

# INVERSE TRIGONOMETRIC FUNCTIONS

---

---

**Inverse Trigonometric Functions:** We know that the equation  $x = \sin y$  ... (i) means that  $y$  is an angle whose sine is  $x$  or  $x$  is the sine of  $y$ .

After solving equation (i) for  $y$ , we get

$$y = \sin^{-1} x \text{ or } y = \arcsin x$$

Similarly  $y = \cos^{-1} x$  if  $\cos y = x$  and  $y = \tan^{-1} x$  if  $x = \tan y$  etc. The functions  $\sin^{-1} x$ ,  $\cos^{-1} x$ ,  $\tan^{-1} x$ ,  $\sec^{-1} x$ ,  $\operatorname{cosec}^{-1} x$  and  $\cot^{-1} x$  are called inverse trigonometric functions. It is important to note that

(i)  $\sin y$  is a number whereas  $\sin^{-1} x$  is an angle

$$(ii) \sin^{-1} \neq (\sin x)^{-1} = \frac{1}{\sin x}$$

**Properties of inverse trigonometric functions**

*I. Some angle can be expressed by different inverse trigonometric functions.*

We know that

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \Rightarrow 60^\circ = \sin^{-1} \left( \frac{\sqrt{3}}{2} \right)$$

$$\cos 60^\circ = \frac{1}{2} \Rightarrow 60^\circ = \cos^{-1} \left( \frac{1}{2} \right)$$

$$\tan 60^\circ = \sqrt{3} \Rightarrow 60^\circ = \tan^{-1} (\sqrt{3})$$

$$\text{Here, } 60^\circ = \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) = \cos^{-1} \left( \frac{1}{2} \right) = \tan^{-1} \sqrt{3} = \dots$$

**II. Inverse property:** We know that

$$x = \cos \theta, \text{ then, } \theta = \cos^{-1} x$$

$$\therefore \theta = \cos^{-1} (\cos \theta) (\because x = \cos \theta)$$

**III. Principle of reciprocity:** Following reciprocal relation exists between inverse trigonometric functions,

$$\operatorname{cosec}^{-1} \frac{1}{x} = \sin^{-1} x$$

$$\sec^{-1} \frac{1}{x} = \cos^{-1} x$$

$$\cot^{-1} \frac{1}{x} = \tan^{-1} x$$

**IV.** Inverse trigonometric functions are odd functions within the principal values

$$(i) \sin^{-1}(-x) = -\sin^{-1} x$$

$$(ii) \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$$

$$(iii) \tan^{-1}(-x) = -\tan^{-1} x$$

**V. Some fundamental formulae**

$$(i) \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$(ii) \tan^{-1} x + \cot^{-1} x = \pi/2$$

$$(iii) \operatorname{cosec}^{-1} x + \sec^{-1} x = \pi/2$$

$$(iv) \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left( \frac{x+y}{1-xy} \right)$$

$$(v) \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left( \frac{x-y}{1+xy} \right)$$

$$(vi) 2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2}$$

$$= \cos^{-1} \frac{1-x^2}{1+x^2}$$

$$= \tan^{-1} \frac{2x}{1-x^2}$$

**VI. To express one inverse trigonometric function in terms of other ones:**

$$(i) \sin^{-1} x = \cos^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$$

$$(ii) \cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{\sqrt{1-x^2}}{x}$$

$$(iii) \operatorname{cosec}^{-1} \frac{1}{x} = \sin^{-1} \frac{1}{\sqrt{1-x^2}} \\ = \cot^{-1} \frac{\sqrt{1-x^2}}{x}$$

**VII. Some Important Deductions:**

$$(a) \sin (\sin^{-1} x) = x ; -1 \leq x \leq 1$$

$$\tan (\tan^{-1} x) = x ; -\infty < x < \infty$$

$$\cos (\cos^{-1} x) = x ; -1 \leq x \leq 1$$

$$\cot (\cot^{-1} x) = x ; -\infty < x < \infty$$

$$\sec (\sec^{-1} x) = x ; x \leq -1 \text{ or } x \geq 1$$

$$\operatorname{cosec} (\operatorname{cosec}^{-1} x) = x ; x \leq -1 \text{ or } x \geq 1$$

$$(b) \sin^{-1} (\sin \theta) = \theta ; -\pi/2 \leq \theta \leq \pi/2$$

$$\cos^{-1} (\cos \theta) = \theta ; 0 \leq \theta \leq \pi$$

$$\tan^{-1}(\tan \theta) = \theta ; -\pi/2 < \theta < \pi/2$$

$$\cot^{-1}(\cot \theta) = \theta ; 0 < \theta < \pi$$

$$\sec^{-1}(\sec \theta) = \theta ; 0 \leq \theta < \pi, \theta \neq \pi/2$$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} \theta) = \theta ; -\pi/2 \leq \theta \leq \pi/2, \theta \neq 0$$

$$(c) \sin^{-1}(-x) = -\sin^{-1} x ; -1 \leq x \leq 1$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x ; -1 \leq x \leq 1$$

$$\tan^{-1}(-x) = -\tan^{-1} x ; x \in \mathbb{R}$$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x ; x \in \mathbb{R}.$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x ; x \geq 1 \text{ or } x \leq -1$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x ; x \geq 1 \text{ or } x \leq -1.$$

