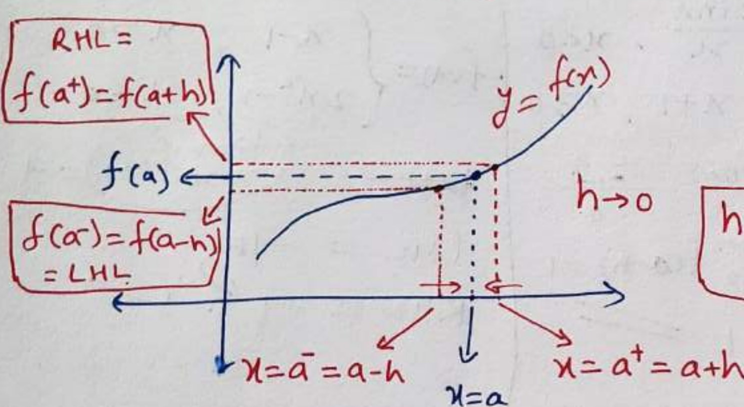
Class 12 Maths [Continuity and Differentiability]

In this Video

- Limit (सीमा)
- Continuity (सततता)

Limit of a function $f(x)$ at $x = a$ = [Value of function in the neighbourhood of $x = a$.]
($x = a$ के आस पास)



$h \to 0$

h = very very small Positive real number

$$\lim_{x \to a} f(x) \rightarrow \text{Right Hand limit at } x = a = RHL = \lim_{h \to 0} f(a+h) = f(a^+)$$

$$\lim_{x \to a} f(x) \rightarrow \text{Left hand limit at } x = a = LHL = \lim_{h \to 0} f(a-h) = f(a^-)$$

{limit of a function at $x = a$}

Limit exists at $x = a$ if $LHL = RHL$ = finite real no.

How to Deal with Questions of 'Limit'?

Find $\lim_{x \to a} f(x)$

$\lim_{x \to a} f(x)$ से ही काम चल जाता है

Normal Functions

$\lim_{x \to 1} x^2+5$

(Direct put)

$= 1^2+5$

$= 6$

$\lim_{x \to 3} \frac{x^2-9}{x-3}$

$= \frac{9-9}{3-3} = \frac{0}{0}$

$= \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$= \lim_{x \to 3} (x+3)$

$= 6$

LHL, RHL अलग-अलग निकालना पड़ता है

Piecewise Functions $[x], |x|, sgn, \{x\}$

$f(x) = \begin{cases} \frac{\sin x}{x}, & x<0 \\ x+1, & x \geq 0 \end{cases}$

$\lim_{x \to 0} f(x)$

$LHL = \lim_{h \to 0} f(0-h) = 1$

$RHL = 1$

$f(x) = \begin{cases} x-1, & x>0 \\ 2x^2-1, & x \leq 0 \end{cases}$

$\lim_{x \to 0} f(x) = -1$

$LHL = -1$

$RHL = -1$

Indeterminate Forms

$\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, 0.\infty, 1^{\infty}, 0^0, \infty^0$

↳ use Factorisation

↳ use standard Forms

- $\lim_{x \to a} \frac{x^n-a^n}{x-a} = na^{n-1}$
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{\tan x}{x} = 1$
- etc.

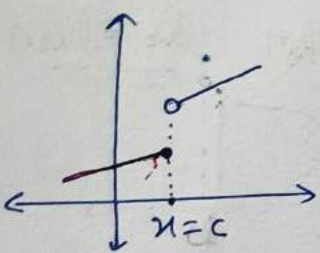
↳ use L-Hospital Rule (only for $\frac{0}{0}, \frac{\infty}{\infty}$)

Continuity (सतत्ता)

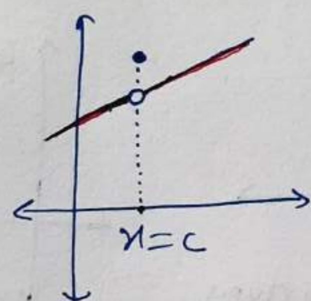
(I) Continuity at a Point ($x=c$)

Continuity at a Point ($x=c$) → Continuous (सतत)
Continuity at a Point ($x=c$) → Discontinuous (असतत)

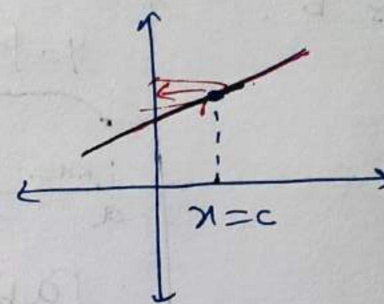
—o exclude (हटा)
—• include (शामिल)



Discontinuous



Discontinuous



Continuous

Continuous at Fixed Point → if we can draw the graph of the function around that point without lifting the pen from the plane of the paper.

Mathematically LHL = $f(c)$ = RHL

$x=c$ पर

Continuous

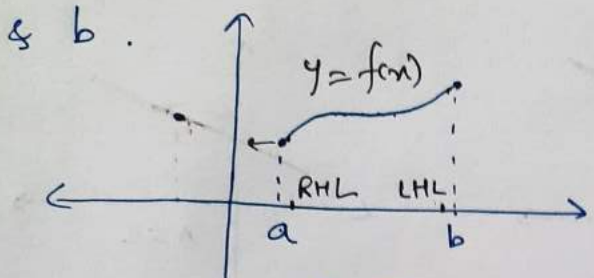
$$\lim_{h\to 0} f(c-h) = f(c) = \lim_{h\to 0} f(c+h)$$

$$\boxed{\lim_{x\to c} f(x) = f(c)}$$

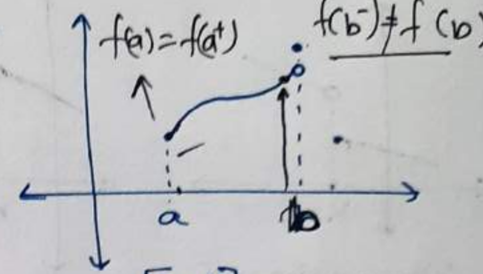
↑ limit ↑ Exact.

(II) Continuity In an Interval $\rightarrow [a, b]$ $\quad$ (a, b)

f is said to be continuous in interval $[a, b]$, if it is continuous at every point in $[a, b]$ including a & b.



$[a,b]$ Continuous in $[a, b]$



$[a,b]$ Discontinuous

Continuity at 'a' means

$$f(a) = RHL$$
$$f(a) = f(a^+)$$

Continuity at 'b' means

$$LHL = f(b)$$
$$f(b^-) = f(b)$$

A real function f is said to be continuous if it is continuous at every point in the domain of f.

