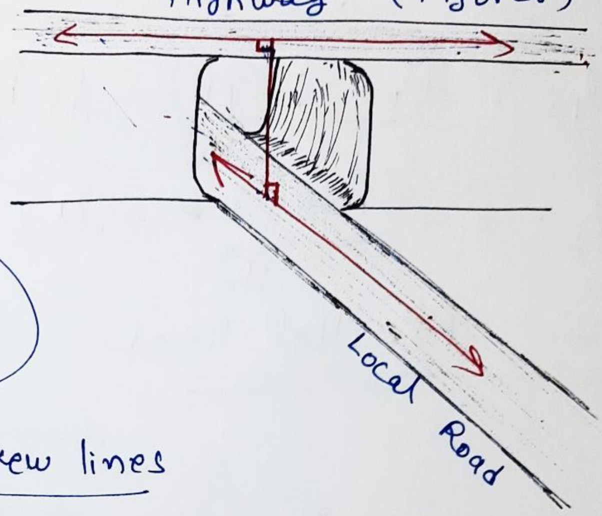
Non Coplanar

(एक plane में हो ही नही सकती)

Skew Lines



Highway (Flyover)



Shortest Distance b/w Skew lines

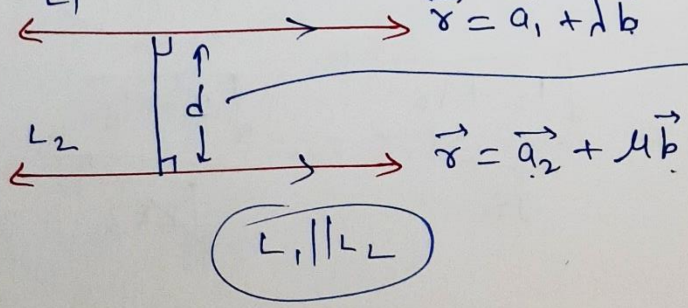
$$d = \left| \frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|} \right|$$

← modulus

magnitude

Remember only one Formula (vector form only)

Distance b/w Parallel lines



$$d = \left| \frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|\vec{b}|} \right|$$

e.g. Find the shortest distance between the lines l_1 & l_2 given by

$$\boxed{\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})} \leftarrow l_1$$

&

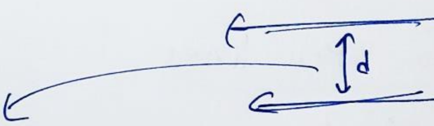
$$\boxed{\vec{r} = \hat{i} + 2\hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})} \leftarrow l_2$$

$$l_1 : \vec{r} = \underbrace{(\hat{i} + 2\hat{j} - 4\hat{k})}_{\vec{a_1}} + \lambda(\underline{\hat{i} - \hat{j} + 2\hat{k}}) \rightarrow \boxed{\vec{b}}$$

$$l_2 : \vec{r} = \underbrace{(\hat{i} + 2\hat{j})}_{\vec{a_2}} \cancel{+} - \mu(\underline{\hat{i} - \hat{j} + 2\hat{k}})$$

Parallel lines.

$(-\mu) = (\cdot\lambda_2)$

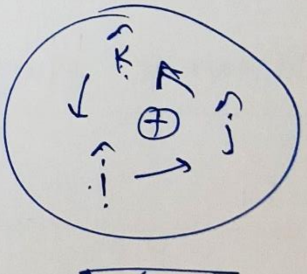


$$\vec{r} = \vec{a_1} + \lambda\vec{b}$$

$$\vec{r} = \vec{a_2} + \mu\vec{b}$$

$$d = \left|\frac{(\vec{a_2} - \vec{a_1}) \times \vec{b}}{|\vec{b}|}\right| = \left|\frac{(0.\hat{i} + 0.\hat{j} + 4\hat{k}) \times (\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{1^2 + (-1)^2 + (2)^2}}\right|$$

$$d = \left|\frac{4(\hat{k} \times \hat{i}) - 4(\hat{k} \times \hat{j}) + 8(0)}{\sqrt{1+1+4}}\right|$$



$$d = \left|\frac{4\hat{j} + 4\hat{i}}{\sqrt{6}}\right| = \frac{\sqrt{16+16}}{\sqrt{6}} = \frac{\sqrt{2 \times 16}}{\sqrt{2 \times 3}}$$

$$\boxed{d = \frac{4}{\sqrt{3}}}$$

e.g. Find the shortest distance between the lines l_1 & l_2 whose vector equations are

$$\vec{r} = \overset{a_1}{(\hat{i}+\hat{j})} + \lambda(2\hat{i}-\hat{j}+\hat{k}) \;\&\; \longrightarrow \vec{b_1}$$

$$\vec{r} = \underset{\vec{a_2}}{(2\hat{i}+\hat{j}-\hat{k})} + \mu(3\hat{i}-5\hat{j}+2\hat{k}) \longrightarrow \vec{b_2}$$

Skew lines

$\vec{r} = \vec{a_1} + \lambda \vec{b_1}$

$\vec{r} = \vec{a_2} + \lambda \vec{b_2}$

$$d = \left| \frac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|} \right|$$

$$\vec{b_1}\times\vec{b_2} = (2\hat{i}-\hat{j}+\hat{k}) \times (3\hat{i}-5\hat{j}+2\hat{k})$$

Cross Products

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$$

$$\vec{b_1}\times\vec{b_2} = 3\hat{i}-\hat{j}-7\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{9+1+49} = \sqrt{59}$$

$$d = \left| \frac{(\hat{i}-\hat{k})\cdot(3\hat{i}-\hat{j}-7\hat{k})}{\sqrt{59}} \right| = \left| \frac{3+7}{\sqrt{59}} \right|$$

$$\boxed{d = \frac{10}{\sqrt{59}}} \checkmark$$

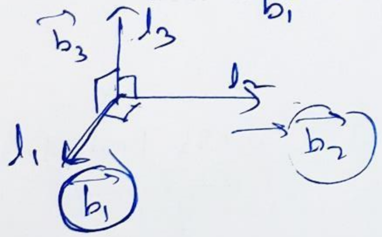
Exercise 11.2 Line (3D)

$$\boxed{\vec{r} = \underset{\text{Point}}{\vec{a}} + \lambda \underset{\text{Direction}}{\vec{b}}}$$

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$

Q.1 Show that the three lines with direction cosines

$\underbrace{\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}}_{\vec{b_1}}$; $\underbrace{\frac{4}{13}, \frac{12}{13}, \frac{3}{13}}_{\vec{b_2}}$; $\underbrace{\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}}_{\vec{b_3}}$ are mutually perpendicular.



$$\vec{b_1} = \frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} - \frac{4}{13}\hat{k}$$

$$\vec{b_2} = \frac{4}{13}\hat{i} + \frac{12}{13}\hat{j} + \frac{3}{13}\hat{k}$$

$$\vec{b_3} = \frac{3}{13}\hat{i} - \frac{4}{13}\hat{j} + \frac{12}{13}\hat{k}$$

$$\vec{b_1}\cdot\vec{b_2} = \left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} - \frac{4\hat{k}}{13}\right)\cdot\left(\frac{4}{13}\hat{i} + \frac{12}{13}\hat{j} + \frac{3}{13}\hat{k}\right)$$

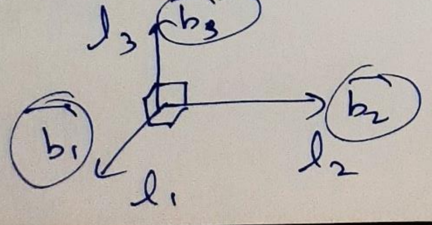
$$= \frac{12\times4}{13\times13} - \frac{3\times12}{13\times13} \cancel{+} - \frac{4\times3}{13\times13} = \frac{\cancel{48} - \cancel{36} - \cancel{12}}{13\times13}$$

$$= 0$$

$\boxed{\vec{b_1}\cdot\vec{b_2} = 0}$ $\quad \vec{b_1} \perp \vec{b_2} \quad$ $l_1 \perp l_2$

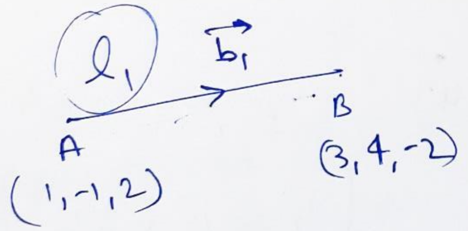
$$\vec{b_2}\cdot\vec{b_3} = \frac{4\times3}{13^2} - \frac{12\times4}{13^2} + \frac{3\times12}{13^2} = \frac{\cancel{12} - \cancel{48} + \cancel{36}}{13^2} = 0$$

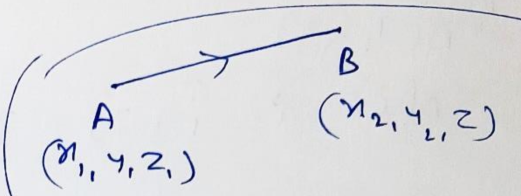
$$\vec{b_1}\cdot\vec{b_3} = \frac{12\times3}{13^2} + \frac{3\times4}{13^2} - \frac{4\times12}{13^2} = \frac{\cancel{36} + \cancel{12} - \cancel{48}}{13^2} = 0$$



$l_1 \perp l_2$, $l_2 \perp l_3$, $l_3 \perp l_1$

Q.2 Show that the line through the points (1,−1,2), (3,4,−2) is perpendicular to the line through the point (0,3,2) and (3,5,6).



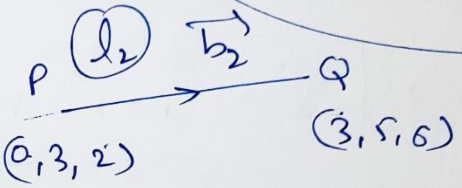


DR: $x_2-x_1,\ y_2-y_1,\ z_2-z_1$

$DR_1 = 3-1,\ 4+1,\ -2-2$

$= 2,\ 5,\ -4$

$\vec{b_1} = 2\hat{i} + 5\hat{j} - 4\hat{k}$



$DR_2 = 3,\ 2,\ 4$

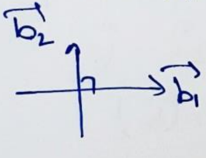
$\vec{b_2} = 3\hat{i} + 2\hat{j} + 4\hat{k}$

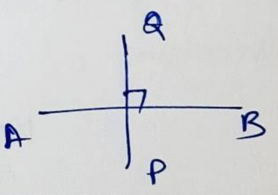
$\vec{b_1} \cdot \vec{b_2}$

$= (2\hat{i} + 5\hat{j} - 4\hat{k}) . (3\hat{i} + 2\hat{j} + 4\hat{k})$

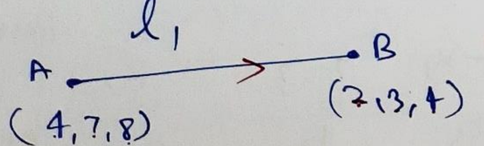
$= 6 + 10 - 16$

$= 16 - 16 = 0$

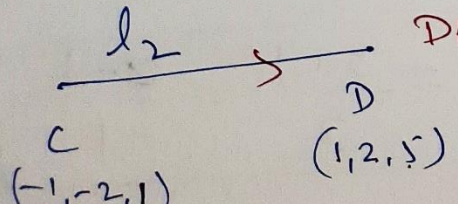




Q.3 Show that the line through the points (4,7,8), (2,3,4) is parallel to the line through the points (−1,−2,1), (1,2,5).



$DR_1 = -2, -4, -4$



$DR_2 = 2, 4, 4$

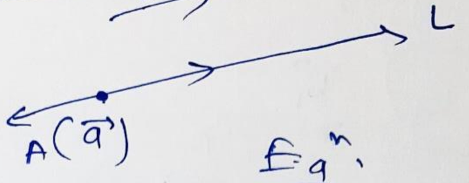
$\frac{-2}{2} = \frac{-4}{4} = \frac{-4}{4}$

$-1 = -1 = -1$

DRs proportional

∴ lines are ∥.

Q.4 Find the equation of the line which passes through the point (1,2,3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$ = $\vec{b}$



Point
A(1,2,3)

$A(\vec{a})$ $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Direction, $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

Eq^n.

$$\boxed{\underset{\text{Variable}}{\vec{r}} = \underset{\text{Point}}{\vec{a}} + \lambda \underset{\text{Direction}}{\vec{b}}}$$

$$\Rightarrow \boxed{\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})}$$

Q.5 Find the equation of the line in vector and in Cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of $\hat{i} + 2\hat{j} - \hat{k}$.

Point (2, −1, 4)

Direction ($\vec{b}$)

D.R. = 1, 2, −1

Eq^n. of line

Vector form,

$$\vec{r} = \underset{\text{Point}}{\vec{a}} + \lambda \underset{\text{Dir.}}{\vec{b}}$$

$$\boxed{\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})}$$

Cartesian form

Point

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Dir.

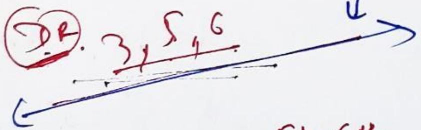
$$\boxed{\frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z - 4}{-1}}$$

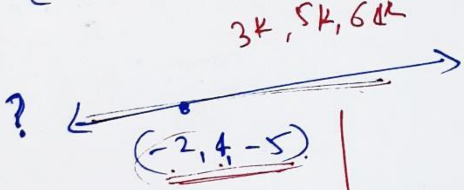
Q.6 Find the Cartesian eqⁿ. of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by

Point Dir.

$$\boxed{\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}}.$$

Dir.





$$\frac{x-\overset{-2}{x_1}}{a} = \frac{y-\overset{4}{y_1}}{b} = \frac{z-\overset{-5}{z_1}}{c}$$

Point

Dir.

$a \to 3$, $b \to 5$, $c \to 6$

$$\boxed{\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}}$$

Q.7 The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form

$$\boxed{\vec{r} = \vec{a} + \lambda \vec{b}}$$

Point Direction

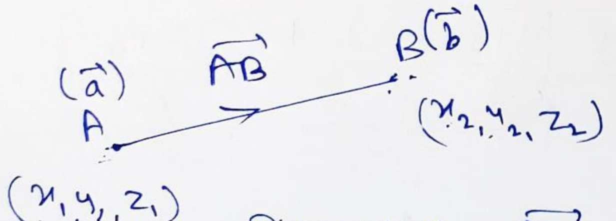
$(5, -4, 6)$ Point

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$$

Direction DR

$$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

Q.8 Find the Vector and the Cartesian equations of the lines that passes through the origin (0,0,0) and (5,−2,3).



1 Point, 1 Dir.

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\frac{x-x_1}{a}$

Direction $= \vec{AB}$

$= \vec{b} - \vec{a}$

Eqn. of line,

Vector form,

$$\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$$

Cartesian form,

$$\boxed{\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}}$$

ATQ A($\vec{a}$) (0,0,0)

B($\vec{b}$) (5,−2,3)

Vector Form

$$\boxed{\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})}$$

$\downarrow$

$$\boxed{\vec{r} = \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})}$$

Cartesian Form

$$\frac{x-0}{5-0} = \frac{y-0}{-2-0} = \frac{z-0}{3-0}$$

$$\Rightarrow \boxed{\frac{x}{5} = \frac{y}{-2} = \frac{z}{3}}$$

Q.9 Find the vector and the Cartesian equations of the line that passes through the points $(3,-2,-5)$, $(3,-2,6)$.

A $\qquad$ B.

