CARTESIAN SYSTEM OF RECTANGULAR COORDINATES

Coordinate geometry is that branch of geometry in which two numbers called co-ordinates, are used to indicate position of a point in a plane and which makes the use of algebraic methods in the study of geometric figures.

Distance Formula: Distance between two points P and Q whose coordinates are (x_1, y_1) and (x_2, y_2) is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance of a point *p* (x_1, y_1) from Origin: The distance of a point P (x_1, y_1) from the origin (0, 0) is

$$\sqrt{(x_1 - 0)^2 + (y_1 - 0)^2} = \sqrt{x_1^2 + y_1^2}$$

Remember:

(*i*) Distance between two given points

$$= \sqrt{(\text{Difference of Abscissa})^2 + (\text{Difference of Ordinates})^2}$$

(*ii*) We take only the positive square root as we are usually interested only in the magnitude of the line PQ.

Application of Distance formula to Geometry:

(A) When three points are given

Case I: To show that the given three points A, B, C form an isosceles triangle

Working Rule

Step I: Find the length of the sides AB, BC, CA with the help of distance formula.

Step II: Show that two of the three sides AB, BC, CA are equal. Then the triangle is Isosceles triangle.

Case II: To show the given three points A, B and C form a right angled triangle.

Working Rule:

Step I: Find the sides AB, BC,CA.

Step II: Show that the square of one side is equal to the sum of the squares of the other two. Then ΔABC is a Right angled triangle.

Case III: To show that the given three points A, B, C form an equilateral triangle.

Working Rule:

Step I: Find the sides AB, BC, CA.

Step II: Show that AB = BC = CA. Then DABC is an Equilateral triangle.

Case IV: To show that given three points A, B, C form a right-angled isosceles triangle.

Working Rule:

Step I: Find the sides AB, BC, CA.

Step II: Show that two of the three sides are equal.

Step III: Show that the square of one side is equal to the sum of the squares of the other two sides (Pythagorus Theorem)

The DABC is a Right-angled Isosceles triangle.

Case V: To show that the three poins A, B, C are collinear.

Step I: Find AB, BC, CA.

Step II: Show that the sum of the distances between two pairs of points is equal to the distance between the third pair. Or, show that the area of the triangle formed by these three points is zero.

(B) When four points are given

Case VI: To show that the four points A, B, C, D form a parallelogram.

Working Rule:

Step I: Find AB, BC, CD, DA.

Step II: Show that the opposite sides are equal, for example, AB = DC and BC = DA, then the four points A, B, C, D form a parallelogram. Or, to show that the mid-point of the diagonal AC is equal to mid point of the diagonal BD.

Case VII: To show that the four points form a rectangle.

Step I: Find AB, BC, CD, DA.

Step II: Show that the opposite diagonals are equal. Or, to show that DABC is a right angled triangle i.e.,

$AB^2 + BC^2 = CA^2$

Hence, the quadrilateral is a Rectangle.

Case VIII: To show that the four points A, B, C, D form a rhombus.

Working Rule:

Step I: Find the sides AB, BC, CD and DA.

Step II: Show that all the four sides are equal *i.e.,* AB = BC = CD = DA.

Hence, the quadrilateral ABCD is a rhombus.

Case IX: To show that the four points A, B, C, D form a square.

Working Rule:

Step I: Find the sides AB, BC, CD, DA.

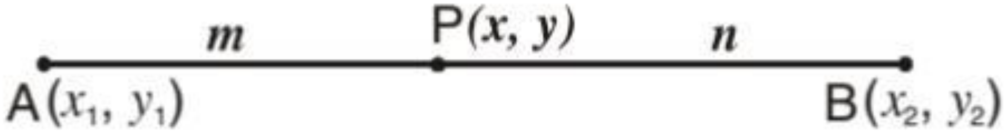
Step II: Show that all the sides are equal.

Step III: Also, show that the diagonals are equal. Or, to show that DABC is a right angled triangle. Hence, the quadrilateral is a square.

Section Formulae:

(i) The coordinates of the point P (x, y), dividing the line joining the points A (x_1, y_1) and B (x_2, y_2) in the ratio $m : n$ (Internally and Externally).

(a) If P divides AB internally, then



$$x = \frac{mx_2 + nx_1}{m+n}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

(*b*) If P divides AB externally, then

$$x = \frac{mx_2 - nx_1}{m-n}$$

$$y = \frac{my_2 - ny_1}{m-n}$$

(*ii*) The coordinates of the mid-point of line joining A (x_1, y_1) and B (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