" Draw graph without lifting pen in the domain "

$f(x) = 2x+3$

$f(x) = x^2 - x^3 + 1$

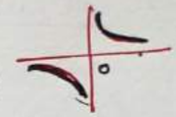
$f(x) = |x|$ Continuous

$f(x) = K$ $x \in R$

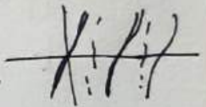
$f(x) = x$

$f(x) = \sin x$

$f(x) = \frac{1}{x}$, $x \neq 0$



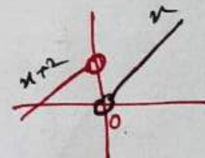
$f(x) = \tan x$, $x \neq (2n+1)\frac{\pi}{2}$



$f(x) = \frac{1}{\sin x}$, $x \neq n\pi$

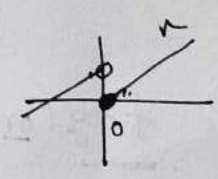
$$f(x) = \begin{cases} x+2, & x<0 \\ x, & x>0 \end{cases}$$

Continuous



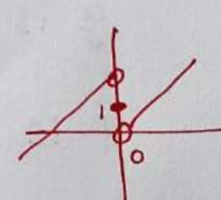
$$f(x) = \begin{cases} x+2, & x<0 \\ x, & x\geq 0 \end{cases}$$

Discont.



$$f(x) = \begin{cases} x+2, & x<0 \\ 1, & x=0 \\ x, & x>0 \end{cases}$$

Discont



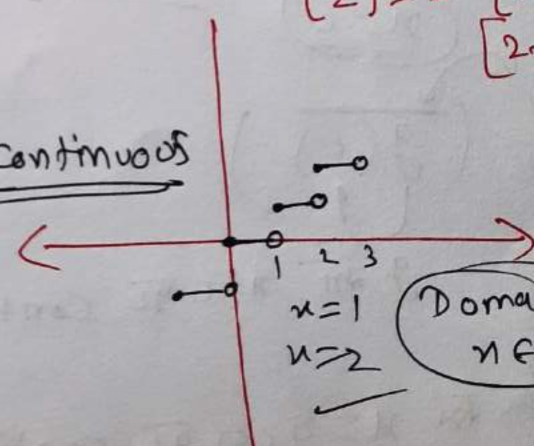
$f(x) = [x]$ = greatest integer function.

$[2]=2$ $[2.1]=2$

$[2.9]=2$

$[3]=3$

Discontinuous



$x=1$

$x=2$

Domain $x \in R$

Important Points for Continuity

① Suppose f & g be two functions continuous at a real number $x = c$.

→ $f+g$ is continuous at $x=c$ $\quad (f(x)+g(x))$

→ $f-g$ is continuous at $x=c$ $\quad f(x)-g(x)$

→ $f \cdot g$ is continuous at $x=c$ $\quad f(x)\cdot g(x)$

→ $\frac{f}{g}$, $g(c) \neq 0$ is continuous at $x=c$. $\quad \frac{f(x)}{g(x)}$

② $f \circ g$ (or $f(g(x))$) is continuous at $x=c$ if

g is continuous at $x=c$ and

f is continuous at $x = g(c)$.

(Composite Functions)

f in f

$f(x)$ input

$\sin(\cos x)$

$f(g(x))$ Continuity

$x=c$

$f(g(c))$

$\cancel{f(x)}$ $\quad \cancel{f(c)}$

g का $x=c$ पर continuous होना जरूरी है

f का $x=g(c)$ पर continuous होना जरूरी है

$f(2)=5$, $g(1)=2$ $\quad | \; f(g(1)) = f(2) = 5$

Class 12 maths [Continuity & Differentiability]

Exercise - 5.1 Chapter 5

Q.1 Prove that the function $f(x) = 5x-3$ is continuous at $x=0$, at $x=-3$ and at $x=5$.

Ans. $f(x) = 5x-3$

$\boxed{x=0}$

$f(0) = -3$ (Exact)

$\lim_{x\to 0} f(x) = \lim_{x\to 0}(5x-3)$

$= 5(0)-3$

$= -3$

$\therefore f(0) = \lim_{x\to 0} f(x)$

$\therefore$ at $x=0$, f is continuous.

$\boxed{x=-3}$

$f(-3) = 5(-3)-3$

$= -18$

$\lim_{x\to -3} f(x) = \lim_{x\to -3}(5x-3)$

$= 5(-3)-3$

$= -18$

$f(-3) = \lim_{x\to -3} f(x)$

$\therefore$ at $x=-3$, f is continuous.

$\boxed{x=5}$

Q.2 $f(x) = 2x^2-1$ at $x=3$

$f(3) = 2(3)^2-1 = 17$

$\lim_{x\to 3} f(x) = \lim_{x\to 3}(2x^2-1) = 17$

$\boxed{f(3) = \lim_{x\to 3} f(x)}$ Continuous at $x=3$

Q.3 Examine the Continuity.

Continuous Function
↓
Continuous in their Domain

(a) $f(x) = x-5$

Domain $= R \leftarrow$ Real no.

$x \in R$

let $x = c \in R$

$f(c) = c-5$, $\lim_{x\to c} f(x) = \lim_{x\to c} (x-5)$

$= c-5$

$$\boxed{f(c) = \lim_{x\to c} f(x) \,,\ c \in R}$$

$\therefore$ $f(x)$ is continuous fn.

(b) $f(x) = \dfrac{1}{x-5}$, $x \neq 5$

Domain $x-5 \neq 0$

$\Rightarrow x \neq 5$

$\hookrightarrow x \in R-\{5\}$

let $c \in R-\{5\}$

$\boxed{f(c) = \dfrac{1}{c-5}}$, $c \in R-\{5\}$

$\lim_{x\to c} f(x) = \lim_{x\to c} \left(\dfrac{1}{x-5}\right)$

$= \dfrac{1}{c-5}$

$$\boxed{f(c) = \lim_{x\to c} f(x), \ c \in \underbrace{R-\{5\}}_{\text{Domain}}}$$

$\therefore$ $f(x)$ is continuous fun.

(C) $f(x) = \dfrac{x^2-25}{x+5}$, $x \neq -5$

Domain $x+5 \neq 0$

$\Rightarrow x \neq -5$

$x \in \underbrace{R - \{-5\}}_{\text{Domain}}$

$c \in \underline{R - \{-5\}}$

$f(c) = \dfrac{c^2-25}{c+5}$ ✓

$\lim_{x\to c} f(x) = \lim_{x\to c} \dfrac{x^2-25}{x+5}$

$= \dfrac{c^2-25}{c+5}$ ✓

$\boxed{f(c) = \lim_{x\to c} f(x)}$, $c \in \underline{R - \{-5\}}$

Continuous fn.

(D) $f(x) = |x-5|$

Domain $= R$

Let $\underline{x = c \in R}$

$f(c) = |c-5|$ ✓

$\lim_{x\to c} f(x) = \lim_{x\to c} |x-5|$

$= |c-5|$ ✓

$\boxed{f(c) = \lim_{x\to c} f(x)}$, $c \in R$

Continuous fn.

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(4) $\underline{f(x) = x^n}$ at $\underline{x = n}$, $\underline{n \in \text{Positive integer}} = \{1, 2, 3 \ldots\}$

$n \in N \longleftrightarrow$

$f(n) = n^n$ ✓

$\lim_{x\to n} f(x) = \lim_{x\to n} x^n = n^n$ ✓

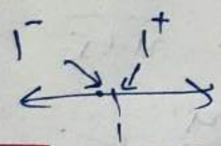
$\boxed{f(n) = \lim_{x\to n} f(x)}$, $n \in N$

Continuous fn.

Q.5 Is the function f defined by

$$f(x) = \begin{cases} x, & x \le 1 \\ 5, & x > 1 \end{cases}$$; continuous at $x=0$? at $x=1$? at $x=2$?

$x=1$ critical point



$\boxed{x=1}$

$\lim_{x\to 1} f(x) \neq f(1)$

LHL $\neq$ RHL $\neq$ $f(1)$

$f(1^-) = f(1^+) = f(1)$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

exact

x

$\Rightarrow \lim_{x\to 1^-}[x] = \lim_{x\to 1^+}[5] = [1]$

$\Rightarrow \underline{1} \neq \underline{5} \neq \underline{1}$

LHL RHL exact.

not continuous at $x=1$

$x=0$ के लिए

$f(x)=x$

$\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow 0 = 0$

Continuous

$x=2$ के लिए

$f(x)=5$

$\lim_{x\to 2} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2} 5 = 5$

$\Rightarrow 5=5$

Continuous

Class 12 Maths [Continuity & Differentiability]

Exercise-5.1 Chapter - 5

Find all points of discontinuity of f, where f is defined by —

Q.6 $f(x) = \begin{cases} 2x+3, & \text{if } x \le 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$ Piecewise Function

($x=2$ Critical point)

$x=2$ पर Continuous

$\text{LHL} = \text{RHL} = f(2)$

limit: $\lim_{x\to 2} f(x)$ — Exact: $f(2)$

LHL: 2^- RHL: 2^+ (2^-, 2, 2^+)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2) \quad (x=2 \text{ exact})$$

$$\Rightarrow \lim_{x\to 2^-} (2x+3) = \lim_{x\to 2^+} (2x-3) = 2(2)+3$$

$$\Rightarrow 4+3 \neq 4-3 \neq 4+3$$

$$\Rightarrow 7 \neq 1 \neq 7$$

$\therefore$ Point of Discontinuity is $x=2$.

