CIRCLES AND FAMILY OF CIRCLES

Circle: A circle is a locus of a point which moves in a plane so that it is always at a constant distance from a fixed point in the plane. The fixed point is called the centre and the constant distance is called the radius of the circle.

Equation of a Circle: The equation of a circle whose centre is C (h, k) and radius r is

$$(x-h)^2 + (y-k)^2 = r^2$$

Particular Case: If a centre of the circle is origin (0, 0), then the above equation reduces to

$$(x-0)^2 + (y-0)^2 = r^2$$

or

$$x^2 + y^2 = r^2$$

General Equation of a Circle: $x^2 + y^2 + 2gx + 2fy + c = 0$ is the equation of a circle whose centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$

(A) To find the centre and radius of circle

(i) make the coefficient of x^2 and y^2 each

equal to 1 by dividing throughout (if necessary) by the coefficients of x^2 and y^2.

(ii) The coordinates of the centre are

$$\left(-\frac{1}{2}\text{coeff. of } x,\ \frac{-1}{2}\text{coeff. of } y\right)$$

(iii) Radius

$$= \sqrt{\left(-\frac{1}{2}\text{coeff. of } x\right)^2 + \left(-\frac{1}{2}\text{coeff. of } y\right)^2 - \text{constant term}}$$

(B) Nature of the circle

(i) If $g^2 + f^2 - c = 0$, the radius is zero and the circle becomes a point, which is known as a point circle.

(ii) If $g^2 + f^2 - c > 0$, the radius is real and hence, the cicle is a real circle.

(iii) If $g^2 + f^2 - c < 0$, the radius is imaginary and therefore, the circle is an imaginary circle and is called a virtual circle.

Condition of a Circle: Any equation satisfying the following conditions must be a circle.

(i) The coeff. of x^2 = coeff. of y^2

(ii) It contains no term involving the product xy.

(iii) It is an equation of second degree in x and y.

Diameter form of a Circle: The equation of a circle whose end diameter are (x_1, y_1) and (x_2, y_2) is given by $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Intersection of a line and a circle:

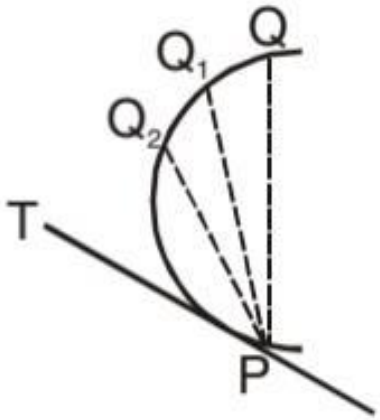
(i) The line $y = mx + c$ will intersect the circle $x^2 + y^2 = r^2$ in two distinct and real points if $c^2 < r^2 (1 + m^2)$

(ii) The line $y = mx + c$ will intersect the circle $x^2 + y^2 = r^2$ at one point if $c^2 = r^2 (1 + m^2)$

(iii) The line $y = mx + c$ will intersect the circle $x^2 + y^2 = r^2$ at two imaginary points if $c^2 > r^2 (1 + m^2)$

Tangent to a Circle:

Let P be any point on a curve and Q be any other point on the curve, join PQ and that PQ is called a chord of the Circle.



Let the point Q moves along the curve towards P taking the position Q_1, Q_2 ... and finally coincide with P such that the chord PQ takes the position PT. The limiting position PT of the line PQ as Q tends to P is called the tangent to the curve at P.

Equation of a tangent

(i) The equation of the tangent at point (x_1, y_1) on the circle is $x\,x_1 + y\,y_1 = r^2$

(ii) Equation of the tangent at the point (x_1, y_1) on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

Working Rule:

(i) Change x^2 into xx_1 and y^2 into yy_1

(ii) Change x into $\frac{x + x_1}{2}$ and y to $\frac{y + y_1}{2}$ and keep constant term same.

Normal: The normal at a point of a curve is the straight line through the point and perpendicular to the tangent at the point.

Equation of a Normal

(i) Equation of a normal at the point (x_1, y_1) to the circle $x^2 + y^2 = r^2$ is

$$xy_1 - yx_1 = 0$$

Note: (a) The normal at any point $(x_1 y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$x(y_1 + f) - y(x_1 + g) + (gy_1 - fx_1) = 0$$

A Circle and a line: A line $y = mx + c$ will be a tangent to the circle $x^2 + y^2 = a^2$ is $c = \pm a\sqrt{1 + m^2}$

Rule to find the equation of radical axis:

(i) Subtract the equations of two circles, after making coefficients of x^2 and y^2 unity in both the equations.

(ii) If the two circles intersects, then radical axis is the common chord of the two circles.

(iii) If the two circles touch each other, then radical axis is the common tangent.

Radical Centre of three circle: The point of intersection of radical axes of three circles taken in pairs is called the radical centre of the three circles.

Family of Circles: A collection of circles is called a family of circles.

The general equation of a circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ where g, f, c are three arbitrary constants (called parameters).We also know that three constants g, f, c can have fixed value only when three independent conditions satisfied by a circle are given.