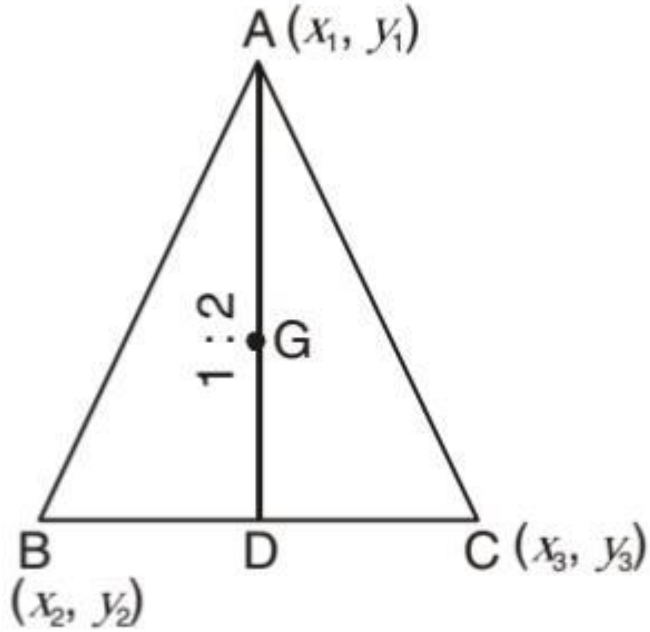
Remember:

(*i*) The line joining any vertex of a triangle to the mid-point of the opposite side is called a median of the triangle. Therefore, a triangle has three medians.

(*ii*) The point of intersection of the medians of a triangle is called the centroid of a triangle.

(*iii*) The centroid of a triangle divides each median in the ratio 2 : 1.

The coordinates of the centroid G of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is $\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$



Trapezium: The area of a Trapezium

$= \frac{1}{2}$ (sum of parallel sides) × (perpendicular distance between them)

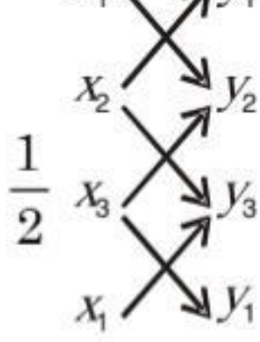
Area of a triangle: The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\,[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3)]$$

Remember: An easier method of writing down the area of a triangle quickly is as follows:

(*i*) Write down the coordinates of the vertices as shown repeating at the end the coordinates at the top.

(*ii*) Multiply the abscissa by the ordinate of next row and the ordinate by the abscissa of the next row and put the minus sign between them as in cross-multiplication; add these results and divide the sum by 2.



Thus 

$= x_1 y_2 - x_2 y_1 + \ldots$

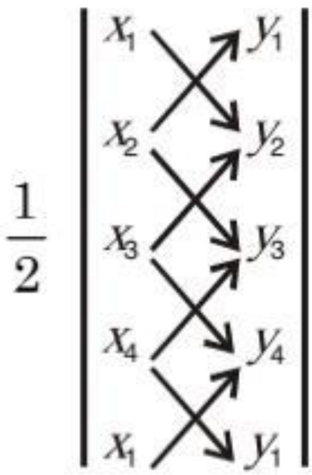
Condition for collinearity of three points: The condition that the three points lie in a straight line is

$$x_1 y_2 - x_2 y_1 + x_2 y_3 - x_3 y_2 + x_3 y_1 - x_1 y_3 = 0$$

[If three points are collinear, the area of the triangle formed by them is zero].

Area of the quadrilateral: The area of the quadrilateral whose vertices (x_1, y_1) (x_2, y_2) (x_3, y_3) and (x_4, y_4) is given by $[x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_4 - x_4y_3 + x_4y_1 - x_1y_4]$

Remember: The rule to write the area of a quadrilateral is the same as the rule for triangle.



Locus and its Equation: *Locus:* The path described by a point which moves under a given condition or conditions is called its locus.

Equation of a Locus: The equation of a locus is an equation in x and y which is satisfied by the coordinates of any and every point on the locus and by the coordinates of no other point.

Method to find the equation of the locus of a moving point: To find the locus of a point proceed as follows:

1. Let P (x, y) be any point on the locus.
2. Write down the given condition under which the point P moves.
3. Express the said condition in terms of x and y and simplify the results.
4. The simplified result so obtained is the required equation of the locus.
5. In a triangle ABC, if AD is the median drawn to BC, then
   $AB^2 + AC^2 = 2 (AD^2 + BD^2)$
6. A triangle is isosceles if any two of its medians are equal.
7. In a right angled triangle, the midpoint of the hypotenuse is equidistant from the vertices.
8. The diagonals in Rhombus, Square, Rectangle and Parallelogram bisect each other.
9. The figure obtained by joining the middle points of a quadrilateral in order is parallelogram.

10. If (x, y) are the coordinates of a point P w.r.t. the axes OX and OY and (x', y') its coordinates w.r.t. the axes OX′.
OY′ obtained by rotating the original axes OX, OY through an angle θ in anticlockwise direction, keeping the same origin, then

$$x = x' \cos\theta - y' \sin\theta$$
$$y = x' \sin\theta + y' \cos\theta$$

Thus, to find the equation of the curve referred to the turned axes, through angle θ, replace
x by $x' \cos\theta - y' \sin\theta$
andy by $x' \sin\theta + y' \cos\theta$
The relations connecting (x, y) and (x', y') can be written in the following scheme

	x'	y'
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

11. To remove the term of xy in the equation, $ax^2 + 2hxy + by^2 = 0$, the angle θ through which the axes must be turned (rotated) is given by

$$\theta = \frac{1}{2} \tan^{-1} \{2h/(a - b)\}.$$