$2x+3$ $2x-3$

2

Dis.

Always continuous. $x<2$

Always continuous $x>2$

$\boxed{Q.7}$ $f(x) = \begin{cases} |x|+3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$

we will check continuity at $x=-3$ & $x=3$ only. and at rest of values of x, the function $f(x)$ will be continuous.

$\boxed{x=-3}$ LHL = RHL = $f(-3)$ $x=-3$

$$\lim_{x\to -3^-} (|x|+3) = \lim_{x\to -3^+} (-2x) = |-3|+3$$

$$\Rightarrow |-3|+3 = -2(-3) = 3+3$$

$$\Rightarrow 6 = 6 = 6$$

at $x=-3$ Continuous

$\boxed{x=3}$ LHL $\neq$ RHL = $f(3)$ exact. $x=3$

$$\Rightarrow \lim_{x\to 3^-} (-2x) = \lim_{x\to 3^+} (6x+2) = (6(3)+2)$$

$$\Rightarrow -2(+3) = 6(3)+2 = 20$$

$$\Rightarrow -6 \neq 20 = 20$$

at $x=3$ discontinuous.

Point of Discontinuity = $x=3$

$\boxed{Q.8}$ $f(x)=\begin{cases}\dfrac{|x|}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$

we have to check at $x=0$.

LHL = RHL = $f(0)$

Exact.

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x) = 0$

$\Rightarrow \lim\limits_{x\to 0^-} \dfrac{|x|}{x} = \lim\limits_{x\to 0^+} \dfrac{|x|}{x} = 0$

$\Rightarrow \dfrac{-x}{x} = \dfrac{+x}{x} = 0$

$\ominus$ 0^- 0^+ $\oplus$

0

$\Rightarrow -1 \neq +1 \neq 0$

LHL $\neq$ RHL $\neq f(0)$

$x=0$ पर Discontinuous

$\boxed{Q.9}$ $f(x)=\begin{cases}\dfrac{x}{|x|}, & \text{if } x<0\\ -1, & \text{if } x\geq 0\end{cases}$

we have to check at $x=0$

LHL = RHL = $f(0)$

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x) = (-1)$

Exact.

$\Rightarrow \lim\limits_{x\to 0^-} \dfrac{x}{|x|} = \lim\limits_{x\to 0^+} (-1) = (-1)$

$\Rightarrow \dfrac{x}{-x} = -1 = -1$

$\Rightarrow -1 = -1 = -1$

LHL = RHL = $f(0)$

Continuous at $x=0$

$x\in R$

no Point of Discont.

$\boxed{Q.10}$ $f(x)=\begin{cases}x+1, & x\geq 1\\ x^2+1, & x<1\end{cases}$

$x=1$ — check

$x\in R-\{1\}\rightarrow$ cont.

LHL = RHL = $f(1)$

$\Rightarrow \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) = (1+1)$

$\Rightarrow 1^2+1 = 1+1 = 2$

$\Rightarrow 2 = 2 = 2$

Conti. $\rightarrow$ every where

$\boxed{Q.11}$ $f(x)=\begin{cases} x^3-3, & \text{if } x\le 2 \\ x^2+1, & \text{if } x>2 \end{cases}$

Check at only $x=2$

$LHL = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^3-3$

$= 5$

$RHL = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2+1$

$= 5$

$f(2) = 2^3-3 = 5$

Continuous at $x=2$

Continuous every where

$\boxed{Q.12}$

$f(x)=\begin{cases} x^{10}-1, & x\le 1 \\ x^2, & x>1 \end{cases}$

Check at only $x=1$

$LHL = \lim_{x\to 1^-} f(x)$

$= \lim_{x\to 1^-} (x^{10}-1)$

$= 1-1 = 0$

$RHL = \lim_{x\to 1^+} (x^2)$

$= 1$

$f(1) = 1^{10}-1 = 0$

$\underset{0}{LHL} = \underset{0}{f(1)} \neq \underset{1}{RHL}$

Point of Discontinuity

$x=1$

~~Q.13~~ $\cancel{f(x)=\begin{cases} x+8, & \text{if } x\le 5 \\ x-5, & \end{cases}}$

$\boxed{Q.13}$ $f(x) = \begin{cases} x+5, & x \le 1 \\ x-5, & x > 1 \end{cases}$

exact

$f(1) = x+5$
$= 1+5 = 6$

$-\infty$ ∞ 1

Check at only $x = 1$

$LHL = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x+5) = 1+5 = 6$

$RHL = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x-5) = 1-5 = -4$

$f(1) = LHL \ne RHL$ Discontinous at $x = 1$

$\boxed{Q.14}$ $f(x) = \begin{cases} 3, & \text{if } 0 \le x \le 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \le x \le 10 \end{cases}$

f का Domain

0 1 3 10

Closed $\rightarrow [0, 10]$

Check at $x = 0, 1, 3, 10$

$\boxed{x=0}$ $f(0) = RHL$ ($x \to 0^+$)

$\Rightarrow 3 = 3$ Continuous

$\boxed{x=1}$ $f(1) = \underset{1^-}{LHL} \ne \underset{1^+}{RHL}$

$\Rightarrow 3 = 3 \ne 4$ (Discontinuos)

$\boxed{x=3} \longrightarrow$ Discont.

$\boxed{x=10} \rightarrow \underset{10^-}{LHL} = f(10)$

$\Rightarrow 5 = 5$ Continuous

Discontinuous

Q.15

$$f(x)=\begin{cases}2x, & x<0\\ 0, & 0\le x\le 1\\ 4x, & x>1\end{cases}$$

Check at $x=0, 1$

$\boxed{x=0}$ $LHL = RHL = f(0)$

0^- 0^+

$\Rightarrow 2(0) = (0) = 0$

$\Rightarrow 0 = 0 = 0$

Continuous.

$\boxed{x=1}$ $LHL \neq RHL \neq f(1)$

$x=1^-$ $x=1^+$ $x=1$

$\Rightarrow 0 \neq 4(1) \neq 0$

$0 \neq 4 \neq 0$

Discontinuous.

Q.16

$$f(x)=\begin{cases}-2, & x\le -1\\ 2x, & -1<x\le 1\\ 2, & x>1\end{cases}$$

Check at $x=-1, 1$

$\boxed{x=-1}$ $LHL = RHL = f(-1)$

$\Rightarrow (-2) = 2(-1) = -2$

$\Rightarrow -2 = -2 = -2$

Continuous.

$\boxed{x=1}$ $LHL = RHL = f(1)$

$2(1) = 2 = 2(1)$

$\Rightarrow 2 = 2 = 2$

Continuous

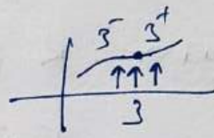
Continuous.

Exercise 5.1 Chapter - 5

Q.17 Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$$ is continuous at $x=3$.