$(x_1, y_1, z_1) \qquad (x_2, y_2, z_2)$

Cartesian form,

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \frac{x-3}{3-3} = \frac{y-(-2)}{-2+2} = \frac{z-(-5)}{6+5}$$

$$\Rightarrow \boxed{\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}}$$

& vector form

$$\vec{r} = \vec{a} + \lambda(\vec{b}-\vec{a})$$

Point

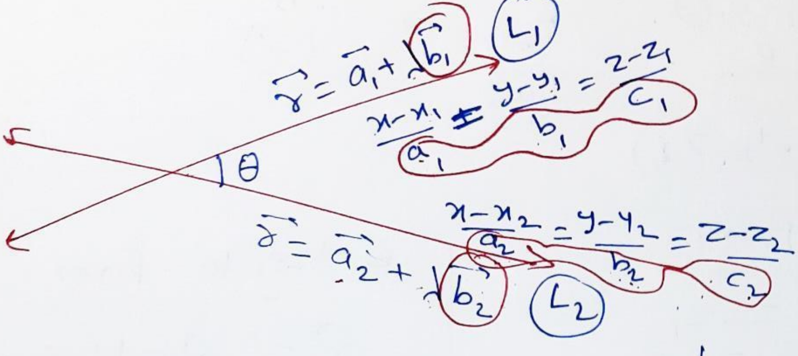
$$\Rightarrow \vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda((3-3)\hat{i} + (-2+2)\hat{j} + (6+5)\hat{k})$$

$$\rightarrow \vec{r} = (3\hat{i} - 2\hat{j} - 5\hat{k}) + \lambda(0\hat{i} + 0\hat{j} + 11\hat{k})$$

$$\boxed{\vec{r} = (3\hat{i} - 2\hat{j} - 5\hat{k}) + \lambda(11\hat{k})}$$

[redacted]	Class 12 maths	Three Dimensional Geometry

Exercise 11.2 (Angle & Distance b/w Two lines)



$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

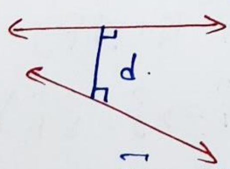
$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

$\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$

$$\cos\theta = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \right|$$

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2} \sqrt{a_2^2+b_2^2+c_2^2}} \right|$$

Skew lines

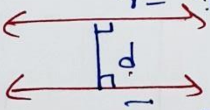


$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$

$\vec{r} = \vec{a}_2 + \mu \vec{b}_2$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

Parallel lines



$\vec{r} = \vec{a}_1 + \lambda \vec{b}$

$\vec{r} = \vec{a}_2 + \mu \vec{b}$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

Q.10 Find the $\overset{\theta}{\text{angle}}$ b/w the following pairs of lines:

(i) $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ $\quad \vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ $\quad \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|} = \frac{(3\hat{i}+2\hat{j}+6\hat{k}) \cdot (\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{9+4+36} \cdot \sqrt{1^2+4+4}}$$

$$\cos\theta = \frac{3+4+12}{7 \times 3} = \frac{19}{21} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{19}{21}\right)}$$

(ii) $\vec{r} = \underset{\times}{(3\hat{i} + \hat{j} - 2\hat{k})} + \lambda\underbrace{(\hat{i} - \hat{j} - 2\hat{k})}_{\vec{b_1}}$ | $\vec{r} = \underset{\times}{(2\hat{i} - \hat{j} - 56\hat{k})} + \mu\underbrace{(3\hat{i} - 5\hat{j} - 4\hat{k})}_{\vec{b_2}}$

$$\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|} = \frac{3+5+8}{\sqrt{1+1+4}\,\sqrt{9+25+16}}$$

$$\cos\theta = \frac{16}{\sqrt{6}\,\sqrt{50}} = \frac{16}{\sqrt{300}} = \frac{16}{\sqrt{100\times 3}} = \frac{\cancel{16}\,8}{\underset{5}{\cancel{10}}\sqrt{3}}$$

$$\cos\theta = \frac{8}{5\sqrt{3}} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)}$$

(Q.1) Find the angle between the following pair of lines :

Cartesian form

(i) $\frac{x-2}{\underset{a_1}{2}} = \frac{y-1}{\underset{b_1}{5}} = \frac{z+3}{\underset{c_1}{-3}}$; $\frac{x+2}{\underset{a_2}{-1}} = \frac{y-4}{\underset{b_2}{8}} = \frac{z-5}{\underset{c_2}{4}}$

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}\right| = \left|\frac{-2+40-12}{\sqrt{4+25+9}\,\sqrt{1+64+16}}\right|$$

$$\Rightarrow \cos\theta = \frac{26}{\sqrt{38}\,\sqrt{81}} = \frac{26}{9\sqrt{38}} \Rightarrow ~~\cos\theta =~~ \boxed{\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)}$$

(ii) $\frac{x}{\underset{a_1}{2}} = \frac{y}{\underset{b_1}{2}} = \frac{z}{\underset{c_1}{1}}$ & $\frac{x-5}{\underset{a_2}{4}} = \frac{y-2}{\underset{b_2}{1}} = \frac{z-3}{\underset{c_2}{8}}$

$$\cos\theta = \left|\frac{8+2+8}{\sqrt{4+4+1}\,\sqrt{16+1+64}}\right| = \frac{\overset{2}{\cancel{18}}}{3\times\underset{1}{\cancel{9}}} = \frac{2}{3}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{2}{3}\right)}$$

Q.12 Find the value of $\textcircled{P}$ so that the lines

$$\underbrace{\frac{1-x}{3} = \frac{7y-14}{2P} = \frac{z-3}{2}}\ \&\ \underbrace{\frac{7-7x}{3P} = \frac{y-5}{1} = \frac{6-z}{5}}$$ are at Right angle.

C.F.

Standard forms

$\theta = 90^\circ$

$\cos 90^\circ = 0$

$$\Rightarrow \frac{-(x-1)}{3} = \frac{7(y-2)}{2p} = \frac{z-3}{2}$$

$$\Rightarrow \boxed{\frac{x-1}{\underset{a_1}{-3}} = \frac{y-2}{\underset{b_1}{\left(\frac{2P}{7}\right)}} = \frac{z-3}{\underset{c_1}{2}}}$$

$$\Rightarrow \frac{-7(x-1)}{3P} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

$$\Rightarrow \boxed{\frac{x-1}{\underset{a_2}{\left(\frac{3P}{-7}\right)}} = \frac{y-5}{\underset{b_2}{1}} = \frac{z-6}{\underset{c_2}{-5}}}$$

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}} \right|$$

$\perp\ \theta = 90^\circ$

$\theta = 90^\circ$, $\cos\theta = \cos 90^\circ = \underline{0}$

$$\Rightarrow 0 = \left| \frac{(-3)\cdot\left(\frac{3P}{-7}\right) + \frac{2P}{7} + 2(-5)}{\cancel{\sqrt{}\ \sqrt{}}} \right|$$

$$\Rightarrow 0 = \frac{9P}{7} + \frac{2P}{7} - 10$$

$$\Rightarrow 10 = \frac{11P}{7}$$

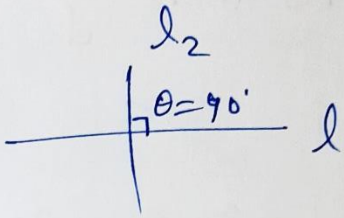
$$\Rightarrow \boxed{\frac{70}{11} = P}$$

Q.13 Show that the lines $\underbrace{\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}}_{a_1 \quad b_1 \quad c_1}$ & $\underbrace{\frac{x}{1} = \frac{y}{2} = \frac{z}{3}}_{a_2 \quad b_2 \quad c_2}$ are perpendicular to each other.

l_1 l_2

$$\cos\theta = \frac{\cancel{7} + (-\cancel{10}) + (\cancel{3})}{\sqrt{49+25+1}\cdot\sqrt{1+4+9}} = 0$$

$$\cos\theta = 0 \Rightarrow \boxed{\theta = 90^\circ}$$



Q.14 Find the shortest distance b/w the lines

$$\vec{r} = \underbrace{(\hat{i}+2\hat{j}+\hat{k})}_{\text{Point } \vec{a_1}} + \lambda\underbrace{(\hat{i}-\hat{j}+\hat{k})}_{\text{Direction } \vec{b_1}} \ \& \ \vec{r} = \underbrace{(2\hat{i}-\hat{j}-\hat{k})}_{\text{Point } (\vec{a_2})} + \mu\underbrace{(2\hat{i}+\hat{j}+2\hat{k})}_{\text{Direction } \vec{b_2}}$$

Skew lines,

$$d = \left|\frac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

$$d = \left|\frac{(\hat{i}-3\hat{j}-2\hat{k})\cdot(-3\hat{i}+3\hat{k})}{3\sqrt{2}}\right|$$

$$\Rightarrow d = \left|\frac{-3-6}{3\sqrt{2}}\right| = \frac{+\cancel{9}\,3}{\cancel{3}\sqrt{2}}$$

$$\Rightarrow \boxed{d = \frac{3}{\sqrt{2}}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow \boxed{d = \frac{3\sqrt{2}}{2}}$$

$$\vec{b_1}\times\vec{b_2} = (\hat{i}-\hat{j}+\hat{k}) \times (2\hat{i}+\hat{j}+2\hat{k})$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} \xrightarrow{} R_1$$

expand

$$= \hat{i}(-3) - \hat{j}(0) + \hat{k}(3)$$

$$= -3\hat{i} + 3\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{9+9}$$

$$= \sqrt{9\times 2}$$

$$= 3\sqrt{2}$$

$\boxed{Q.15}$ Find the shortest distance b/w the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \quad \& \quad \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Point $\vec{a_1} = -\hat{i}-\hat{j}-\hat{k}$ ✓

Direction $\vec{b_1} = 7\hat{i}-6\hat{j}+\hat{k}$ ✓

Point $\vec{a_2} = 3\hat{i}+5\hat{j}+7\hat{k}$ ✓

Direction $\vec{b_2} = \hat{i}-2\hat{j}+\hat{k}$ ✓

Skew lines,

$$d = \left| \frac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|} \right|$$

$$d = \left| \frac{(4\hat{i}+6\hat{j}+8\hat{k})\cdot(-4\hat{i}-6\hat{j}-8\hat{k})}{2\sqrt{29}} \right|$$

$$d = \left| \frac{-16-36-64}{2\sqrt{29}} \right|$$

$$d = \frac{+116}{2\sqrt{29}} = \frac{\overset{2}{\cancel{4}}\times\cancel{29}\sqrt{29}}{\cancel{2}\cancel{\sqrt{29}}}$$

$$\boxed{d = 2\sqrt{29}}$$

$$\vec{b_1}\times\vec{b_2} = (7\hat{i}-6\hat{j}+\hat{k}) \times (\hat{i}-2\hat{j}+\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}(-4) - \hat{j}(6) + \hat{k}(-8)$$

$$\vec{b_1}\times\vec{b_2} = -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{16+36+64}$$

$$= \sqrt{116}$$

$$= \sqrt{4\times 29}$$

$$= 2\sqrt{29}$$

$\boxed{\text{Q.16}}$ Find the shortest distance between the lines whose vector equatios are

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}), \quad \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

Point $(\vec{a_1})$ $\boxed{\text{Direction } (\vec{b_1})}$ Point $(\vec{a_2})$ $\boxed{\text{Direction } (\vec{b_2})}$

Skew lines

$$d = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$d = \left| \frac{(3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (-9\hat{i} + 3\hat{j} + 9\hat{k})}{3\sqrt{19}} \right|$$

$$d = \left| \frac{-\cancel{27} + 9 + \cancel{27}}{3\sqrt{19}} \right|$$

$$d = \frac{3}{\sqrt{19}}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} \rightarrow$$

$$= \hat{i}(-9) - \hat{j}(-3) + \hat{k}(9)$$

$$= -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{81 + 9 + 81}$$

$$= \sqrt{171}$$

$$= \sqrt{9 \times 19}$$

$$= 3\sqrt{19}$$

Q.17 Find the shortest distance b/w the lines whose vector equations are $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ -

& $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

$L_1: \vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$

$$(L_1)\ \boxed{\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})}$$

Point($\vec{a_1}$) Direction($\vec{b_1}$)

Similarly

$$(L_2): \boxed{\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})}$$

Point($\vec{a_2}$) Direction($\vec{b_2}$)

Skew lines,

$$d = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$d = \left| \frac{(\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})}{\sqrt{29}} \right|$$

$$d = \left| \frac{-4 + 12}{\sqrt{29}} \right|$$

$$\boxed{d = \frac{8}{\sqrt{29}}}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}(2) - \hat{j}(4) + \hat{k}(-3)$$

$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

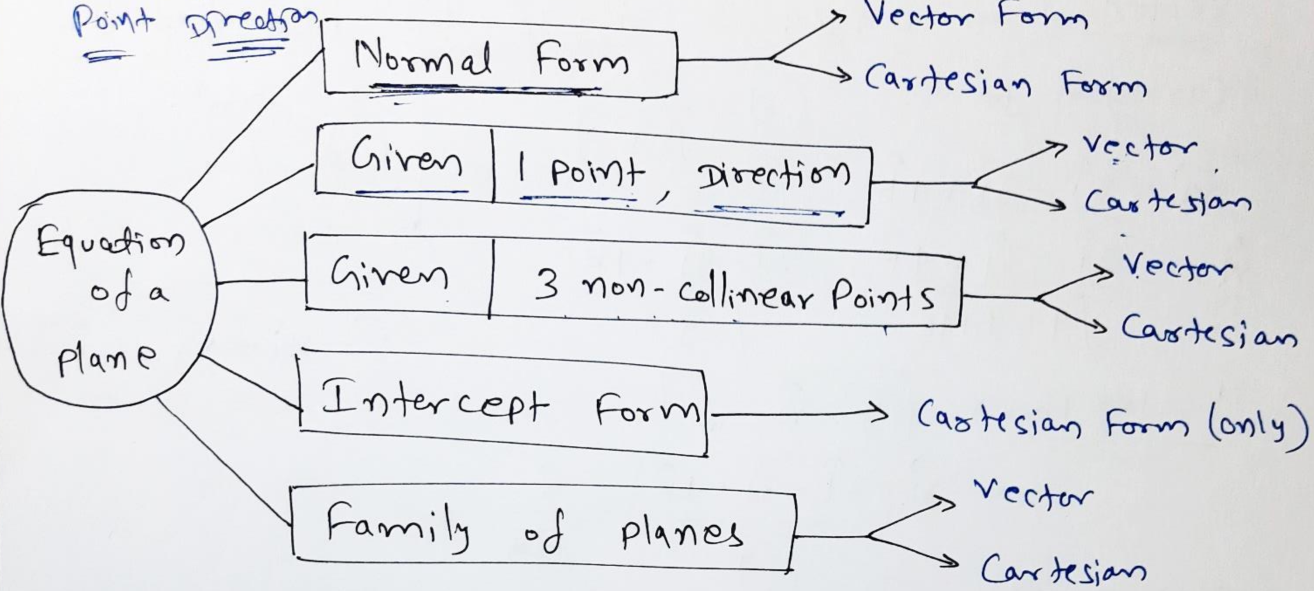
$$|\vec{b_1} \times \vec{b_2}| = \sqrt{4 + 16 + 9}$$

$$= \sqrt{29}$$

[redacted] | Class 12 Maths | Three Dimensional Geometry

Plane (in 3D) [समतल]

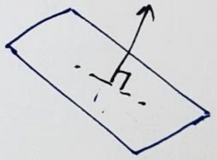
Point Direction



Note: Line मे Direction का मतलब

(Line के along या parallel vector की Direction)

Plane मे Direction का मतलब

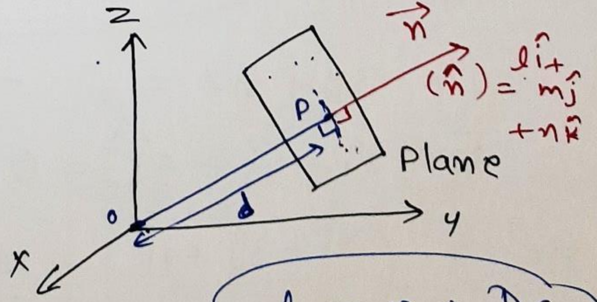


(Plane के perpendicular vector की Direction)

Normal Form of a Plane

Plane के perpendicular unit vector = $\hat{n}$ (given)

Plane की Origin से Distance = d (given)



$l, m, n \rightarrow$ D.C.