Ans. Given, Continuous at $x=3$



$$LHL = f(3) = RHL$$

$$\Rightarrow \lim_{x\to 3^-} f(x) = f(3) = \lim_{x\to 3^+} f(x)$$

$$\lim_{x\to 3^-} (ax+1) = a(3)+1 = \lim_{x\to 3^+} (bx+3)$$

$$\Rightarrow 3a+1 = 3a+1 = 3b+3 \Rightarrow 3a+1 = 3b+3$$

$$\Rightarrow \boxed{3a = 3b+2}$$

Q.18 For what value of λ, $f(x) = \begin{cases} \lambda(x^2-2x), & x \le 0 \\ 4x+1, & x > 0 \end{cases}$ is continuous at $x=0$? What about continuity at $x=1$?

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$x=0$

$$\Rightarrow \lim_{x\to 0^-} \lambda(x^2-2x) = \lim_{x\to 0^+} (4x+1) = \lambda(x^2-2x)$$

$$\Rightarrow 0 \ne 1 \ne 0$$ no λ

$x=1$ के लिए.

$f(x) = 4x+1$

$LHL = RHL = f(1)$

Continuous.

$\lambda \in R$

Q.19 Show that the function $g(x) = x - [x]$ is discontinuous at at all integral points.

($[x] \rightarrow$ greatest integer function)

Continuous ✓

$LHL = f(c) = RHL$

Domain = R ($x \in R$)

$g(x) = x - [x]$

Let integer $I \in R$ ($x = I$)

$LHL = \lim_{x \to I^-} g(x)$

$= \lim_{x \to I^-} x - [x]$

$= I^- - [I^-]$

$= I - (I - 1)$

$= \cancel{I} - \cancel{I} + 1 = 1$

$RHL = \lim_{x \to I^+} g(x)$

$= \lim_{x \to I^+} (x - [x])$

$= I^+ - [I^+]$

$= I - I = 0$

$g(I) = x - [x] = I - [I]$

$= I - I = 0$

$f(x) = [x]$ (Just छोटा & बराबर)

I ← x → I+1

1 2 3 4

5.5 5.4

$[1] = 1$

$[1.1] = 1$

$[1.999] = 1$ — LHL

$[2] = 2$ — f(2) ≠

$[2.1] = 2$ — RHL

$[2.2] = 2$

$[2.999] = 2$ — LHL (3^-)

$[3.0] = 3$ — f(3) (I)

$[3.1] = 3$ — RHL (I^+)

$[3.2] = 3$

⋮

$[3.9] = 3$

$[4] = 4$

4

$LHL = 1$, $RHL = 0$, $g(I) = 0$

Discontinuous

Q.20 Is the function defined by $\underbrace{f(x) = x^2 - \sin x + 5}$ continuous at $x = \pi$?

Ans.

$$LHL = RHL = f(\pi)$$

$$\lim_{x\to\pi} f(x) = f(\pi)$$

limit, exact

$$\lim_{x\to\pi} f(x) = \lim_{x\to\pi}(x^2 - \sin x + 5)$$

$$= \pi^2 - \sin\pi + 5$$

$$= \pi^2 + 5$$

$$f(\pi) = \pi^2 - \sin\pi + 5$$

$$= \pi^2 - 0 + 5$$

$$= \pi^2 + 5$$

Yes

Q.21 Discuss the Continuity

(a) $f(x) = \sin x + \cos x$

(b) $f(x) = \sin x - \cos x$

(c) $f(x) = \sin x \cdot \cos x$

We know that $\sin x$, $\cos x$ are continuous functions in their Domain ($x \in R$).

$\therefore$ $\sin x + \cos x$, $\sin x - \cos x$, $\sin x \cdot \cos x$ are also continuous function.

Q.22 cosine, cosecant, secant, cotangent functions

	cosine	cosecant	secant	cotangent
	$\cos x$	$\operatorname{cosec} x$	$\sec x$	$\cot x$
Domain	R	$R - \{n\pi\}$	$R - \{(2n+1)\frac{\pi}{2}\}$	$R - \{n\pi\}$

$x = c \in R - \{n\pi\}$

$f(c) = \operatorname{cosec} c$

$\lim_{x\to c} f(x) = \operatorname{cosec} c$

Contin—

V. in

Q.23 Find all points of discontinuity of f, where $f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0 \\ x+1, & x \geq 0 \end{cases}$

Ans. we have to check continuity at only $x = 0$.

$0^- \quad 0 \quad 0^+$

$LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin x}{x}$

Standard Form

Class 11

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$= 1$.

$RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x+1) = 1$

$f(0) = (0)+1 = 1$

$x=0$

$LHL = RHL = f(0)$

Continuous.

Continuous everywhere

$\boxed{Q.24}$ Determine if f defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

is a continuous function?

$\boxed{x=0}$ पर करो check.

$\boxed{LHL = RHL = f(0)}$??

$x \to 0^-$, $x \to 0^+$

$\lim_{x \to 0} f(x)$?

$f(0) = 0$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(x^2 \sin\frac{1}{x}\right)$$

$\sin\frac{1}{x} \in [-1, 1]$

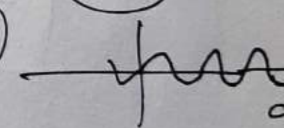
$(0)^2 \cdot \sin\left(\frac{1}{0}\right)$

$0 \times \boxed{\sin(\infty)}$

$0 \times [-1, 1] = 0$

$$= (0)^2 \times \begin{pmatrix} \text{Quantity} \\ \text{in } [-1,1] \end{pmatrix}$$

$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \sin, \frac{\sin x}{x}$



$$= 0$$

$$\therefore \lim_{x \to 0} f(x) = f(0) = 0$$

$\therefore f(x)$ is a continuous function.

$\boxed{Q.25}$ Examine the continuity of f, where f is defined by $f(x) = \begin{cases} \sin x - \cos x, & x \neq 0 \\ -1, & x = 0 \end{cases}$

~~lim~~ Check continuity at $x = 0$ only.

{ at rest of the values of x, $f(x)$ will be continuous. }

LHL = RHL = $f(0)$

$\lim_{x \to 0} f(x) = f(0)$

$f(0) = -1$

$\lim_{x \to 0} f(x)$

$= \lim_{x \to 0} (\sin x - \cos x)$

$= \sin 0 - \cos 0$

$= 0 - 1$

$= -1$

$\Rightarrow \boxed{-1 = -1}$

Continuous at $x = 0$

Continuous ~~at~~ everywhere

Class 12 Maths [Continuity & Differentiability]

Exercise 5.1 Chapter 5

Find the value of 'K' so that 'f' is continuous at the indicated point.

Q.26 $f(x) = \begin{cases} \dfrac{K\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ at $x = \frac{\pi}{2}$

Continuous

$LHL = RHL = f(\frac{\pi}{2})$ $\quad f(\frac{\pi}{2}) = 3$

$f(\frac{\pi}{2}^-) = f(\frac{\pi}{2}^+) = f(\frac{\pi}{2})$

$\underbrace{\lim_{x \to \frac{\pi}{2}} f(x)} = f(\frac{\pi}{2})$

$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$

$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{K\cos x}{\pi - 2x} = 3$

$\dfrac{K\cdot\cos(\frac{\pi}{2}) \to 0}{\pi - 2\cdot(\frac{\pi}{2}) \to 0}$ ($\frac{0}{0}$ form) $\to$ indeterminate Form

$x \to \frac{\pi}{2}$

$x = \frac{\pi}{2} + h$ $\quad \lim_{h \to 0}$ $\quad \lim_{x \to \frac{\pi}{2}}$ $\quad \lim (\cancel{\frac{\pi}{2}} + h) \to \cancel{\frac{\pi}{2}}$ $\quad \lim_{h \to 0}$

Substitution

$\Rightarrow \lim_{h \to 0} \frac{K\cdot\cos(\frac{\pi}{2} + h)}{\pi - 2(\frac{\pi}{2} + h)} = 3 \Rightarrow \boxed{\lim_{h \to 0} \frac{-K\sin h}{\cancel{\pi} - \cancel{\pi} - 2h} = 3}$

$$\Rightarrow \lim_{h\to 0} \frac{\cancel{+}K \sin h}{\cancel{+}2h} = 3$$

$\to 1$

Standard Form

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

$$\Rightarrow \frac{K}{2} = 3$$

$$\Rightarrow \boxed{K = 6}$$

Q.27 $f(x) = \begin{cases} Kx^2, & x \le 2 \\ 3, & x > 2 \end{cases}$ at $x = 2$.

Continuous

$$LHL = f(2) = RHL$$

$$\Rightarrow f(2^-) = f(2) = f(2^+)$$

$$\lim_{x\to 2^-} Kx^2 = K(2)^2 = \lim_{x\to 2^+} (3)$$

$$\Rightarrow K(2)^2 = K(2)^2 = 3$$

$$\Rightarrow 4K = 3 \Rightarrow K = 3/4$$

Q.28 $f(x) = \begin{cases} Kx+1, & x \le \pi \\ \cos x, & x > \pi \end{cases}$ at $x = \pi$.

$$LHL = f(\pi) = RHL$$

$$\Rightarrow K(\pi)+1 = K(\pi)+1 = \cos(\pi)$$

$$\Rightarrow K\pi + 1 = -1$$

$$\Rightarrow K\pi = -2$$

$$K = \frac{-2}{\pi}$$

Q.29 $f(x) = \begin{cases} Kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ at $x=5$ Continuous

$LHL = f(5) = RHL$

$\Rightarrow (K(5)+1) = K(5)+1 = (3(5)-5)$

$\Rightarrow 5K+1 = 10 \Rightarrow K = \frac{9}{5}$

Q.30 Find the value of a & b such that the function $f(x) = \begin{cases} 5, & x \le 2 \\ ax+b, & 2 < x < 10 \\ 21, & x \ge 10 \end{cases}$ is a continuous function.

$x=2$

$LHL = RHL = f(2)$

$\Rightarrow f(2^-) = f(2^+) = f(2)$

$\Rightarrow 5 = a(2)+b = 5$

$\Rightarrow \boxed{2a+b=5}$ — (1)

$x=10$

$LHL = RHL = f(10)$

$f(10^-) = f(10^+) = f(10)$

$\Rightarrow a(10)+b = 21 = 21$

$\Rightarrow \boxed{10.a+b = 21}$ — (2)

eqn (2) − eqn (1)

$10.a + \cancel{b} = 21$

$2a + \cancel{b} = 5$

$- \quad - \quad -$

$8a = 16$

$a = 2$

$2a+b = 5$

$2(2)+b = 5$

$\Rightarrow b = 1$

Q.31 Show that $f(x) = \cos(x^2)$ is a continuous function.

Concept - Composite Function (function in function)

$f(g(x))$ $\quad f(x),\ g(x)$

$g(x) \rightarrow$ Continuous $x \in R$ $\quad f(x) \rightarrow$ Continuous $x \in R$

$f(g(x))$ $\quad$ f of g of x $\rightarrow f \circ g(x)$

$f(g(x))$ या $g(f(x)) \rightarrow$ continuous. $x \in R$

$f(x) = \cos(x^2)$

Let $g(x) = x^2 \rightarrow$ Continuous

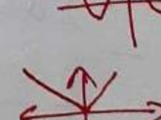
$h(x) = \cos x \rightarrow$ Continuous

$\Rightarrow h(g(x)) = \cos(x^2) \rightarrow$ Continuous

$\therefore f(x) = \cos(x^2) \rightarrow$ Continuous Fn.

Q.32 Show that $f(x) = |\cos x|$ is a continuous Fn.

Let $g(x) = \cos x \rightarrow$ continuous 

$h(x) = |x| \rightarrow$ Continuous 

$\Rightarrow h(g(x)) = |\cos x| \rightarrow$ Continuous Fn.

$f(x)$

Q.33 Examine that $\sin|x|$ is a continuous function.

$f(x) = \sin|x|$

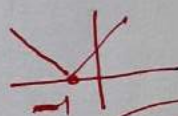
let $g(x) = |x| \rightarrow$ continuous

$h(x) = \sin x \rightarrow$ continuous

$h(g(x)) = \sin|x| \rightarrow$ continuous

Q.34 Find all the points of discontinuity of f defined by $f(x) = |x| - |x+1|$.

Ans. $g(x) = |x| \longrightarrow$ Continuous  $x \in R$

$h(x) = |x+1| \rightarrow$ Continuous.  $x \in R$

$g(x) - h(x) = |x| - |x+1|$

Continuous

$f(x) = |x| - |x+1|$ is also continuous for $x \in R$.

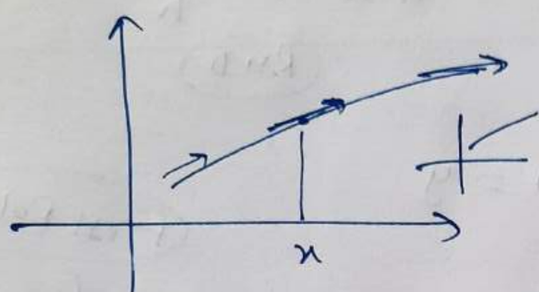
No points of discontinuity

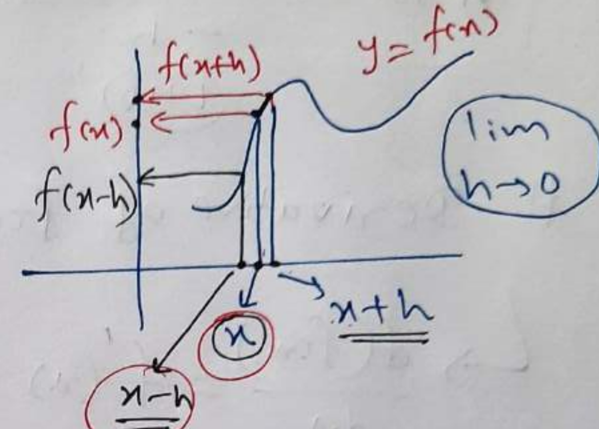
Class 12 Maths. [Continuity & Differentiability]

Derivatives (अवकलन)

→ Rate of change (Daily life)

→ (Graph) Slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$





final - initial

Derivative = Rate of change $= \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{(x+h) - (x)}$

$$\boxed{\text{Right Hand Derivative} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \text{RHD}}$$

Left Hand Derivative $(\text{LHD}) = \lim\limits_{h \to 0} \dfrac{f(x) - f(x-h)}{(x) - (x-h)}$

$= \lim\limits_{h \to 0} \dfrac{f(x) - f(x-h)}{\cancel{x} - \cancel{x} + h}$

$$\boxed{\text{LHD} = \lim_{h \to 0} \frac{f(x-h) - f(x)}{-h}}$$

A function $f(x)$ is said to be differentiable at 'x' if (I) Continuous at x

(II) LHD = RHD (at x) = finite

$$\boxed{\lim_{h\to 0} \frac{f(x-h)-f(x)}{-h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}$$

LHD RHD

Derivative of $f(x) = y$

First Rule

$\hookrightarrow \frac{d(f(x))}{dx} = f'(x) = \frac{d(y)}{dx} = y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

y_1

↳ Diff. of $f(x)$ w.r.t. x

Standard Formulas

Constant $= x^0$

① $\frac{d(x^n)}{dx} = n.x^{n-1}$

$\frac{d(K)}{dx} = 0$

$\frac{d(x)}{dx} = 1$

$\frac{d(x^2)}{dx} = 2x$

$\frac{d(\frac{1}{x})}{dx} = -\frac{1}{x^2}$

$\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

② Trigonometric Functions.

$$\frac{d(\sin x)}{dx} = \cos x \quad , \quad \frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x \quad , \quad \frac{d(\sec x)}{dx} = \sec x \cdot \tan x$$

$$\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x \quad , \quad \frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cdot \cot x$$

Rules: ① $(u+v)' = u' + v'$

$\frac{d(u+v)}{dx}$

$u, v \rightarrow u(x)$, $\rightarrow v(x)$

function

② $(u-v)' = u' - v'$

③ $(u \cdot v)' = u' \cdot v + u \cdot v'$ — Leibnitz Rule

$(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$

④ $\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$

⑤ $(K \cdot u)' = K(u)'$

$$\frac{d(Ku)}{dx} = K \frac{d(u)}{dx}$$

(K = constant)

Chain Rule (For composite Function)

(function in function)

$g(x)$ in $f(x) = f(g(x))$

e.g. $\sin(x^2) = g(p(x))$

$p(x) = x^2$, $g(x) = \sin x$

$\sin(\tan(\cos x))$

$p(g(x)) = p(\sin x)$

$= (\sin x)^2 = \sin^2 x$

Composite Fn.