Vector Form $\boxed{\vec{r}\cdot\hat{n}=d}$ ★

variable = $x\hat{i}+y\hat{j}+z\hat{k}$

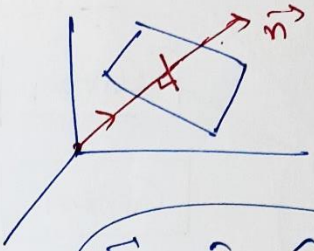
Cartesian Form $\boxed{lx+my+nz=d}$

$(l^2+m^2+n^2=1)$

$\boxed{\vec{r}\cdot\hat{n} = d}$

e.g. Find the vector equation of a plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also find its Cartesian form.

$\boxed{d = \frac{6}{\sqrt{29}}}$



$\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\hat{n} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4+9+16}} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$

$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$

Vector form, $\boxed{\vec{r}\cdot\hat{n} = d}$

$\Rightarrow \vec{r}\cdot\left(\frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$ ✓

Cartesian form:,

$$\boxed{\frac{2x}{\sqrt{29}} - \frac{3y}{\sqrt{29}} + \frac{4z}{\sqrt{29}} = \frac{6}{\sqrt{29}}}$$ ✓

e.g. Find the distance of the plane $\boxed{2x - 3y + 4z - 6 = 0}$ from origin. Also find D.R. & D.C. of vector perpendi-cular to it.

$\boxed{\text{D.R.} = 2, -3, 4}$

$2x - 3y + 4z = 6 \longrightarrow \frac{2x - 3y + 4z}{\sqrt{29}} = \frac{6}{\sqrt{29}}$

$\vec{r}\cdot(2\hat{i} - 3\hat{j} + 4\hat{k}) = 6$

$\Rightarrow \vec{r}\cdot\left(\frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$

$\frac{2x}{\sqrt{29}} - \frac{3y}{\sqrt{29}} + \frac{4z}{\sqrt{29}} = \boxed{\frac{6}{\sqrt{29}}}$

$\boxed{\vec{r}\cdot\hat{n} = d}$

$\boxed{\text{D.C. } \frac{2}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{4}{\sqrt{29}}}$

Distance from origin $= \frac{6}{\sqrt{29}}$

Ⓟ Point

e.g. Find the coordinate of the foot of the perpendicular drawn ~~fo~~ from the origin to the plane $2x-3y+4z-6=0$

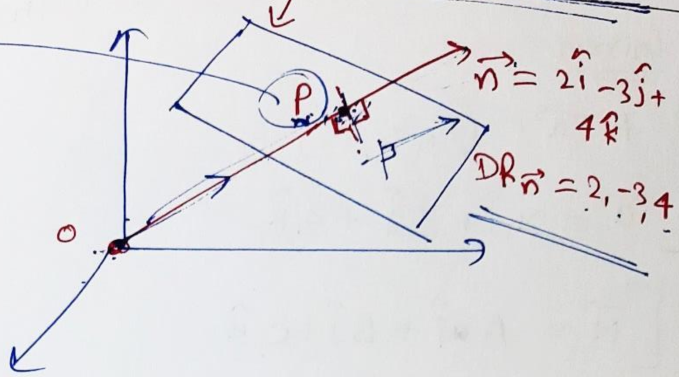
Ans: (let) $P(x_1, y_1, z_1) = ?$

$O(0,0,0)$

$\vec{OP} \rightarrow$ Direction Ratios

$\Downarrow$

$DR_{op} = (x_1, y_1, z_1)$



Plane $(2x-3y+4z-6 = 0)$ का perpendicular vector $(\vec{n})$

का DR. $= (2, -3, 4)$

$\vec{OP} \parallel \vec{n} \longrightarrow \boxed{DR_{\vec{OP}} \propto DR_{\vec{n}}}$

$\quad x_1, y_1, z_1 \qquad 2, -3, 4$

$$\Rightarrow \frac{x_1}{2} = \frac{y_1}{-3} = \frac{z_1}{4} = \lambda$$

$\boxed{x_1 = 2\lambda}$ $\boxed{y_1 = -3\lambda}$ $\boxed{z_1 = 4\lambda}$

∵ Point 'P' (x_1, y_1, z_1) lies on the plane also. (Satisfy.)

$2x - 3y + 4z - 6 = 0$

$\Rightarrow 2x_1 - 3y_1 + 4z_1 - 6 = 0$

$\Rightarrow 2(2\lambda) - 3(-3\lambda) + 4(4\lambda) - 6 = 0$

$\Rightarrow 4\lambda + 9\lambda + 16\lambda = 6$

$\Rightarrow 29\lambda = 6$

$\lambda = 6/29$

$P(x_1, y_1, z_1)$

$P(2\lambda, -3\lambda, 4\lambda)$

$P\left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$

Equation of a plane (Given – 1 Point, Direction)

(Plane passes through this)

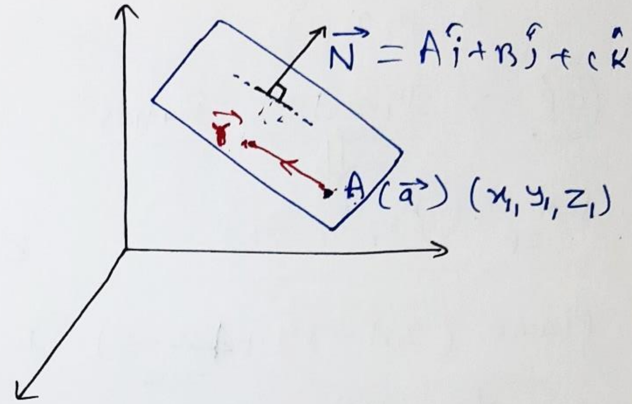
(Perpendicular to plane)

Given

A($\vec{a}$) (x_1, y_1, z_1)

$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$



Plane:

Vector Form

$$\boxed{(\vec{r}-\vec{a}).\vec{N}=0}$$

Point

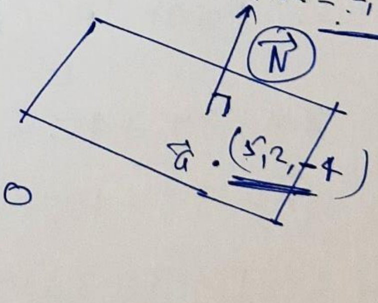
Direction (Perpendicular)

DR

Cartesian Form

$$\boxed{A(x-x_1) + B(y-y_1) + C(z-z_1) = 0}$$

e.g. Find the vector and Cartesian eqn. of the plane which passes through the point (5, 2, −4) and perpendicular to the line with direction ratios 2, 3, −1.



Cartesian Form.

$$2(x-5) + 3(y-2) - 1(z+4) = 0$$

$$\Rightarrow \boxed{2x\ 3y - z = 20}$$

vector Form: $(\vec{r}-\vec{a}).\vec{N}=0$

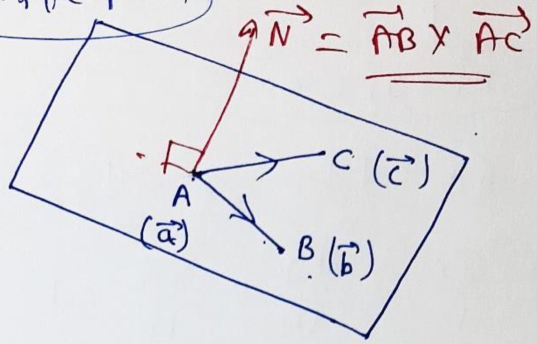
$$\Rightarrow \boxed{(\vec{r} - 5\hat{i} - 2\hat{j} + 4\hat{k}).(2\hat{i} + 3\hat{j} - \hat{k}) = 0}$$

Equation of a plane

Given	3 non-collinear ~~plane~~ Points

Unique plane

Points	Position Vector	Coordinates
A	$\vec{a}$	(x_1, y_1, z_1)
B	$\vec{b}$	(x_2, y_2, z_2)
C	$\vec{c}$	(x_3, y_3, z_3)



Plane

Vector Form

$$(\vec{r}-\vec{a}) \cdot \left[(\vec{b}-\vec{a}) \times (\vec{c}-\vec{a})\right] = 0$$

Cartesian form

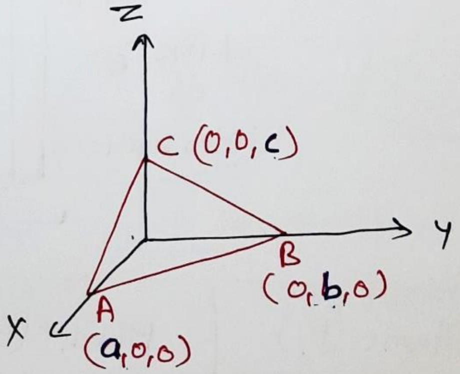
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Intercept Form

x-intercept $= a$

y-intercept $= b$

z-intercept $= c$



Equation of plane in Intercept form

$$\boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$$

e.g. Find x, y and z- intercepts of the plane

$$\boxed{\vec{r}\cdot(6\hat{i}+4\hat{j}+3\hat{k})=12.}$$

$$x\hat{i}+y\hat{j}+z\hat{k}$$

$\Rightarrow 6x+4y+3z=\textcircled{12}$

$\Rightarrow \frac{6x}{12}+\frac{4y}{12}+\frac{3z}{12}=1$

$\Rightarrow \boxed{\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1}$

x - intercept = 2

y ——— = 3

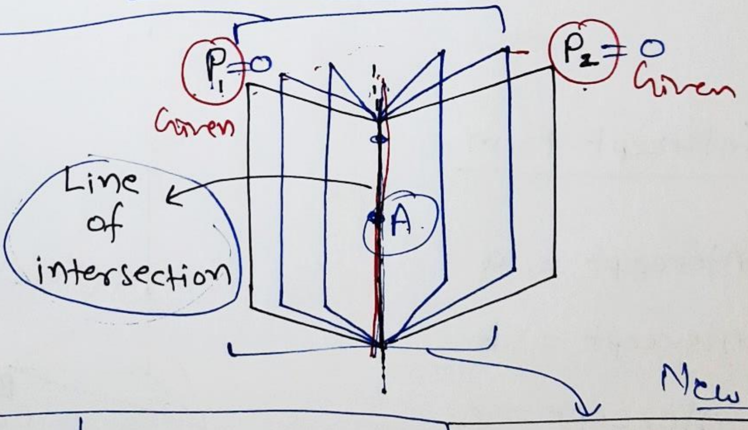
z ——— = 4

Family of Planes

New (plane passing through the intersection of two given planes)

New Plane

$$\boxed{\underbrace{P_1+\lambda P_2}_{P_3}=0}$$



New Plam

	Plane (P_1)	Plane (P_2)	Family of planes ($P_1+\lambda P_2=0$)
vector Form	$\vec{r}\cdot\vec{n}_1=d_1$	$\vec{r}\cdot\vec{n}_2=d_2$	$(\vec{r}\cdot\vec{n}_1-d_1)+\lambda(\vec{r}\cdot\vec{n}_2-d_2)=0$
Cartesian Form	$A_1x+B_1y+C_1z=d_1$	$A_2x+B_2y+C_2z=d_2$	$(A_1x+B_1y+C_1z-d_1)+\lambda(A_2x+B_2y+C_2z-d_2)=0$

e.g. Find the cartesion equation of the plane (New) passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=6$ (P_1) and $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k})=-5$ (P_2), and also passes through origin. $(0,0,0)$

$$\left(\begin{aligned} P_1 &\equiv x+y+z-6=0 \\ P_2 &\equiv 2x+3y+4z+5=0 \end{aligned} \right)$$

New plane $(P_3)=0$

$\Rightarrow P_1+\lambda(P_2)=0$

$\Rightarrow \boxed{(x+y+z-6)+\lambda(2x+3y+4z+5)=0}$

(0,0,0) से satisfy —

$\Rightarrow (0-6)+\lambda(0+5)=0$

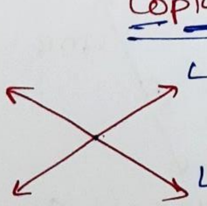
$\Rightarrow \boxed{\lambda=\frac{6}{5}}$

New Plane:

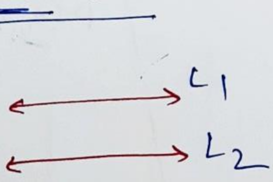
$$(x+y+z-6)+\frac{6}{5}(2x+3y+4z+5)=0$$

Coplanarity of Two lines

Coplanar Lines



Yes, Point of intersection



Parallel

Vector Form

$L_1: \vec{r}=\vec{a}_1+\lambda\vec{b}_1$

$L_2: \vec{r}=\vec{a}_2+\lambda\vec{b}_2$

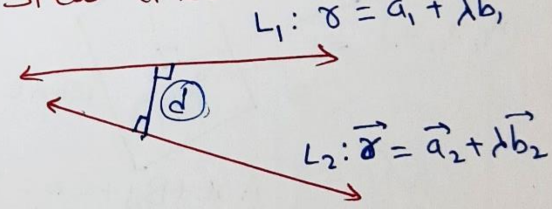
(d=0) (Point) (Direction)

Condition of coplanarity of two lines

$$\boxed{(a_2-a_1)\cdot(b_1\times b_2)=0}$$

(Non-Coplanar)

Skew lines



- No point of intersection
- not parallel

$$d=\left|\frac{(a_2-a_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right| \neq 0$$

$d\neq 0.$

e.g. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \to b_1$ and

$\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5} \to b_2$ are coplanar. $(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})=0$

$\vec{b_1} = -3\hat{i}+\hat{j}+5\hat{k}$

$\vec{b_2} = -\hat{i}+2\hat{j}+5\hat{k}$

$\vec{a_1} = -3\hat{i}+\hat{j}+5\hat{k}$

$\vec{a_2} = -\hat{i}+2\hat{j}+5\hat{k}$

$\boxed{\vec{a_2}-\vec{a_1} = 2\hat{i}+\hat{j}+0.\hat{k}}$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$$

$\boxed{\vec{b_1}\times\vec{b_2} = -5\hat{i}+10\hat{j}-5\hat{k}}$

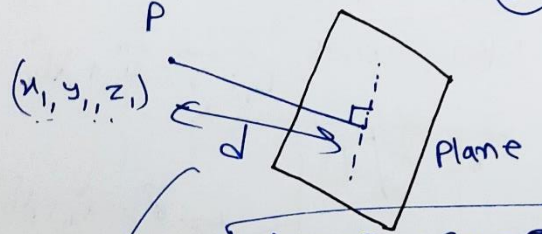
Cond^n. of Coplanarity.

$(2\hat{i}+\hat{j})\cdot(-5\hat{j}+10\hat{j}-5\hat{k})$

$= -10+10 = 0$

Distance of a Point from a Plane.

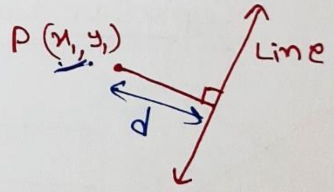
3D



$\boxed{Ax+By+Cz=D}$

✯ $$d = \left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

2D



$ax+by+c=0$

$$\boxed{d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|}$$

e.g. Find the distance b/w Point $(2,5,-3)$ and plane $\vec{r}\cdot(6\hat{i}-3\hat{j}+2\hat{k})=4$

$6x-3y+2z-4=0$

$$d = \left|\frac{12-15-6-4}{\sqrt{6^2+(-3)^2+2^2}}\right|$$

$$= \left|\frac{-13}{7}\right| = \frac{13}{7}$$

Angle between ☐ & ☐

Dot Product

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

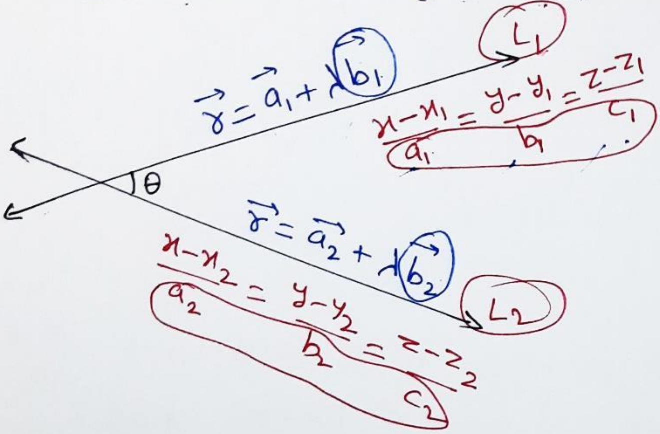
Line — Line → $\cos\theta$

Plane — Plane → $\cos\theta$

Plane — line → $\sin\theta$?