$\frac{d\left[f(g(x))\right]}{dx} = f'(g(x)) \times g'(x)$

$\frac{d\left[f(g(h(p(x))))\right]}{dx} = f'(g(h(p(x)))) \cdot g'(h(p(x))) \cdot h'(p(x)) \cdot p'(x).$

e.g. $f(x) = \sin(\tan(x^2))$

$$\frac{d(f(x))}{dx} = \frac{d(\sin(\tan(x^2)))}{dx}$$

fastest way.

$$= \cos(\tan(x^2)) \times \sec^2(x^2) \times 2x$$

II-way

$$\frac{d(\sin(\tan x^2))}{d(\tan x^2)} \cdot \frac{d(\tan x^2)}{d(x^2)} \cdot \frac{d(x^2)}{dx}$$

e.g. $f(x) = \sqrt{\cot(2x+3)}$

$$\frac{d(f(x))}{dx} = \frac{d\left(\sqrt{\cot(2x+3)}\right)}{dx}$$

$$= \left(\frac{1}{\cancel{2}\sqrt{\cot(2x+3)}}\right) \cdot \left(-\operatorname{cosec}^2(2x+3)\right) \cdot (\cancel{2})$$

$$= \frac{-\operatorname{cosec}^2(2x+3)}{\sqrt{\cot(2x+3)}}$$

$$\frac{d(f(x))}{dx} = \frac{d\sqrt{\cot(2x+3)}}{dx}$$

$$= \frac{d\sqrt{\cot(2x+3)}}{d(\cot(2x+3))} \cdot \frac{d(\cot(2x+3))}{d(2x+3)} \cdot \frac{d(2x+3)}{dx}$$

$$= \frac{1}{\cancel{2}\sqrt{\cot(2x+3)}} \cdot -\operatorname{cosec}^2(2x+3) \cdot (\cancel{2})$$

$\sqrt{\cot(2x+3)}$

$$\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

$$\frac{d(2x+3)}{dx}$$

$$= \frac{d(2x)}{dx} + \frac{d(3)}{dx}$$

$$= 2\frac{(d(x))}{dx} + 0$$

$$= 2$$

Class 12 maths [Continuity & Differentiability]

Exercise 5.2 Chapter 5

Chain Rule $\frac{d(f(g(x)))}{dx} = f'(g(x)).g'(x)$

$(u.v)' = u'.v + u.v'$

$\left(\frac{u}{v}\right)' = \frac{u'.v - u.v'}{v^2}$

Q.1 Differentiate the function with respect to 'x'.

$\sin(x^2+5)$

$\frac{d(x^n)}{dx} = n.x^{n-1}$

(I) $\frac{d[\sin(x^2+5)]}{dx} = \cos(x^2+5) . \frac{d(x^2+5)}{dx}$

$= \cos(x^2+5).(2x+0)$

$= 2x\cos(x^2+5)$

(II) $\frac{d(\sin(x^2+5))}{dx} = \frac{d(\sin(x^2+5))}{d(x^2+5)} . \frac{d(x^2+5)}{dx}$

$= \cos(x^2+5).(2x+0)$

$\boxed{Q.2}$ $\cos(\sin x)$

$$\frac{d(\cos \bigcirc)}{d\bigcirc} = -\sin x$$

I $\dfrac{d(\cos(\sin x))}{dx} = -\sin(\sin x) \cdot \cos x$ ✓

II $\dfrac{d(\cos(\sin x))}{dx} = \dfrac{d(\cos(\sin x))}{d(\sin x)} \cdot \dfrac{d(\sin x)}{dx}$

$= -\sin(\sin x) \cdot \cos x$ ✓

$\boxed{Q.3}$ $\dfrac{d[\sin(ax+b)]}{dx} = \cos(ax+b) \cdot (a \cdot 1 + 0)$

$= a \cdot \cos(ax+b)$

$\boxed{Q.4}$ $\dfrac{d[\sec(\tan\sqrt{x})]}{dx} = \sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot \sec^2(\sqrt{x}) \cdot \dfrac{1}{2\sqrt{x}}$

$\dfrac{d(\sec x)}{dx} = \sec x \cdot \tan x$

$\dfrac{d(x^{1/2})}{dx} = \dfrac{1}{2} \cdot x^{\frac{1}{2}-1}$

$$\frac{d(\sec(\tan\sqrt{x}))}{dx} = \underbrace{\frac{d(\sec(\tan\sqrt{x}))}{d(\tan\sqrt{x})}} \cdot \frac{d(\tan\sqrt{x})}{d(\sqrt{x})} \cdot \frac{d(\sqrt{x})}{dx}$$

✓

$\boxed{Q.5}$ $\dfrac{\sin(ax+b)}{\cos(cx+d)}$ $\left(\dfrac{u}{v}\right)' = \dfrac{u'.v - u.v'}{v^2}$

$$\frac{d\left(\dfrac{\sin(ax+b)}{\cos(cx+d)}\right)}{dx} = \frac{\overbrace{\dfrac{d(\sin(ax+b))}{dx}}^{\text{Chain Rule.}} . \cos(cx+d) - \sin(ax+b) . \overbrace{\dfrac{d(\cos(cx+d))}{dx}}^{\text{Chain}}}{\cos^2(cx+d)}$$

$$= \frac{\cos(ax+b).(a).\cos(cx+d) - \sin(ax+b).(-\sin(cx+d)).(c)}{\cos^2(cx+d)}$$

$$= \frac{a.\cos(ax+b).\cos(cx+d) + c.\sin(ax+b).\sin(cx+d)}{\cos^2(cx+d)}$$

$\boxed{Q.6.}$ $\cos x^3 . \sin^2(x^5) = \underbrace{\cos(x^3)}_{u} . \underbrace{[\sin(x^5)]^2}_{v}$

$(u.v)' = u'.v + u.v'$

$$\frac{d\left[\cos x^3 . (\sin x^5)^2\right]}{dx} = \underbrace{\frac{d(\cos x^3)}{dx}}_{\text{Chain}} . (\sin x^5)^2 + (\cos x^3) . \underbrace{\frac{d[(\sin x^5)^2]}{dx}}_{\text{Chain}}$$

$$= \underbrace{-\sin(x^3) . 3x^{3-1}}_{\text{Chain}} . (\sin x^5)^2 + (\cos x^3) . \underbrace{2(\sin x^5).\cos x^5 . 5.x^4}_{\text{Chain}}$$

$$= \underline{-3x^2(\sin x^3) . \sin^2(x^5)} + \underline{10x^4 . \cos x^3 . (\sin x^5) . \cos x^5}$$

$\boxed{Q.7}$ $2\sqrt{\cot(x^2)}$

$$\left(\begin{aligned} \frac{d(\sqrt{x})}{dx} &= \frac{1}{2\sqrt{x}} \\ \frac{d(\cot x)}{dx} &= -\operatorname{cosec}^2(x) \\ \frac{d(x^2)}{dx} &= 2x \end{aligned}\right)$$

$$\frac{d\left(2\sqrt{\cot x^2}\right)}{dx} = \cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{\cot x^2}}\cdot -\operatorname{cosec}^2(x^2)\cdot 2x$$

$$= \frac{-2x\cdot\operatorname{cosec}^2 x^2}{\sqrt{\cot x^2}}$$

$\boxed{Q.8}$ $\cos\sqrt{x}$

$$\frac{d(\cos\sqrt{x})}{dx} = -\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$$

$\boxed{Q.9}$ Prove that the function $f(x) = |x-1|$, $x \in R$ is not differentiable at $x=1$.

Solution

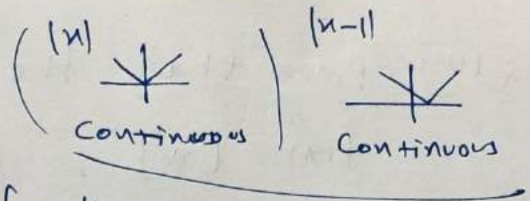
Condition for Differentiability at point $x=1 \rightarrow$

(I) Continuous at $x=1$ ✓

(II) At $x=1$, LHD = RHD $\quad$ $1-h$, 1, $1+h$ $\quad$ ($h>0$)

$$\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$f(x) = |x-1|$



we know that

$|x-1|$ is a continuous function in $x \in \mathbb{R}$.

$\therefore |x-1|$ is also continuous at $x=1$.

at $x=1$ Left Hand Derivative

$$LHD = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$$

$f(x) = |x-1|$

$f(1-h) = |(1-h)-1|$

$= |\cancel{1}-h-\cancel{1}|$

$= |-h|$

$$= \lim_{h\to 0} \frac{|1-h-1| - |1-1|}{-h}$$

$$= \lim_{h\to 0} \frac{|-h| - 0}{-h}$$

$|-3| = 3 = -(-3)$

$|-h| = -(-h) = h$

$$= \lim_{h\to 0} \frac{|-h|}{-h}$$

$h>0$

$$= \lim_{h\to 0} \frac{h}{-h} = -1 = LHD$$

$$RHD = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$f(x) = |x-1|$

$$= \lim_{h\to 0} \frac{|\cancel{1}+h-\cancel{1}| - |1-1|}{h}$$

$$= \lim_{h\to 0} \frac{|h| - 0}{h} = \lim_{h\to 0} \frac{h}{h} = 1 = RHD.$$

$\therefore LHD \neq RHD. \Rightarrow$ $\boxed{\text{not Diff. at } x=1}$

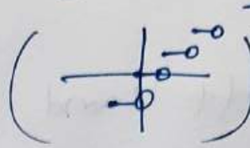
$\boxed{Q.10}$ Prove that the greatest integer function

$f(x) = [x]$, $0 < x < 3$ is not differentiable at $\underline{x=1}$ & $\underline{x=2}$.

$\boxed{x=1}$

(I) Continuity at $x=1$

$[x] \rightarrow$ Piecewise Fun^n.



$$\lim_{x\to 1} f(x) = f(1)$$

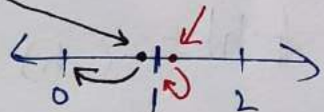
$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

LHL RHL (Exact)

$$\Rightarrow \lim_{x\to 1^-} [x] = \lim_{x\to 1^+} [x] = [1]$$

$$\Rightarrow [1^-] = [1^+] = [1]$$

$\Rightarrow$



$0 \neq 1 \cancel{=} 1$

$0 \neq 1$

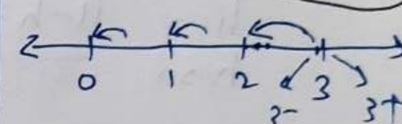
LHL $\neq$ RHL

$\therefore f(x) = [x]$ is not continuous at $x=1$

$\Rightarrow f(x) = [x]$ is not differentiable at $x=1$.

Int.

$[x] =$ या बराबर / just छोटा



$[2] = 2$

$[2.1] = 2$

$[2.2] = 2$

$[2.999] = 2$

3^-

Similarly $x=2$ ——————〃——————

Class 12 Maths [Continuity & Differentiability]

Derivatives of

Implicit Functions | Inverse Trigonometric Fun^n. ($\sin^{-1}x$, $\cos^{-1}x$, ...)

→ $xy - 3y^3 \sin x = 8$

→ $x - \sin(xy) + x^2 = y^2$

Explicit Fn^n. $y = f(x)$

$y = \sin x$, $y = x^2 - 3x + \cos x$, $y = |x| - 3x^2$

Differentiation of Implicit functions. $\frac{dy}{dx}$

→ Start Diff. from one end to another

e.g. $x + \sin(xy) - y = 0$ find $\frac{dy}{dx}$

Find derivative of y with respect to 'x'

{ Chain ✓, u.v ✓, u/v ✓ }

$x + \sin(xy) - y = 0$

by Diff. w.r.t. 'x'

$$\boxed{\frac{d(x)}{dx} = 1, \frac{d(y)}{dx}}$$

$$\Rightarrow \frac{d(x)}{dx} + \frac{d(\sin(xy))}{dx} - \frac{d(y)}{dx} = 0$$

(Chain Rule)

$$\Rightarrow 1 + \cos(xy) \cdot \left\{ \frac{dx}{dx} \cdot y + x \cdot \frac{dy}{dx} \right\} - \frac{dy}{dx} = 0$$

① u.v

$$\Rightarrow 1 + \cos(xy) \cdot \left\{ y + x \cdot \frac{dy}{dx} \right\} - \frac{dy}{dx} = 0$$

$$\Rightarrow 1 + \cos(xy) \cdot \left\{ y + x \frac{dy}{dx} \right\} - \frac{dy}{dx} = 0$$

$$\Rightarrow 1 + y \cdot \cos xy + x \cdot \cos(xy) \cdot \boxed{\frac{dy}{dx}} - \boxed{\frac{dy}{dx}} = 0$$

$$\Rightarrow \frac{dy}{dx} \cdot \left\{ x\cos(xy) - 1 \right\} = -(1 + y\cos xy)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(1 + y\cos xy)}{x\cos xy - 1} = \frac{(1 + y\cos xy)}{1 - x\cos xy}$$

Derivatives of Inverse Trigonometric Functions

$\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$	$\leftrightarrow$ $\frac{d(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$	$\leftrightarrow$ $\frac{d(\cot^{-1}x)}{dx} = \frac{-1}{1+x^2}$
$\frac{d(\sec^{-1}x)}{dx} = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d(\csc^{-1}x)}{dx} = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$

e.g. Proof, $y = \sin^{-1}x$ (माना) $\qquad \frac{dy}{dx} = ?$

$\Rightarrow \sin y = x$

$\cos^2 y = 1 - \sin^2 y$

~~sin y~~ $\cos y = \sqrt{1-\sin^2 y}$

10^{th}

Diff. w.r.t. x

$$\Rightarrow \frac{d(\sin y)}{dx} = \frac{d(x)}{dx} \Rightarrow \cos y \cdot \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}}$$

$$\Rightarrow \boxed{\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}}$$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \rightarrow$ Property

$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$\rightarrow$ Differentiation

☆ e.g. Find $\frac{dy}{dx}$ in $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

(V.V. Imp.)