Already Covered

I Angle between Two lines



$\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

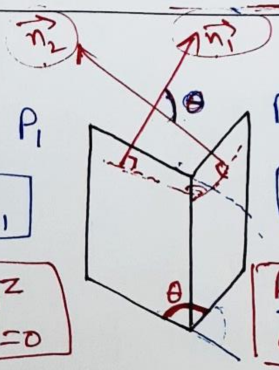
$\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

$$\cos\theta = \left|\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

II Angle between Two Planes



$\vec{r}\cdot\vec{n}_1 = d_1$

$A_1x+B_1y+C_1z + D_1 = 0$

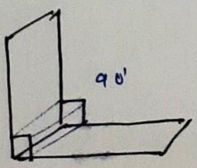
$\vec{r}\cdot\vec{n}_2 = d_2$

$A_2x+B_2y+C_2z+D_2 = 0$

$$\cos\theta = \left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$$

$$\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$$

Note: ~~Rep~~ Perpendicular Planes

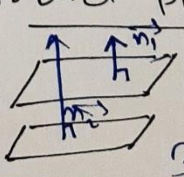


$\theta = 90°$

$\cos\theta = 0$

$\boxed{A_1A_2+B_1B_2+C_1C_2=0}$

Parallel planes

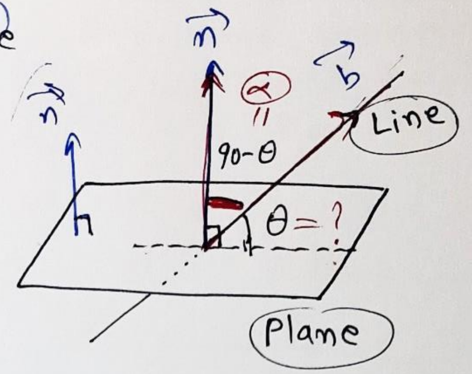


$\vec{n}_1 \parallel \vec{n}_2$

DR → Propor.

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

(III) Angle between a Line & a Plane



Line $\vec{r} = \vec{a} + \lambda \vec{b}$

Plane. $\vec{r} \cdot \vec{n} = d$

$$\cos(\alpha) = \cos(90-\theta) = \left| \frac{\vec{b} \cdot \vec{n}}{|\vec{b}||\vec{n}|} \right|$$

$$\Rightarrow \boxed{\sin\theta = \left| \frac{\vec{b} \cdot \vec{n}}{|\vec{b}||\vec{n}|} \right|}$$

e.g. Find the angle between the planes $2x+y-2z=5$ and $3x-6y-2z=7$. (P_1) (P_2)

$$\cos\theta = \left| \frac{(2\times3)+1\times(-6)+(-2)(-2)}{\sqrt{2^2+1^2+(-2)^2}\sqrt{3^2+(-6)^2+(-2)^2}} \right| = \left| \frac{\cancel{6}-\cancel{6}+4}{3\times7} \right|$$

$$\cos\theta = \frac{4}{21} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{4}{21}\right)}$$

e.g. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x+2y-11z=3$.

Line की Direction, → D.R. 2, 3, 6 $(\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k})$

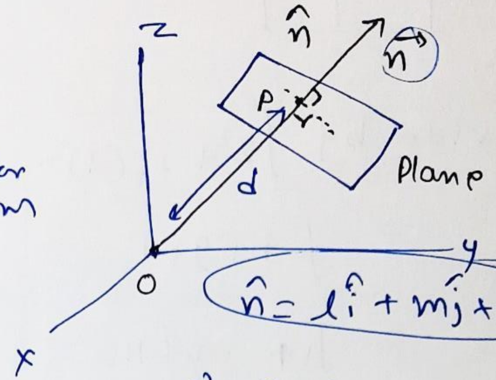
Plane की Direction, (D.R.) 10, 2, −11 $\vec{n} = 10\hat{i}+2\hat{j}-11\hat{k}$

$$\sin\theta = \left| \frac{\vec{b} \cdot \vec{n}}{|\vec{b}||\vec{n}|} \right| = \frac{(2\hat{i}+3\hat{j}+6\hat{k}) \cdot (10\hat{i}+2\hat{j}-11\hat{k})}{\sqrt{2^2+3^2+6^2}\sqrt{10^2+2^2+(-11)^2}}$$

$$\sin\theta = \left| \frac{20+6-66}{7\times15} \right| \Rightarrow \sin\theta = \frac{40}{7\times15} \Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{8}{21}\right)}$$

Exercise 11.3

Plane (3D)



Eqⁿ. of plane, $\boxed{\vec{r}\cdot\hat{n}=d}$ Vector Form

Direction ✓ (Unit vector Perpendicular to the plane)

Distance from origin

$\hat{n}=l\hat{i}+m\hat{j}+n\hat{k}$

$l^2+m^2+n^2=1$

$\boxed{lx+my+nz=d}$ Cartesian Form.

D.C.

Distance from origin

e.g. $Ax+By+Cz=d$

DR. → A, B, C

D.C.

$$\left(\frac{A}{\sqrt{A^2+B^2+C^2}}, \frac{B}{\sqrt{A^2+B^2+C^2}}, \frac{C}{\sqrt{A^2+B^2+C^2}}\right)$$

Exercise 11.3

Q.1 In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

(a) $z=2$

$\Rightarrow 0.x+0y+1.z=2$

D.C. $\boxed{l=0,\ m=0,\ n=1}$

$l^2+m^2+n^2=1$

Distance from origin $=2$

(b) $x+y+z=1$ D.R. 1, 1, 1

Divide by $\sqrt{1^2+1^2+1^2}=\sqrt{3}$ on both sides.

$$\Rightarrow \frac{x}{\sqrt{3}}+\frac{y}{\sqrt{3}}+\frac{z}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$lx+my+nz=d$

DC. $=\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Distance from (0,0,0) $=\frac{1}{\sqrt{3}}$

ⓒ $2x+3y-z=5$

$\uparrow \quad \uparrow \quad \uparrow$

DR.

Divide by $\sqrt{2^2+3^2+(-1)^2}$

$=\sqrt{4+9+1}$

$=\sqrt{14}$ in LHS & RHS

Eqn. $\boxed{\frac{2x}{\sqrt{14}}+\frac{3y}{\sqrt{14}}-\frac{z}{\sqrt{14}}=\frac{5}{\sqrt{14}}}$

D.C. $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}$

$d=\frac{5}{\sqrt{14}}$

ⓓ $5y+8=0$

$\Rightarrow 5y=-8$

$\Rightarrow -5y=8$

$\Rightarrow 0.x-5.y+0.z=8$

DR.

Divide by $\sqrt{0^2+(-5)^2+0^2}$

$=\sqrt{25}=5$

$\Rightarrow \frac{0.x}{5}-\frac{5y}{5}+\frac{0.z}{5}=\frac{8}{5}$

D.C. 0, -1, 0

$d=\frac{8}{5}$

Q.2 Find the vector equation of a plane which is at a distance 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

$(d=7)$

normal vector $(\vec{n})$

$\vec{r}\cdot\hat{n}=d$

$\uparrow$ unit vector

$$\hat{n}=\frac{\vec{n}}{|\vec{n}|}=\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{9+25+36}}=\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}$$

$$\Rightarrow \boxed{\vec{r}\cdot\left(\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}\right)=7}$$

$\boxed{\text{Q.3}}$ Find the Cartesian ~~product~~ equation of the following planes:

ⓐ $\boxed{\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k})=2}$ Vector Form.

$(\vec{r}=x\hat{i}+y\hat{j}+z\hat{k})$ put.

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+\hat{j}-\hat{k})=2$

$\Rightarrow \boxed{x+y-z=2}$

Cartesian Form.

ⓑ $\vec{r}\cdot(2\hat{i}+3\hat{j}-4\hat{k})=1$

Cartesian Form,

$\boxed{2x+3y-4z=1}$

ⓒ $\vec{r}\cdot\left[(s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}\right]=5$ Vector Form

Cartesian form, $\boxed{(s-2t)x+(3-t)y+(2s+t)z=5}$

~~Q.4 In the following cases, find the coordinates~~

[redacted] | Class 12 Maths | Three Dimensional Geometry

Exercise 11.3

Plane (3D)

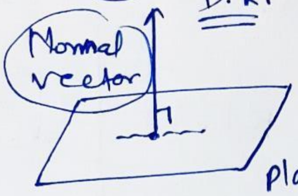
$A\hat{i}+B\hat{j}+C\hat{k} = \boxed{\vec{n}}$

D.R. A, B, C

Plane

$\boxed{\vec{r}\cdot\vec{n}=d}$

Vector form



$\boxed{Ax+By+Cz=d}$

Cartesian Form

Q.4 In the following cases, find the coordinates of the foot of perpendicular drawn from the origin.

$A(x_1,y_1,z_1) \rightarrow B(x_2,y_2,z_2)$

$DR_{AB} = x_2-x_1,\ y_2-y_1,\ z_2-z_1$

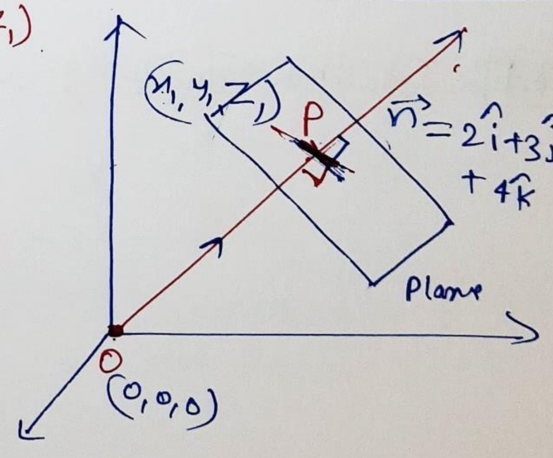
(i) $2x+3y+4z-12=0$

Foot of Perpendicular drawn from Origin ? $= P(x_1,y_1,z_1)$ (Let)

$\vec{n} = 2\hat{i}+3\hat{j}+4\hat{k}$

$\boxed{DR_{\vec{n}} = 2,3,4}$

$\boxed{DR_{\vec{OP}} = x_1,\ y_1,\ z_1}$



$\vec{n} \parallel \vec{OP}$ → DR → (Proportional)

$\frac{x_1}{2} = \frac{y_1}{3} = \frac{z_1}{4} = \lambda$ (Let)

$x_1=2\lambda,\ y_1=3\lambda,\ z_1=4\lambda$

$P(x_1,y_1,z_1)$?

$P(2\lambda,\ 3\lambda,\ 4\lambda)$

↑ ↑ ↑

Plane $2x+3y+4z-12=0$

Foot of Perpendicular

$P(x_1, y_1, z_1)$

or

$P(2\lambda, 3\lambda, 4\lambda)$

Point 'P' lies on plane also.

$\Rightarrow$ Point 'P' $(2\lambda, 3\lambda, 4\lambda)$ will satisfy the eqⁿ. of plane $\boxed{2x+3y+4z-12=0}$

$$\Rightarrow 2(2\lambda)+3(3\lambda)+4(4\lambda)-12=0$$

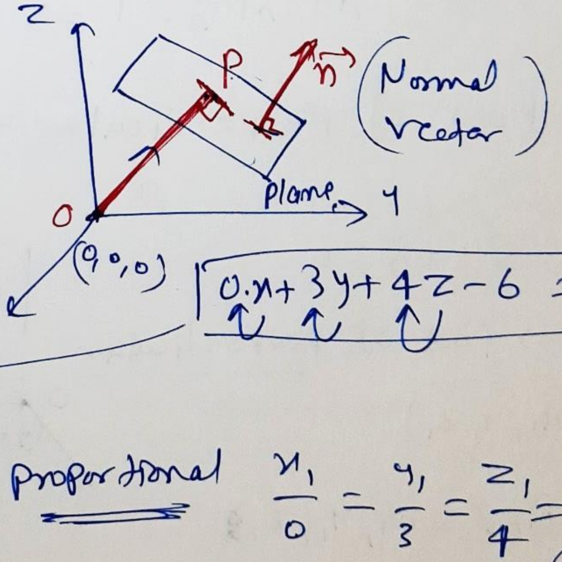
$$\Rightarrow 4\lambda+9\lambda+16\lambda=12 \Rightarrow 29\lambda=12 \Rightarrow \boxed{\lambda=\frac{12}{29}}$$

Foot of perpendicular drawn from origin

$$P(x_1, y_1, z_1) \equiv P(2\lambda, 3\lambda, 4\lambda)$$

$$= P\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$$

(b) $\boxed{3y+4z-6=0}$



Foot of Perpendicular

$P(x_1, y_1, z_1)$

$DR_{\vec{OP}} = x_1, y_1, z_1$

$\boxed{0.x+3y+4z-6=0}$

$DR_{\vec{n}} = 0, 3, 4$

$\because \vec{OP} \parallel \vec{n} \longrightarrow DR \rightarrow$ proportional $\frac{x_1}{0}=\frac{y_1}{3}=\frac{z_1}{4}=\lambda$ (let)

$x_1=0, y_1=3\lambda, z_1=4\lambda$

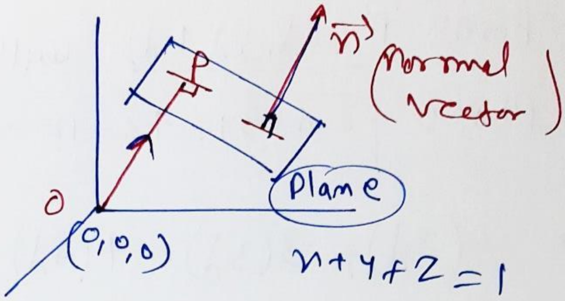
$P(0, 3\lambda, 4\lambda)$ satisfy the eqⁿ. of plane. $3y+4z-6=0$

$$\Rightarrow 3(3\lambda)+4(4\lambda)-6=0 \Rightarrow 25\lambda=6 \Rightarrow \boxed{\lambda=\frac{6}{25}}$$

Foot of Perpendicular $P(0, 3\lambda, 4\lambda)$

$\lambda = \frac{6}{25}$ $\quad \therefore P\left(0, \frac{18}{25}, \frac{24}{25}\right)$

Ⓒ $x+y+z=1$



Foot of Perpendicular
$P(x_1, y_1, z_1)$

$x+y+z=1$

$DR_{\vec{OP}} = x_1, y_1, z_1$

$DR_{\vec{n}} = 1, 1, 1$

$\vec{OP} \parallel \vec{n} \rightarrow$ DR = Proportional $\rightarrow \frac{x_1}{1} = \frac{y_1}{1} = \frac{z_1}{1} = \lambda$ (let)

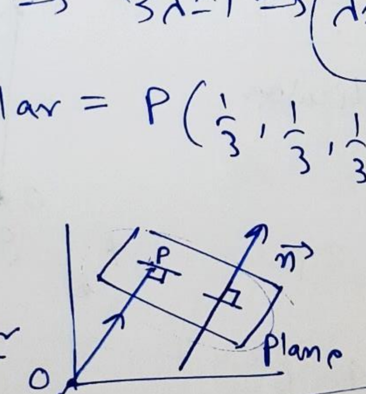
$P(x_1, y_1, z_1) \equiv P(\lambda, \lambda, \lambda)$

Point $P(\lambda, \lambda, \lambda)$ lies on $(x+y+z=1)$ plane.

$\Rightarrow \lambda + \lambda + \lambda = 1 \Rightarrow 3\lambda = 1 \Rightarrow \lambda = \frac{1}{3}$

Foot of Perpendicular = $P\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

ⓓ $5y+8=0$



P → Foot of Perpendicular
(x_1, y_1, z_1)

$0x + 5y + 0.z + 8 = 0$

$DR_{\vec{OP}} = x_1, y_1, z_1$

$DR_{\vec{n}} = 0, 5, 0$

$\vec{OP} \parallel \vec{n} \rightarrow$ DR → Proportional $\rightarrow \frac{x_1}{0} = \frac{y_1}{5} = \frac{z_1}{0} = \lambda$

$x_1 = 0, \; y_1 = 5\lambda, \; z_1 = 0$

Foot of Perpendicular $P(x_1, y_1, z_1)$ $5\lambda = \cancel{5} \times \frac{-8}{\cancel{25}\,5}$

or $P(0, 5\lambda, 0)$

Satisfy,

Plane $\boxed{5y+8=0}$

$\Rightarrow 5(5\lambda)+8=0$

$\Rightarrow 25\lambda = -8$

$\Rightarrow \boxed{\lambda = \frac{-8}{25}}$

$\boxed{P\left(0, -\frac{8}{5}, 0\right)}$

Q.5 Find the Vector & Cartesian eqⁿ. of the planes

ⓐ that passes through the point $(1, 0, -2)$ and the normal to the plane is $\hat{i}+\hat{j}-\hat{k}$.

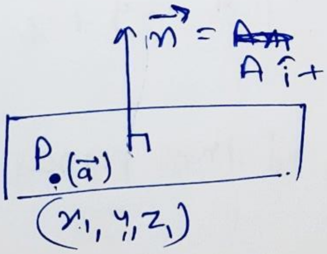
normal vector $(\vec{n})$

Standard form,

$\vec{n} = \cancel{AAA}\ A\hat{i}+B\hat{j}+C\hat{k}$ DR(A, B, C)

Vector form

$(\bar{r}-\bar{a}).\vec{n}=0$



Cartesian form,

$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$

Point $(1, 0, -2)$

$\vec{a} = \hat{i}-2\hat{k}$

$\vec{n} = \hat{i}+\hat{j}-\hat{k}$ DR → 1, 1, −1

Vector form,

$(\bar{r}-\bar{a}).\vec{n}=0$

$[\bar{r}-(\hat{i}-2\hat{k})].(\hat{i}+\hat{j}-\hat{k})=0$

Cartesian form,

$1(x-1)+1(y-0)-1(z+2)=0$

$\Rightarrow x-1+y-z-2=0$

$\Rightarrow \boxed{x+y-z=3}$

ⓑ that passes through the point (1, 4, 6) and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$.

Point (1, 4, 6)

$\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

Normal vector

$\vec{n} = \hat{i} - 2\hat{j} + \hat{k}$

$(D.R. = \underset{A}{①}, \underset{B}{\bigcirc\!\!\!\!-2}, \underset{C}{①})$

Vector form:

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$\Rightarrow \boxed{[\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0}$

Cartesian Form

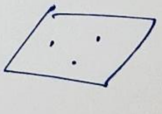
$Ⓐ(x - x_1) + Ⓑ(y - y_1) + Ⓒ(z - z_1) = 0$

$\Rightarrow 1(x-1) + (-2)(y-4) + 1(z-6) = 0$

$\Rightarrow (x-1) - 2y + 8 + z - 6 = 0$

$\Rightarrow \boxed{x - 2y + z + 1 = 0}$

Q.6 Find the equations of the planes that passes through three points.



ⓐ $\underset{A}{(1, 1, -1)}, \underset{B}{(6, 4, -5)}, \underset{C}{(-4, -2, 3)} \longrightarrow$

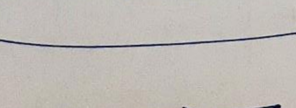
$DR_{AB} = 5, 3, -4$

$DR_{BC} = -10, -6, 8$

$\frac{5}{-10} = \frac{3}{-6} = \frac{-4}{8}$

DR → Proportional

$-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$



C → Collinear → ∞ no. of planes are possible

ⓑ $\underset{A}{(1,1,0)}, \underset{B}{(1,2,1)}, \underset{C}{(-2,2,-1)}$

$$\left.\begin{array}{l} DR_{AB} = 0, 1, 1 \\ DR_{BC} = -3, 0, -2 \end{array}\right\} \frac{0}{-3} \neq \frac{1}{0} \neq \frac{1}{-2}$$

A, B, C → non-collinear Points.

↓

Unique Plane ✓ → A (1,1,0)
B (1,2,1)
C (-2,2,-1)

Eqn. of plane

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z-0 \\ 1-1 & 2-1 & 1-0 \\ -2-1 & 2-1 & -1-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)[-1-1] - (y-1)(3) + z(3) = 0$$

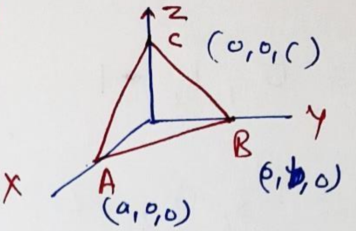
$$\Rightarrow (x-1)(-2) - 3y + 3 + 3z = 0$$

$$\Rightarrow -2x + 2 - 3y + 3 + 3z = 0$$

$$\Rightarrow \boxed{5 = 2x + 3y - 3z}$$

Exercise 11.3

Plane (3D)



Intercept Form.

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

Q.7 Find the intercepts cut off by the plane $2x+y-z=5$.

$$\Rightarrow \frac{2x}{5}+\frac{y}{5}-\frac{z}{5}=1$$

$$\Rightarrow \boxed{\frac{x}{(5/2)}+\frac{y}{(5)}+\frac{z}{(-5)}=1}$$

x-intercept $= \frac{5}{2}$ ✓

y- —— $= 5$ ✓

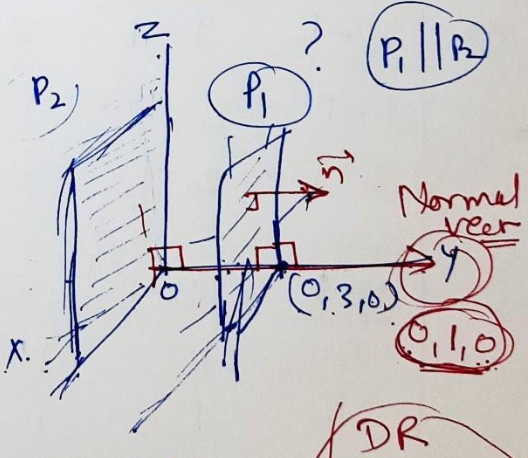
z- —— $= -5$ ✓

Q.8 Find the equation of the plane with intercept 3 on y-axis and parallel to ZOX plane.

y-intercept = 3 = b

Intercept Form.

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$



$$\Rightarrow \boxed{\frac{x}{a}+\frac{y}{3}+\frac{z}{c}=1}$$

x-intercept ↓ ∞

z-intercept ↓ ∞

$$\Rightarrow \frac{x}{\infty}+\frac{y}{3}+\frac{z}{\infty}=1$$

$$\Rightarrow 0+\frac{y}{3}+0=1$$

$$\Rightarrow \boxed{y=3}$$

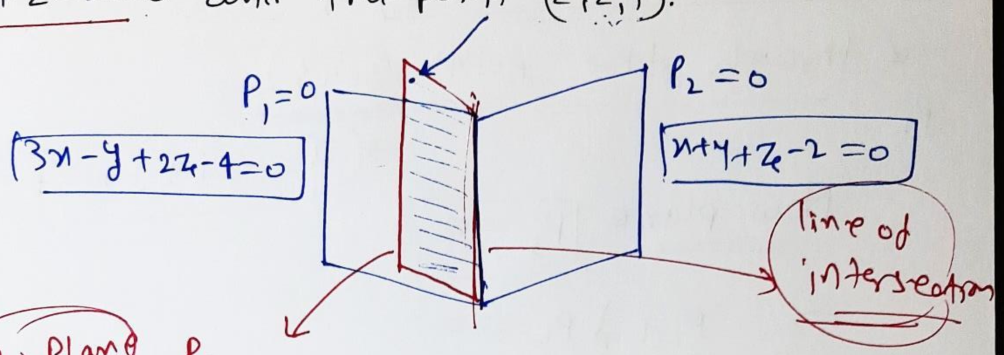
DR

$\frac{1}{a}, \frac{1}{3}, \frac{1}{c}$

$0, 1, 0$

$$\frac{\frac{1}{a}}{0}=\frac{\frac{1}{3}}{1}=\frac{\frac{1}{c}}{0}$$

Q.9 Find the equation of the plane passing through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2,2,1)$.



New Plane $P_3 = 0$

$$\Rightarrow \boxed{P_1 + \lambda P_2 = 0}$$

$$\Rightarrow \boxed{(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0}$$

Also this plane passes through $(2,2,1)$

$$\Rightarrow (6 - \cancel{2} + \cancel{2} - 4) + \lambda(2 + \cancel{2} + 1 - \cancel{2}) = 0$$

$$\Rightarrow 2 + \lambda(3) = 0$$

$$\Rightarrow \lambda = -2/3$$

Required plane, $(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$

$$\Rightarrow \frac{9x - 3y + 6z - 12 - 2x - 2y - 2z + 4}{3} = 0$$

$$\Rightarrow \boxed{7x - 5y + 4z - 8 = 0}$$

$\boxed{Q.10}$ Find the vector equation of the plane passing through the intersection of the planes $\underbrace{\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})=7}_{P_1}$, & $\underbrace{\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})=9}_{P_2}$ and through the point $(2,1,3)$.

$\hookrightarrow \vec{a}=2\hat{i}+\hat{j}+3\hat{k}$

New plane $\boxed{P_3=0}$

$$\Rightarrow P_1+\lambda P_2=0$$

$$\Rightarrow \Big[\underset{\text{variable}}{\vec{r}}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7\Big]+\lambda\Big[\underset{\text{variable}}{\vec{r}}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9\Big]=0$$

(this plane passes through the point $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$)

$$\Rightarrow (2\hat{i}+\hat{j}+3\hat{k})\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7+\lambda\Big[(2\hat{i}+\hat{j}+3\hat{k})\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9\Big]=0$$

$$\Rightarrow 4+2-9-7+\lambda\big[4+5+\cancel{9}-\cancel{9}\big]=0$$

$$\Rightarrow \lambda(9)=10$$

$$\Rightarrow \boxed{\lambda=\frac{10}{9}}$$

Required plane $\Big[\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7\Big]$

$$+\frac{10}{9}\Big[\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9\Big]=0$$

$$\Rightarrow \frac{9\left[\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7\right]+10\left[\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9\right]}{9}=0$$

$$\Rightarrow \vec{r}\cdot(\underline{18\hat{i}}+\underline{18\hat{j}}-\underline{27}\hat{k}+\underline{20\hat{i}}+\underline{50\hat{j}}+30\hat{k})$$

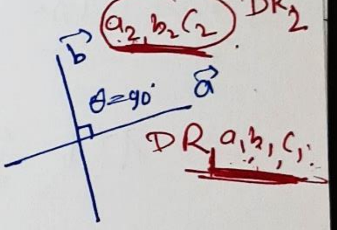
$$-63-90=0$$

$$\Rightarrow \boxed{\vec{r}\cdot(38\hat{i}+68\hat{j}+3\hat{k})=153}$$

Q.11 Find the eq^n. of the plane through the line of intersection of the planes $\overset{P_1}{x+y+z=1}$ and $\overset{P_2}{2x+3y+4z=5}$ which is perpendicular to the plane. $x-y+z=0$.

Ans. $P_1: \underbrace{x+y+z-1}=0$

$P_2: \underbrace{2x+3y+4z-5}=0$



$\bar{a}\cdot\bar{b}=0$

$\boxed{a_1a_2+b_1b_2+c_1c_2=0}$

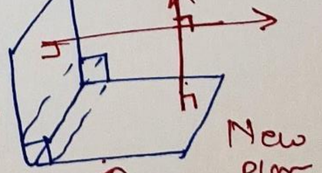
New plane $\boxed{P_3=0}$

$\Rightarrow \boxed{P_1+\lambda P_2=0}$

$\Rightarrow (x+y+z-1)+\lambda(2x+3y+4z-5)=0$

$\Rightarrow \boxed{x(1+2\lambda)+y(1+3\lambda)+z(1+4\lambda)-1-5\lambda=0}$ New Plane

$DR_1: (1+2\lambda,\ 1+3\lambda,\ 1+4\lambda)$



$x-y+z=0$

$DR_2: (1,-1,1)$

New Plane के DR = $\underbrace{1+2\lambda,\ 1+3\lambda,\ 1+4\lambda}$

$(x-y+z=0)$ के DR = $\underbrace{1,\ -1,\ 1}$

Condition of Perpendicular Planes

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow (1+2\lambda)-(1+3\lambda)+(1+4\lambda)=0$$

$$\Rightarrow \cancel{1}+2\lambda-\cancel{1}-3\lambda+1+4\lambda=0$$

$$\Rightarrow 3\lambda+1=0$$

$$\Rightarrow \boxed{\lambda=\frac{-1}{3}}$$

New plane $\quad x(1+2\lambda)+y(1+3\lambda)+z(1+4\lambda)-1-5\lambda=0$

$$\Rightarrow x\left(1-\frac{2}{3}\right)+y\underbrace{\left(1-\frac{\cancel{3}}{\cancel{3}}\right)}_{0}+z\left(1-\frac{4}{3}\right)-1+\frac{5}{3}=0$$

$$\Rightarrow x\cdot\frac{1}{3}+0+z\left(\frac{-1}{3}\right)+\frac{2}{3}=0$$

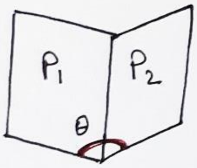
$$\Rightarrow \frac{x}{3}-\frac{z}{3}+\frac{2}{3}=0$$

$$\Rightarrow \boxed{x-z+2=0}$$

[redacted]	Class 12 Maths	Three Dimensional Geometry

Exercise 11.3

Plane (3D)



Angle b/w Two planes

Vector Form

$P_1 : \vec{r}\cdot\vec{n}_1 = d_1$

$P_2 : \vec{r}\cdot\vec{n}_2 = d_2$

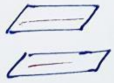
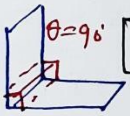
$$\cos\theta = \left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|$$

Cartesian Form

$P_1 : A_1x + B_1y + C_1z = d_1$

$P_2 : A_2x + B_2y + C_2z = d_2$

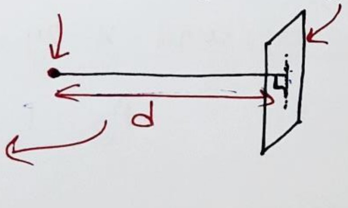
$$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}}\right|$$

Condition for Parallel planes	Condition for Perpendicular Planes
 $\boxed{\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}}$	 $\theta = 90°$ $\boxed{A_1A_2 + B_1B_2 + C_1C_2 = 0}$

Distance Formula [b/w Point & Plane]

Point $\downarrow$ (x_1, y_1, z_1) & Plane $\downarrow$ $(Ax + By + cz = d)$

$$d = \left|\frac{Ax_1 + By_1 + Cz_1 - d}{\sqrt{A^2+B^2+C^2}}\right|$$



Q.12 Find the angle b/w the planes whose vector equations are $\vec{r}\cdot(\underbrace{2\hat{i}+2\hat{j}-3\hat{k}}_{\vec{n}_1}) = 5$ and $\vec{r}\cdot(\underbrace{3\hat{i}-3\hat{j}+5\hat{k}}_{\vec{n}_2}) = 3$.

$$\cos\theta = \left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right| = \left|\frac{(2\hat{i}+2\hat{j}-3\hat{k})\cdot(3\hat{i}-3\hat{j}+5\hat{k})}{\sqrt{4+4+9}\cdot\sqrt{9+9+25}}\right|$$

$$\Rightarrow \cos\theta = \left|\frac{\cancel{6}-\cancel{6}-15}{\sqrt{17}\sqrt{43}}\right| \Rightarrow \cos\theta = \frac{15}{\sqrt{731}} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{15}{\sqrt{731}}\right)}$$

Q.13 In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find angles b/w them.

ⓐ $7x+5y+6z+30=0$ & $3x-y-10z+4=0$

Cond^n. for Parallel Planes $\quad \frac{7}{3} \neq \frac{5}{-1} \neq \frac{6}{-10}$ Not Parallel -

Cond^n. for Perpendicular Planes $\quad 21-5-60 \neq 0$

$\Rightarrow 21-65 \neq 0$

Not ⊥ planes

$$\cos\theta = \left|\frac{21-5-60}{\sqrt{49+25+36}\,\sqrt{9+1+100}}\right| = \left|\frac{44}{\sqrt{110}\,\sqrt{110}}\right|$$

$$\cos\theta = \frac{\cancel{44}\ \cancel{4}\ 2}{\cancel{110}\ \cancel{10}\ 5} \Rightarrow \cos\theta = \frac{2}{5} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{2}{5}\right)}$$

ⓑ $2x+y+3z-2=0$, and $x-2y+5=0$,

$A_1=2,\ B_1=1,\ C_1=3 \qquad A_2=1,\ B_2=-2,\ C_2=0$

Cond^n. for Parallel planes. $\quad \frac{2}{1} \neq \frac{+1}{-2} \neq \frac{3}{0}$ Not Parallel

Condition for Perpendicular Planes. $\quad \cancel{2}-\cancel{2}+0=0$

$\boxed{0=0}$ ✓

Perpendicular planes.

ⓒ $2x-2y+4z+5=0$ and $3x-3y+6z-1=0$

Condition for Parallel planes $\quad \frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$ ✓

$$\Rightarrow \underbrace{\frac{2}{3} = \frac{-2}{-3} = \frac{4}{6}}\Rightarrow \underbrace{\frac{2}{3} = \frac{2}{3} = \frac{2}{3}}$$

∴ Parallel planes.

ⓓ $2x-y+3z-1=0$ & $2x-y+3z+3=0$

Condⁿ for Parallel planes, $\quad \frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$

$$\Rightarrow \underbrace{\frac{2}{2} = \frac{-1}{-1} = \frac{3}{3}} \Rightarrow \underbrace{1 = 1 = 1}$$

∴ Parallel planes.

ⓔ $4x+8y+z-8=0$ and $y+z-4=0$

Condⁿ for Parallel planes, $\quad \frac{4}{0} \neq \frac{8}{1} \neq \frac{1}{1}$ Not Parallels

Condⁿ for ⊥ Planes- $\quad \boxed{A_1A_2 + B_1B_2 + C_1C_2} \neq 0$ $\quad$ (Not ⊥ planes)

$$\Rightarrow 4.0 + 8\times1 + 1\times1 \neq 0$$

$$\Rightarrow 0 + 8 + 1 \neq 0$$

$$\cos\theta = \left| \frac{0+8+1}{\sqrt{16+64+1}\,\sqrt{0+1+1}} \right| \Rightarrow \cos\theta = \frac{9}{\sqrt{81}\,\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\boxed{\theta = 45^\circ}$$

Q.14 In the following cases, find the distance of each of the given points from the corresponding given plane:

(a) Point $(0,0,0)$; plane : $3x-4y+12z=3$

$$\boxed{3x-4y+12z-3=0}$$

(x_1, y_1, z_1) $\quad Ax+By+Cz=d \rightarrow Ax+By+Cz-d=0$

$$\boxed{d=\left|\frac{Ax_1+By_1+Cz_1-d}{\sqrt{A^2+B^2+C^2}}\right|}$$

$$d=\left|\frac{0-0+0-3}{\sqrt{9+16+144}}\right|$$

$$d=\left|\frac{-3}{\sqrt{169}}\right| \Rightarrow \boxed{d=\frac{3}{13}}$$

(b) Point $(3,-2,1)$, plane $2x-y+2z+3=0$

$$d=\left|\frac{6+2+2+3}{\sqrt{4+1+4}}\right|=\frac{13}{\sqrt{9}}=\frac{13}{3}$$

(c) Point $(2,3,-5)$ plane : $x+2y-2z=9$

$$\Rightarrow \boxed{x+2y-2z-9=0}$$

$$d=\left|\frac{2+6+10-9}{\sqrt{1+4+4}}\right|=\frac{9}{\sqrt{9}}=\frac{9}{3}=3$$

(d) Point $(-6,0,0)$ Plane : $2x-3y+6z-2=0$

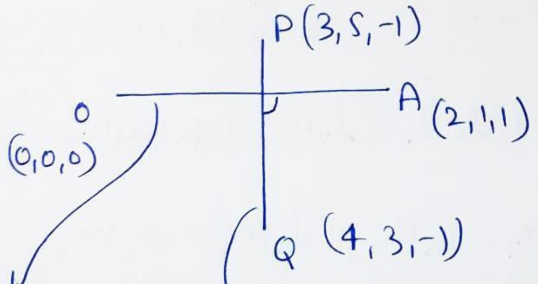
$$d=\left|\frac{-12+0+0-2}{\sqrt{4+9+36}}\right|=\left|\frac{-14}{\sqrt{49}}\right|=\frac{\cancel{14}}{\cancel{7}}=2$$

Miscellaneous Exercise on Chapter 11

Q.1 Show that the line joining the origin to the point (2,1,1) is perpendicular to the line determined by the points (3,5,−1) & (4,3,−1).

A = (2,1,1), P = (3,5,−1), Q = (4,3,−1)

Ans.



DR | a_1, b_1, c_1

$2-0,\ 1-0,\ 1-0$

$\underbrace{2,\ 1,\ 1}_{OA}$

DR | a_2, b_2, c_2

$4-3,\ 3-5,\ -1+1$

$\underbrace{1,\ -2,\ 0}_{PQ}$

Condition of Perpendicular lines

$$\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$$

$$a_1a_2 + b_1b_2 + c_1c_2$$
$$= 2(1) + 1(-2) + 1(0)$$
$$= 2 - 2 + 0$$
$$= 0$$

Perpendicular lines

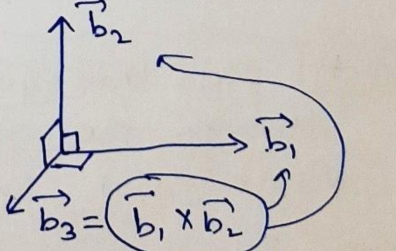
Q.2 If l_1, m_1, n_1 and l_2, m_2, n_2, are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1,\ n_1l_2 - n_2l_1,\ l_1m_2 - l_2m_1$.

$\vec{b_1} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$

$\boxed{l_1^2 + m_1^2 + n_1^2 = 1} \rightarrow |\vec{b_1}| = 1$

$\vec{b_2} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$

$\boxed{l_2^2 + m_2^2 + n_2^2 = 1} \rightarrow |\vec{b_2}| = 1$



Now, $\vec{b_3} = \vec{b_1} \times \vec{b_2}$ will be perpendicular to both $\vec{b_1}$ & $\vec{b_2}$.

$$\vec{b_3} = \vec{b_1} \times \vec{b_2} = (l_1\hat{i} + m_1\hat{j} + n_1\hat{k}) \times (l_2\hat{i} + m_2\hat{j} + n_2\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} \rightarrow \; = \hat{i}(m_1n_2 - m_2n_1) - \hat{j}(l_1n_2 - l_2n_1) + \hat{k}(l_1m_2 - l_2m_1)$$

$$= \hat{i}(m_1n_2 - m_2n_1) + \hat{j}(l_2n_1 - l_1n_2) + \hat{k}(l_1m_2 - l_2m_1)$$

D.C. $m_1n_2 - m_2n_1, \; l_2n_1 - l_1n_2, \; l_1m_2 - l_2m_1$

Q.3 Find the angle between the lines whose direction ratios are a, b, c & $b-c, c-a, a-b$.

D.R

$$\underset{a_1}{\underset{\downarrow}{a}}, \underset{b_1}{\underset{\downarrow}{b}}, \underset{c_1}{\underset{\downarrow}{c}} \quad \& \quad \underset{a_2}{\underset{\downarrow}{b-c}}, \underset{b_2}{\underset{\downarrow}{c-a}}, \underset{c_2}{\underset{\downarrow}{a-b}}$$

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right| = \left|\frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right|$$

$$\cos\theta = \left|\frac{\cancel{ab} - \cancel{ac} + \cancel{bc} - \cancel{ba} + \cancel{ca} - \cancel{cb}}{\sqrt{\ }\sqrt{\ }}\right| = \left|\frac{0}{\sqrt{\ }\cdot\sqrt{\ }}\right|$$

$\cos\theta = 0$ $\quad \boxed{\theta = 90^\circ}$ Perpendicular

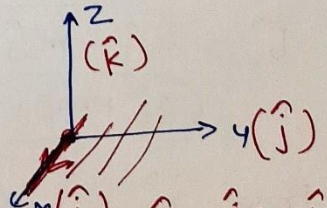
---

Q.4 Find the equation of a line parallel to x-axis & passing through the origin.

Point $(0,0,0)$ Direction: $(1,0,0)$

Eqn. of line $\quad \dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c}$

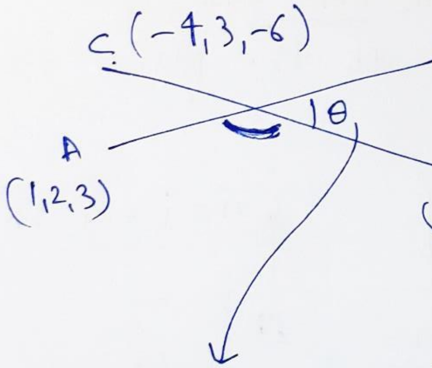
$$\Rightarrow \boxed{\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}} \Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$



$x(\hat{i}) = \hat{i} + 0.\hat{j} + 0.\hat{k}$

$(1, 0, 0)$

Q.5 If the coordinates of the points A, B, C, D are (1,2,3), (4,5,7), (−4,3,−6) and (2,9,2) respectively, then find angle b/w the lines AB & CD.



B (4,5,7) $\Rightarrow DR = 4-1,\ 5-2,\ 7-3$

$= ③,\ 3,\ 4$

$\boxed{a_1,\ b_1,\ c_1}$

D (2,9,2) $\Rightarrow DR = 2+4,\ 9-3,\ 2+6$

$= ⑥,\ 6,\ 8$

$\boxed{a_2,\ b_2,\ c_2}$

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

$$= \left|\frac{3\times6+3\times6+4\times8}{\sqrt{9+9+16}\ \sqrt{36+36+64}}\right| = \left|\frac{18+18+32}{\sqrt{34}\ \sqrt{136}}\right|$$

$$\Rightarrow \cos\theta = \frac{68}{\sqrt{2\times17}\ \sqrt{4\times2\times17}} = \frac{68}{\sqrt{2\times17\times4\times2\times17}}$$

$$\Rightarrow \cos\theta = \frac{\cancel{68}}{\cancel{2\times17\times2}} = 1$$

$$\Rightarrow \cos\theta = 1 \Rightarrow \boxed{\theta = 0^\circ}$$

Q.6 If the lines $\frac{x-1}{\underset{a_1}{-3}} = \frac{y-2}{\underset{b_1}{2K}} = \frac{z-3}{\underset{c_1}{2}}$ and $\frac{x-1}{\underset{a_2}{3K}} = \frac{y-1}{\underset{b_2}{1}} = \frac{z-6}{\underset{c_2}{-5}}$ are perpendicular, find K.

Cond^n. for ⊥ lines:

$$\boxed{a_1a_2+b_1b_2+c_1c_2 = 0}$$

$$\Rightarrow -9K + 2K - 10 = 0$$

$$\Rightarrow -7K = 10$$

$$\Rightarrow \boxed{K = \frac{10}{-7}}$$

[redacted]	Class 12 Maths	Three Dimensional Geometry

Miscellaneous Exercise on Chapter ⑪

Q.7 Find the vector equation of the line passing through (1,2,3) and perpendicular to the plane $\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$.

Line:

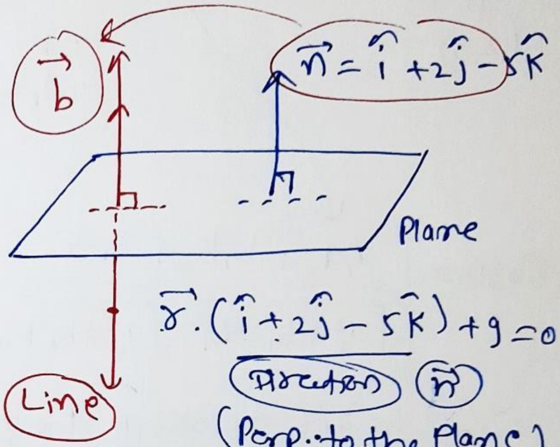
$$\vec{r}=\vec{a}+\lambda\vec{b}$$

Point — Direction

(1,2,3)

$(\hat{i}+2\hat{j}+3\hat{k})$

$\vec{b}\parallel\vec{n}$



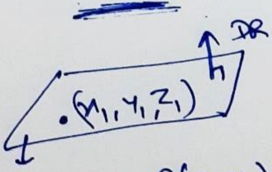
$\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$

Direction $\vec{n}$

(Perp. to the Plane)

Line: $\boxed{\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}+2\hat{j}-5\hat{k})}$

Q.8 Find the equation of the plane passing through (a,b,c) and parallel to the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$.



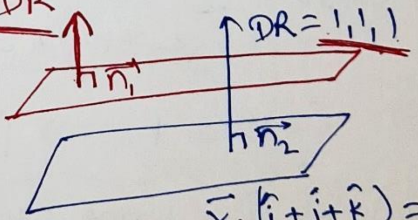
$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$

DR.

(A,B,C) DR

(1,1,1)

$A=B=C=1$



$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$

$\vec{n}_2$ 1,1,1

Required plane

$$1(x-a)+1(y-b)+1(z-c)=0$$

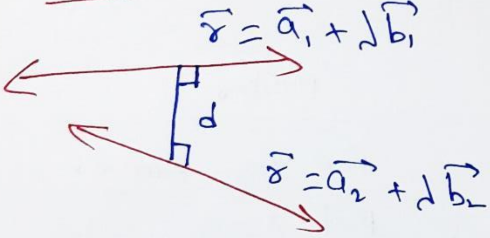
$$\Rightarrow \boxed{x+y+z=a+b+c}$$

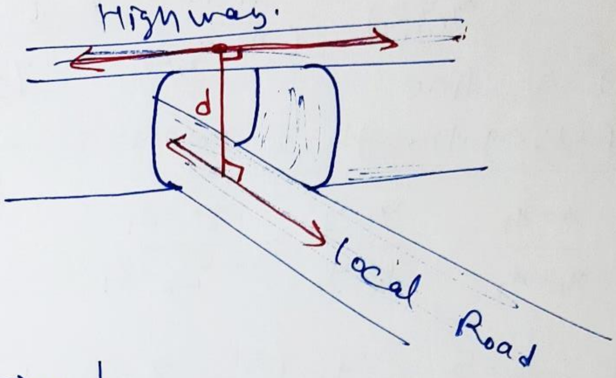
Q.9 Find the shortest distance b/w lines

$$\vec{r}=\underbrace{(6\hat{i}+2\hat{j}+2\hat{k})}_{\text{Point}(\vec{a_1})}+\lambda\underbrace{(\hat{i}-2\hat{j}+2\hat{k})}_{\text{Direction}(\vec{b_1})} \ \& \ \vec{r}=\underbrace{(-4\hat{i}-\hat{k})}_{\text{Point}(\vec{a_2})}+\mu\underbrace{(3\hat{i}-2\hat{j}-2\hat{k})}_{\text{Direction}(\vec{b_2})}.$$

$\vec{b_1} \not\parallel \vec{b_2}$

Skew Lines





$$d=\left|\frac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

$$d=\left|\frac{(-10\hat{i}-2\hat{j}-3\hat{k})\cdot(8\hat{i}+8\hat{j}+4\hat{k})}{12}\right|$$

$$d=\left|\frac{-80-16-12}{12}\right|$$

$$d=\left|\frac{-108}{12}\right|$$

$$d=\frac{\cancel{108}\ 9}{\cancel{12}}$$

$$\boxed{d=9}$$

$\vec{b_1}\times\vec{b_2}$

$$=(\hat{i}-2\hat{j}+2\hat{k})\times(3\hat{i}-2\hat{j}-2\hat{k})$$

$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&2\\3&-2&-2\end{vmatrix}\rightarrow \textcircled{R_1}$$

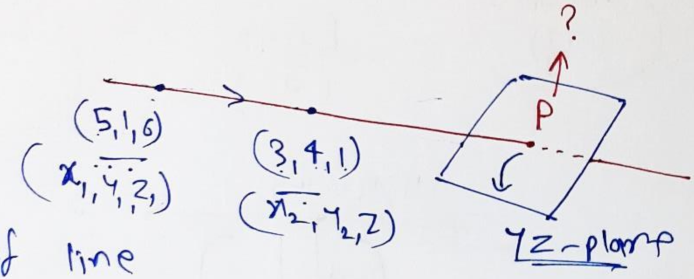
$$=\hat{i}(4+4)-\hat{j}(-2-6)+\hat{k}(-2+6)$$

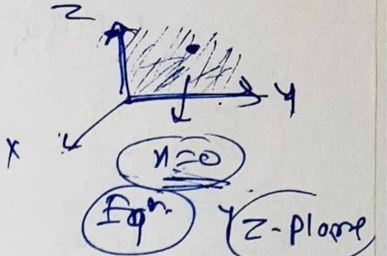
$$=\boxed{8\hat{i}+8\hat{j}+4\hat{k}=\vec{b_1}\times\vec{b_2}}$$

$$|\vec{b_1}\times\vec{b_2}|=\sqrt{64+64+16}$$

$$=\sqrt{144}=12$$

Q.10 Find the coordinates of the point where the line through (5,1,6) and (3,4,1) crosses the YZ- plane.





Eqn. of line passing through 2 - Points.

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} = \lambda$$

for 'P'

Method - II

For YZ- plane,

Put x=0

$$\frac{0-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5}$$

$$\frac{x-5}{-2} = \lambda \Rightarrow \boxed{x = -2\lambda+5}$$

$$\frac{y-1}{3} = \lambda \Rightarrow \boxed{y = 3\lambda+1}$$

$$\frac{z-6}{-5} = \lambda \Rightarrow \boxed{z = -5\lambda+6}$$

Point $P(-2\lambda+5, 3\lambda+1, -5\lambda+6)$

↓

Satisfies the plane (YZ-Plane)

↓

Eqn. $\boxed{x=0}$

$$\Rightarrow -2\lambda+5 = 0$$

$$\Rightarrow \boxed{\lambda = \frac{5}{2}}$$

Required Point $P(-2\lambda+5, 3\lambda+1, -5\lambda+6)$

$$P\left[-2\left(\frac{5}{2}\right)+5, 3\left(\frac{5}{2}\right)+1, -5\left(\frac{5}{2}\right)+6.\right]$$

$$P\left(0, \frac{17}{2}, -\frac{13}{2}\right)$$ ✓

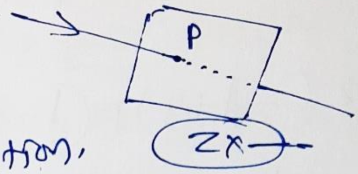
Q.11 Find the coordinates of the point where the line through (5,1,6) & (3,4,1) crosses the ZX-plane.

$\downarrow$

$Y=0$ —①

Ans.

line eqⁿ → same as previous question,



$$\boxed{\frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}} \quad —②$$

By solving eqⁿ ① & ②

$$\underbrace{\frac{x-5}{-2}=\frac{0-1}{3}=\frac{z-6}{-5}}$$

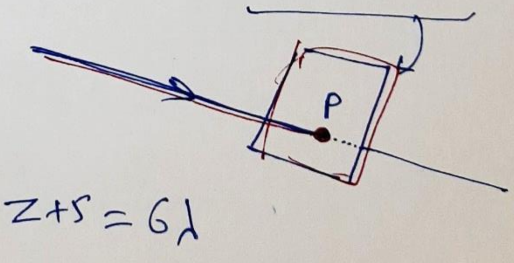
$$\frac{x-5}{-2}=\frac{-1}{3} \Rightarrow x-5=\frac{2}{3} \Rightarrow x=5+\frac{2}{3} \Rightarrow \boxed{x=\frac{17}{3}}$$

$$\frac{-1}{3}=\frac{z-6}{-5} \Rightarrow \frac{5}{3}+6=z \Rightarrow \boxed{\frac{23}{3}=z}$$

Point of intersection $P\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

Q.12 Find the coordinates of the point where the line through (3,-4,-5) and (2,-3,1) crosses the plane $2x+y+z=7$.

Line: $\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=\lambda$



$\Rightarrow \frac{x-3}{-1}=\lambda$ | $\frac{y+4}{1}=\lambda$ | $z+5=6\lambda$

$\Rightarrow x=-\lambda+3$ | $\Rightarrow y=\lambda-4$ | $z=6\lambda-5$

$P(-\lambda+3, \lambda-4, 6\lambda-5) \Rightarrow$ Also satisfies the plane

Point of intersection $P(-\lambda+3,\ \lambda-4,\ 6\lambda-5)$

Satisfies the plane $2\textcircled{x}+\textcircled{y}+\textcircled{z}=7.$

$$\Rightarrow\ 2(-\lambda+3)+(\lambda-4)+(6\lambda-5)=7$$

$$\Rightarrow\ -2\lambda+6+7\lambda-9=7$$

$$\Rightarrow\ 5\lambda=10$$

$$\boxed{\lambda=2}$$

$$P(-2+3,\ 2-4,\ 12-5)$$

$$P(1,-2,7)$$

Miscellaneous Exercise on Chapter ⑪

Q.13 Find the equation of the plane passing through the point $(-1,3,2)$ and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$

Required plane, Ⓟ $A(x-x_1)+B(y-y_1)+C(z-z_1)=0$

$(x_1,y_1,z_1)\equiv(-1,3,2)$

$P: \boxed{A(x+1)+B(y-3)+C(z-2)=0}$ $DR_P = A, B, C$

$P_1: \boxed{x+2y+3z=5} \rightarrow DR_{P_1} = 1,2,3$ $(\vec{n_1} = \hat{i}+2\hat{j}+3\hat{k})$

$P_2: \boxed{3x+3y+z=0} \rightarrow DR_{P_2} = 3,3,1$ $(\vec{n_2} = 3\hat{i}+3\hat{j}+\hat{k})$

ATQ Condition for Perpendicular Planes $\boxed{A_1A_2+B_1B_2+C_1C_2=0}$

$\boxed{P \perp P_1}$

$A\times1+B\times2+C\times3=0$

$\Rightarrow A+2B+3C=0$

$\Rightarrow \boxed{A=-2B-3C}$

$\Rightarrow A=-2\left(-\frac{8C}{3}\right)-3C$

$\Rightarrow A=\frac{16C}{3}-3C$

$\Rightarrow \boxed{A=\frac{7C}{3}}$ ✓

$P \perp P_2$

$3A+3B+C=0$

$\Rightarrow 3(-2B-3C)+3B+C=0$

$\Rightarrow -6B-9C+3B+C=0$

$\Rightarrow -3B-8C=0$

$\Rightarrow -3B=8C$

$\Rightarrow \boxed{B=-\frac{8C}{3}}$ ✓

Required plane $A(x+1) + B(y-3) + C(z-2) = 0$

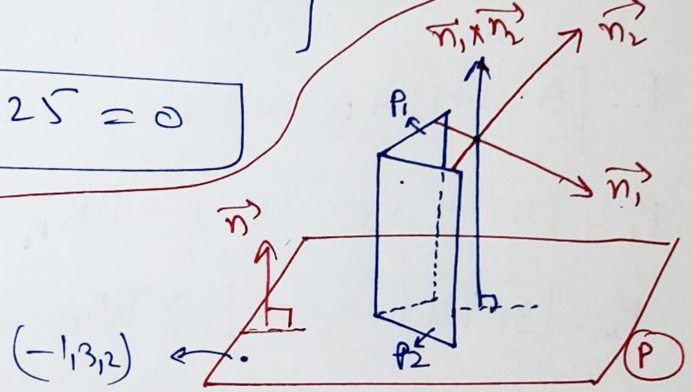
$A = \frac{7C}{3}$, $B = \frac{-8C}{3}$

$\Rightarrow \underbrace{\frac{7C}{3}(x+1)} - \underbrace{\frac{8C}{3}(y-3)} + \underbrace{C\frac{(z-2)}{1}} = 0$

$\Rightarrow C\left\{\frac{7x+7-8y+24+3z-6}{3}\right\} = 0$

$\Rightarrow \boxed{7x - 8y + 3z + 25 = 0}$

II - method
(Hint only)
Vector method

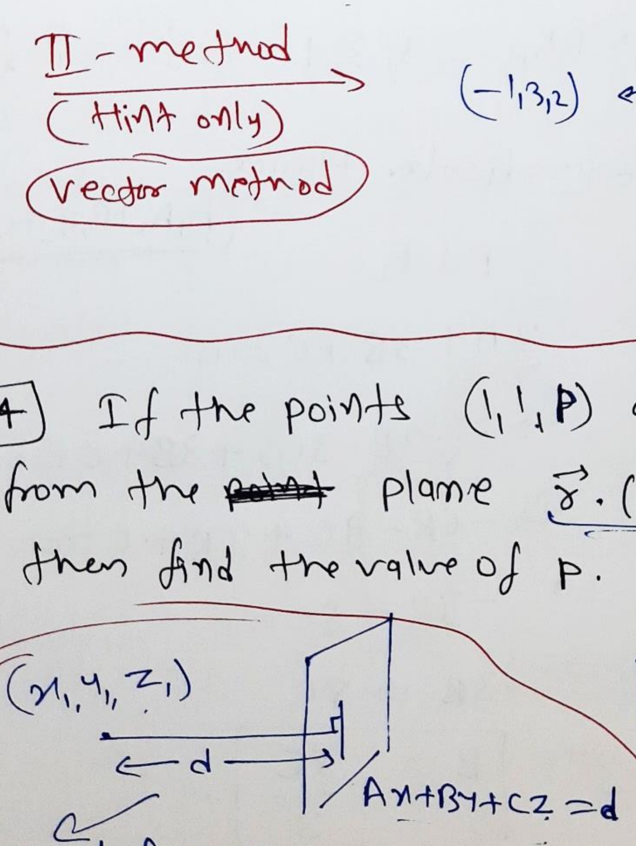


Required plane
$\boxed{\vec{r}\cdot\vec{n} = d}$
normal $= \vec{n_1}\times\vec{n_2}$

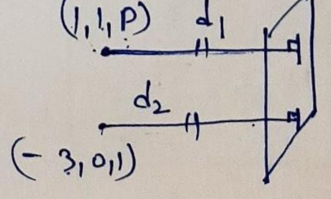
Q.14 If the points (1,1,P) and (−3,0,1) be equidistant from the ~~point~~ plane $\vec{r}\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13=0$, then find the value of P.

$(\vec{r} = x\hat{i}+y\hat{j}+z\hat{k})$

$\boxed{3x+4y-12z+13=0}$



$$d = \left|\frac{Ax_1+By_1+Cz_1-d}{\sqrt{A^2+B^2+C^2}}\right|$$



ATQ. $d_1 = d_2$

$$\Rightarrow \left|\frac{3+4-12P+13}{\sqrt{9+16\cancel{+144}}}\right| = \left|\frac{-9+0-12+13}{\sqrt{9\cancel{+16+}144}}\right|$$

$$\Rightarrow |20-12P| = |-8| = 8$$

$$\Rightarrow \boxed{20-12P = \pm 8}$$

$$20-12P = 8 \qquad\qquad 20-12P=-8$$

$$\Rightarrow 12 = 12P \qquad\qquad \Rightarrow 28 = 12P$$

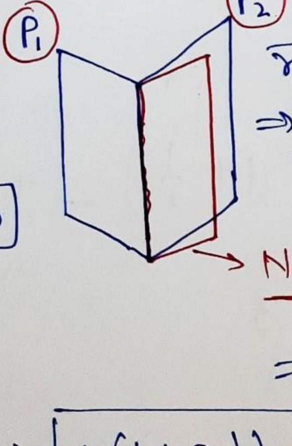
$$\Rightarrow \boxed{P=1} ✓ \qquad\qquad \Rightarrow \boxed{\frac{7}{3} = P} ✓$$

Q.15 Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and Parallel to x-axis.

$\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$

$\Rightarrow x+y+z=1$

$\Rightarrow \boxed{x+y+z-1=0}$ (P_1)

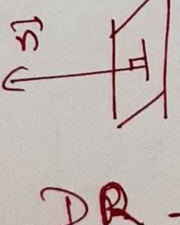


$\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$

$\Rightarrow \boxed{2x+3y-z+4=0}$

$\rightarrow$ New plane $\boxed{P_1+\lambda P_2=0}$

$$\Rightarrow (x+y+z-1)+\lambda(2x+3y-z+4)=0$$

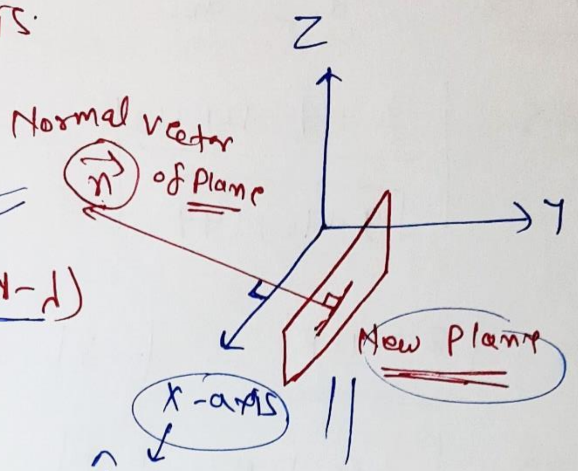


$$\Rightarrow \boxed{x(1+2\lambda)+y(1+3\lambda)+z(1-\lambda)+(4\lambda-1)=0}$$

DR $\longrightarrow$ $1+2\lambda,\ 1+3\lambda,\ 1-\lambda$

New plane || x-axis.

DR ← $(1+2\lambda, 1+3\lambda, 1-\lambda)$



$\vec{n} \to (1+2\lambda, 1+3\lambda, 1-\lambda)$

x-axis → $(1, 0, 0)$

$\hat{i} = \hat{i} + 0.\hat{j} + 0.\hat{k}$

$\boxed{DR = 1, 0, 0}$

By Diagram,

$\vec{n} \perp$ x-axis.

Condition $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$\Rightarrow (1+2\lambda).1 + (1+3\lambda).0 + (1-\lambda).0 = 0$

$\Rightarrow 1+2\lambda = 0$

$\Rightarrow \lambda = -\frac{1}{2}$

New plane, $\boxed{(1+2\lambda)x + (1+3\lambda)y + (1-\lambda)z + (4\lambda-1) = 0}$

Put $\left(\lambda = -\frac{1}{2}\right)$

$\Rightarrow 0.x + \left(1 - \frac{3}{2}\right)y + \left(1 + \frac{1}{2}\right)z + \left(\frac{-\cancel{4}^{2}}{\cancel{2}} - 1\right) = 0$

$\Rightarrow \left(\frac{-1}{2}y\right) + \left(\frac{3}{2}z\right) = 3$

$\Rightarrow \frac{-y+3z}{2} = 3$

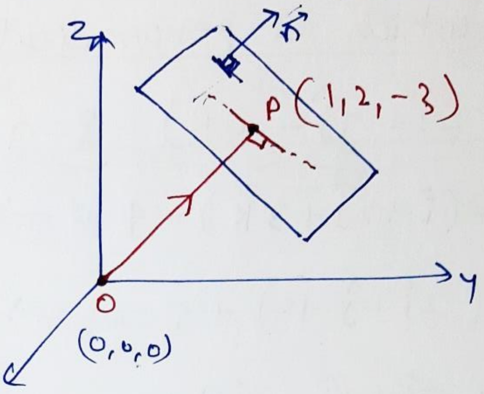
$\Rightarrow \boxed{-y+3z = 6} \Rightarrow \boxed{0 = y - 3z + 6}$

Q.16 If 'o' be the origin and the coordinates of 'p' be P(1,2,-3), then find the equation of the plane passing through P and perpendicular to OP.

O(0,0,0)

P(1,2,-3)

$$\boxed{DR_{op} = 1, 2, -3}$$



Any plane passing through the point (x_1, y_1, z_1)

Eqn ⇒ $\boxed{A(x-x_1) + B(y-y_1) + C(z-z_1) = 0}$

P(1,2,-3)

Plane. $\boxed{A(x-1) + B(y-2) + C(z+3) = 0}$

DR = A, B, C ← DR. of normal vector of Plane

By Diagram, $\vec{OP} \parallel \vec{n}$

DR → $\boxed{1, 2, -3 \;\big|\; A, B, C}$

A=1, B=2, C=-3

Required Plane, $1(x-1) + 2(y-2) + (-3)(z+3) = 0$

$\Rightarrow x - 1 + 2y - 4 - 3z - 9 = 0$

$\Rightarrow \boxed{x + 2y - 3z = 14}$

$\boxed{Q.17}$ Find the equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$, $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane

$$\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0 \rightarrow P_3$$

$$P_1 : \vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0 \Rightarrow \boxed{x+2y+3z-4=0}\ P_1$$

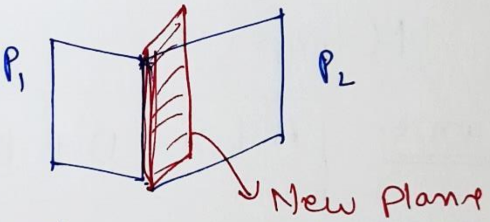
$$P_2 : \vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0 \Rightarrow \boxed{2x+y-z+5=0}\ P_2$$

$$P_3 : \vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0 \Rightarrow \boxed{5x+3y-6z+8=0}\ P_3$$

New plane → passing through the line of intersection of P_1 & P_2

$$\downarrow$$

$$\boxed{P_1+\lambda P_2=0}$$



$$\Rightarrow (x+2y+3z-4)+\lambda(2x+y-z+5)=0$$

$$\Rightarrow \boxed{x(1+2\lambda)+y(2+\lambda)+z(3-\lambda)+(5\lambda-4)=0}\ \text{New plane}$$

$$\downarrow$$

DR $(1+2\lambda, 2+\lambda, 3-\lambda)$

This New plane is perpendicular to the plane $P_3 : \boxed{5x+3y-6z+8=0} \rightarrow$ (DR: 5, 3, −6)

Condition of Perpendicular planes.

$$A_1A_2+B_1B_2+C_1C_2=0$$

$$\Rightarrow (1+2\lambda).5+(2+\lambda)3+(3-\lambda)(-6)=0$$

$$\Rightarrow ⑤+10\lambda+⑥+3\lambda-18+6\lambda=0$$

$$\Rightarrow 19\lambda=7 \Rightarrow \boxed{\lambda=7/19}\ ✓$$

New plane : $x(1+2\lambda)+y(2+\lambda)+z(3-\lambda)+(5\lambda-4)=0$

Put $\left(\lambda=\frac{7}{19}\right)$ ↺

$$\Rightarrow x\left(1+\frac{14}{19}\right)+y\left(2+\frac{7}{19}\right)+z\left(3-\frac{7}{19}\right)+\left(\frac{35}{19}-4\right)=0$$

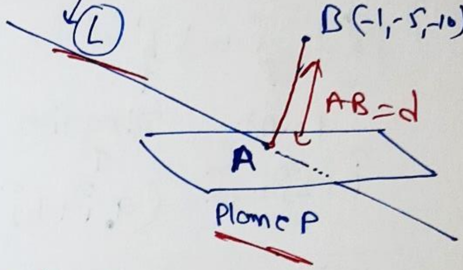
$$\Rightarrow x\left(\frac{33}{19}\right)+y\left(\frac{45}{19}\right)+z\left(\frac{50}{19}\right)+\left(-\frac{41}{19}\right)=0$$

$$\Rightarrow \boxed{33x+45y+50z-41=0}$$

Miscellaneous Exercise on Chapter ⑪

Q.18 Find the distance of the point Ⓑ $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ Ⓛ and the plane $\vec{r}\cdot(\hat{i} - \hat{j} + \hat{k}) = 5$.

Plane P



A → Point of intersection of line L & Plane P

we solve the equations of line L & Plane P for the point of intersection.

Line L: $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ —①

Plane P: $\vec{r}\cdot(\hat{i} - \hat{j} + \hat{k}) = 5$ —②

Substitution method.

$$\Rightarrow \left[(2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})\right]\cdot(\hat{i} - \hat{j} + \hat{k}) = 5$$

(Dot Product)

$$\Rightarrow 2 + 1 + 2 + \lambda(3 - 4 + 2) = 5$$

$$\Rightarrow \cancel{5} + \lambda(1) = \cancel{5}$$

$$\Rightarrow \boxed{\lambda = 0}$$

By eqn ①: $\boxed{\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}}$ Point of intersection Ⓐ

$A(2, -1, 2)$.

$$AB = \sqrt{(3)^2 + 4^2 + 12^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

$B(-1, -5, -10)$ given

[redacted]

Q.19 Find the vector equations of the line passing through $(1,2,3)$ and parallel to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$.

Ans. Vector Eqn. of a line.

$$\vec{r} = \underset{\text{Point}}{\vec{a}} + \lambda \underset{\text{Direction}}{\vec{b}}$$

$\hat{i}+2\hat{j}+3\hat{k}$ $\quad (a\hat{i}+b\hat{j}+c\hat{k})$

$$\Rightarrow \boxed{\vec{r}=(\hat{i}+2\hat{j}+3\hat{k}) + \lambda(\underbrace{a\hat{i}+b\hat{j}+c\hat{k}}_{\vec{b}})}$$

Our required line is parallel to the plane (P_1)

$\vec{r}.(\hat{i}-\hat{j}+2\hat{k}) = 5$

$\Rightarrow \vec{b}.\vec{n}_1 = 0$

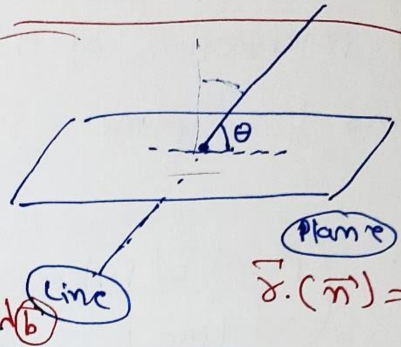
$\Rightarrow (a\hat{i}+b\hat{j}+c\hat{k}).(\hat{i}-\hat{j}+2\hat{k}) = 0$

$\Rightarrow \boxed{a-b+2c=0}$ — ①

Line L $\parallel$ $\vec{r}.(\underbrace{3\hat{i}+\hat{j}+\hat{k}}_{\vec{n}_2})=6$

$\Rightarrow (a\hat{i}+b\hat{j}+c\hat{k}).(3\hat{i}+\hat{j}+\hat{k})=0$

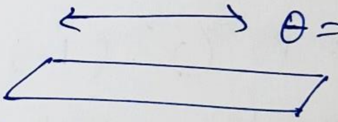
$\Rightarrow \boxed{3a+b+c=0}$ — ②



$\vec{r}=\vec{a}+\lambda\vec{b}$ $\qquad$ $\vec{r}.(\vec{n})=d$

$$\boxed{\sin\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}}$$

Plane $\parallel$ Line



$\theta=0$

$\sin\theta = 0 = \frac{\vec{b}.\vec{n}}{(\ )} \Rightarrow \boxed{\vec{b}.\vec{n}=0}$ ☆

Condn.

By eqn ① & ②

Elimination,

$a-\cancel{b}+2c=0$

$3a+\cancel{b}+c=0$

$4a+3c=0$

$$\boxed{c=-\frac{4a}{3}}$$

by putting the value of $c = -\frac{4a}{3}$ in eq^n ①

$\boxed{a - b + 2c = 0}$

$\Rightarrow a + 2c = b$

$\Rightarrow a + 2\left(-\frac{4a}{3}\right) = b$

$\Rightarrow \frac{3a - 8a}{3} = b \Rightarrow \boxed{b = -\frac{5a}{3}}$

Required Eq^n. of line $\boxed{\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})}$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(\frac{a}{1}\hat{i} - \frac{5a}{3}\hat{j} - \frac{4a}{3}\hat{k}\right)$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(\frac{3a\hat{i} - 5a\hat{j} - 4a\hat{k}}{3}\right)$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \overset{\text{Parameter}}{\underbrace{\lambda\left(\frac{-a}{3}\right)}_{\lambda_1}}(-3\hat{i} + 5\hat{j} + 4\hat{k})$$

$\Rightarrow \boxed{\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda_1(-3\hat{i} + 5\hat{j} + 4\hat{k})}$

Q.20 Find the vector equation of the line passing through the point (1, 2, −4) and perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ (L_1) and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. (L_2)

Vector Eq^n. of line (L)

$\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$

Point ↑ $(\hat{i} + \hat{j} - 4\hat{k})$

Direction ↑ $(a\hat{i} + b\hat{j} + c\hat{k})$

Direction

$\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$

Direction

$\vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$

Concept

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$

$\vec{r} = \vec{a_2} + \mu\vec{b_2}$

$\perp \rightarrow \boxed{\vec{b_1} \cdot \vec{b_2} = 0}$

ATQ Line $L \perp L_1$

$(\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}) \qquad (\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k})$

$\Rightarrow \vec{b} \cdot \vec{b_1} = 0$

$\Rightarrow \boxed{3a - 16b + 7c = 0}$ — ①

ATQ. Line $L \perp L_2$

$\vec{b} = a\hat{i} + b\hat{j} + c\hat{k} \qquad \vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$

$\Rightarrow \vec{b} \cdot \vec{b_2} = 0$

$\Rightarrow \boxed{3a + 8b - 5c = 0}$ — ②

$\Rightarrow$ By eqn. ① − eqn. ②

$$\begin{array}{r} 3a - 16b + 7c = 0 \\ 3a + 8b - 5c = 0 \\ - \quad - \quad + \\ \hline -24b + 12c = 0 \end{array}$$

$\Rightarrow \cancel{12}c = \cancel{24}^{2} b$

$\boxed{c = 2b}$

By eqn ②

$\Rightarrow 3a + 8b - 5(2b) = 0$

$\Rightarrow 3a + 8b - 10b = 0$

$\Rightarrow 3a = 2b \Rightarrow \boxed{a = \frac{2b}{3}}$

Required line $\boxed{\vec{r} = \vec{a} + \lambda\vec{b}}$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\frac{2b}{3}\hat{i} + b\hat{j} + 2b\hat{k})$

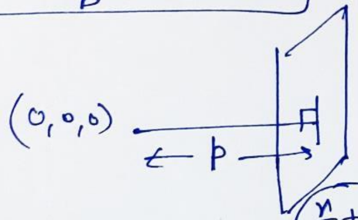
$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + b\lambda\left(\frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{2}\right) \rightarrow$ New λ_1

$\Rightarrow \boxed{\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda_1(2\hat{i} + 3\hat{j} + 6\hat{k})}$

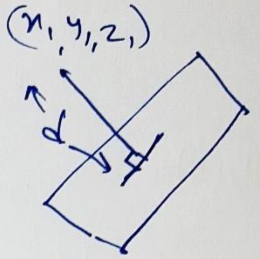
Q.21 **Prove that** if a plane has intercepts a, b, c and is at a distance 'p' units from the origin, then $\boxed{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}}$.

Intercept form of the plane.

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$



$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1=0$$



$Ax+By+Cz=d$

$$d=\left|\frac{Ax_1+By_1+Cz_1-d}{\sqrt{A^2+B^2+C^2}}\right|$$

$$P=\left|\frac{0+0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|$$

$$\Rightarrow P=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} \Rightarrow \frac{1}{P}=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}$$

$$\Rightarrow \boxed{\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}$$

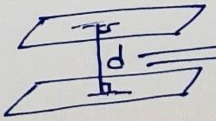
Q.22 Distance b/w the two planes $2x+3y+4z=4$ (P_1) & $4x+6y+8z=12$ (P_2) is —
$\quad\quad\quad\quad\quad\quad\quad\quad 2$

(A) 2 units (B) 4 (C) 8 [✓ (D) $\frac{2}{\sqrt{29}}$]

$P_1: 2x+3y+4z-4=0$

$P_2: 2x+3y+4z-6=0$

A B C



$$\Rightarrow d=\left|\frac{d_1-d_2}{\sqrt{A^2+B^2+C^2}}\right|$$ ☆

I-method.

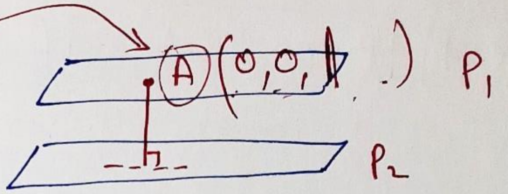
$$d=\left|\frac{(-4)-(-6)}{\sqrt{2^2+3^2+4^2}}\right|=\frac{2}{\sqrt{29}}$$

④+9+⑯

(II - method)

$P_1 : 2x + 3y + 4z - 4 = 0$

$P_2 : 2x + 3y + 4z - 6 = 0$



Point A on 'P₁'

(0,0,1)

let point $\overset{A}{(0,0,z)}$

on 'P₁'

$\Rightarrow 0 + 0 + 4z - 4 = 0$

$\Rightarrow \boxed{z = 1}$

Distance b/w P₁ & P₂

= Distance b/w A & P₂

(Point) (Plane)

(0,0,1) $\boxed{2x + 3y + 4z - 6 = 0}$

$$\Rightarrow \left| \frac{0 + 0 + 4 - 6}{\sqrt{2^2 + 3^2 + 4^2}} \right| = \frac{2}{\sqrt{29}}$$

Q.23 The planes: $\overset{A_1}{2x} - \overset{B_1}{y} + \overset{C_1}{4z} = 5$ and $\overset{A_2}{5x} - \overset{B_2}{2.5y} + \overset{C_2}{10z} = 6$

are Ⓐ perpendicular [✓ Ⓑ Parallel]

Ⓒ intersect y-axis Ⓓ passes through $\left(0, 0, \frac{5}{4}\right)$

Condition of Parallel planes

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

$$\Rightarrow \frac{2}{5} = \frac{+10}{+2,5} = \frac{4}{10}$$

$$\Rightarrow \frac{2}{5} = \frac{\cancel{+0}\,2}{\cancel{25}\,5} = \frac{\cancel{4}\,2}{\cancel{10}\,5}$$

$$\frac{2}{5} = \frac{2}{5} = \frac{2}{5}$$