Solve ↓ (आसान) ~~Substitution~~ (Substitution) → $\frac{dy}{dx}$

$\sqrt{1-x^2} \rightarrow x = \sin\theta \rightarrow \sqrt{1-\sin^2\theta} \rightarrow \cos\theta$

$\sqrt{1+x^2} \rightarrow x = \tan\theta \rightarrow \sqrt{1+\tan^2\theta} \rightarrow \sec\theta$

$\sqrt{x^2-1} \rightarrow x = \sec\theta \rightarrow \sqrt{\sec^2\theta-1} \rightarrow \tan\theta$

(Standard Substitution)

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

replace x by $\tan\theta$ $\quad x = \tan\theta$

$$y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$y = \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right) = \tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}}\right)$$

$$y = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{\cancel{2}\sin^{2}\theta/2}{\cancel{2}\,\cancel{\sin\theta/2}\cos\theta/2}\right)$$

$$y = \tan^{-1}\left(\frac{\sin\theta/2}{\cos\theta/2}\right)$$

$$f^{-1}(f(x)) = x$$

$$y = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$y = \frac{\theta}{2}$$

$x = \tan\theta$ माना

$$\tan^{-1}x = \theta$$

$$y = \frac{\tan^{-1}x}{2}$$

$$\frac{dy}{dx} = \frac{d\left(\frac{\tan^{-1}x}{2}\right)}{dx} = \frac{1}{2}\frac{d(\tan^{-1}x)}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{1+x^2}}$$

Class 12 Maths [Continuity & Differentiability]

Exercise - 5.3 Chapter (5)

Q.1 Find $\frac{dy}{dx}$.

Direct Diff. $\xrightarrow[(u/v)]{(u.v)}$ (Chain) $\frac{dy}{dx}$ अलग

$2x+3y=\sin x$

Diff. w.r.t. 'x'

$\Rightarrow \frac{d(2x)}{dx} + \frac{d(3y)}{dx} = \frac{d(\sin x)}{dx}$

$\Rightarrow 2\times 1 + 3\left(\frac{dy}{dx}\right) = \cos x$ $\Rightarrow \boxed{\frac{dy}{dx} = \frac{\cos x - 2}{3}}$

Q.2 $2x+3y=\sin y$

by Diff. w.r.t. 'x'

$\Rightarrow \frac{d(2x)}{dx} + \frac{d(3y)}{dx} = \frac{d(\sin y)}{dx}$

$\Rightarrow 2 + 3\left(\frac{dy}{dx}\right) = \cos y \cdot \frac{dy}{dx}$

$\Rightarrow 2 = \frac{dy}{dx}(\cos y - 3)$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2}{\cos y - 3}}$

Q3 $ax + by^2 = \cos y$

by diff. w.r.t. 'x'

$$\frac{d(x^2)}{dx} = 2x$$

$$\Rightarrow \frac{d(ax)}{dx} + \frac{d(by^2)}{dx} = \frac{d(\cos y)}{dx}$$

$$\Rightarrow a\times 1 + b.2y.\frac{dy}{dx} = -\sin y.\frac{dy}{dx}$$

$$\Rightarrow 2by.\frac{dy}{dx} + \sin y.\frac{dy}{dx} = -a$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-a}{2by + \sin y}}$$

Q.4 $xy + y^2 = \tan x + y$

by diff. w.r.t. 'x'

$$\Rightarrow \frac{d(xy)}{dx} + \frac{d(y^2)}{dx} = \frac{d(\tan x)}{dx} + \frac{d(y)}{dx}$$

$$\boxed{(u.v)' = u'.v + u.v'}$$

$$\Rightarrow \underbrace{\frac{d(x)}{dx}}_{1}.y + x.\frac{d(y)}{dx} + 2y.\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x - y$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}$$

Q.5 $x^2 + xy + y^2 = 100$

by diff. w.r.t. 'x'

$$\frac{d(C)}{dx} = 0$$

$$\Rightarrow \frac{d(x^2)}{dx} + \frac{d(x.y)}{dx} + \frac{d(y^2)}{dx} = \frac{d(100)}{dx}$$

$$\Rightarrow 2x + 1.y + x\frac{dy}{dx} + 2y.\frac{dy}{dx} = 0$$

$$\Rightarrow (x+2y)\frac{dy}{dx} = -(2x+y)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{(2x+y)}{x+2y}}$$

$$\frac{d(x^n)}{dx} = n.x^{n-1}$$

Q.6 $x^3 + x^2y + xy^2 + y^3 = 81$

by diff. w.r.t. (x)

$$\Rightarrow \frac{d(x^3)}{dx} + \frac{d(x^2.y)}{dx} + \frac{d(x.y^2)}{dx} + \frac{d(y^3)}{dx} = \frac{d(81)}{dx}$$

$$(u.v)' \rightarrow u'.v + u.v'$$

$$\Rightarrow 3.x^2 + \left[(2x).y + x^2.\frac{dy}{dx}\right] + \left[1.y^2 + x.2y.\frac{dy}{dx}\right]$$

$$+ 3y^2.\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}.(x^2 + 2xy + 3y^2) = -(3x^2 + 2xy + y^2)$$

$$\boxed{dy/dx = -\frac{(3x^2 + 2xy + y^2)}{(x^2 + 2xy + 3y^2)}}$$

Q.7 $\sin^2 y + \cos xy = k \leftarrow$ Constant

by diff. w.r.t. 'x'

$(\sin y)^2 \quad x^2 \to 2x$

$$\Rightarrow \frac{d(\sin^2 y)}{dx} + \frac{d(\cos(x.y))}{dx} = \frac{d(k)}{dx}$$

Chain   Chain, u.v

$$\Rightarrow 2\sin y.\cos y.\frac{dy}{dx} + [-\sin(xy)].\frac{d(x.y)}{dx} = 0$$

$$\Rightarrow \underbrace{2\sin y.\cos y}_{11^{th}\text{ class Formula}}.\frac{dy}{dx} - \sin(xy).\left\{1.y + x.\frac{dy}{dx}\right\} = 0$$

$$\Rightarrow \sin 2y.\frac{dy}{dx} - y\sin xy - x\sin(xy).\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(\sin 2y - y\sin xy) = y\sin xy$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{y\sin(xy)}{\sin 2y - y\sin xy}}$$

Q.8 $\sin^2 x + \cos^2 y = 1$

by diff. w.r.t. 'x'

$\rightarrow (\sin x)^2$

$(\cos y)^2$

$x^2 \rightarrow 2x$

$$\Rightarrow \frac{d(\sin^2 x)}{dx} + \frac{d(\cos^2 y)}{dx} = \frac{d(1)}{dx}$$

Chain Chain

$$\Rightarrow 2\sin x \cdot \cos x + 2\cos y \cdot (-\sin y) \cdot \frac{dy}{dx} = 0$$

Square Sin Square Cos y

$2\sin\theta \cdot \cos\theta = \sin 2\theta$

$\ominus \rightarrow \oplus$

$$\Rightarrow \sin 2x = \sin 2y \cdot \frac{dy}{dx}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}}$$

Class 12 Maths [Continuity & Differentiability]

Exercise-5.3 Chapter (5)

ITF

Q9 — Q15

Find $\frac{dy}{dx}$

Q.9 $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(Avoid ~~Chain Rule~~)

(Trigonometric Substitution) ↓ Solve (आसान) → Diff.

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $\boxed{x = \tan\theta}$ ☆ $\tan^{-1}x = \theta$

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}x$

Diff. $\frac{dy}{dx} = 2\frac{d(\tan^{-1}x)}{dx} = \frac{2}{1+x^2}$ ✓

Q.10 $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Subtitution $x = \tan\theta$ $\theta = \tan^{-1}x$

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$

$y = \tan^{-1}(\tan 3\theta)$

$y = 3\theta = 3\tan^{-1}x$

$y = 3\tan^{-1}x$

Diff.

$\frac{dy}{dx} = \frac{3}{1+x^2}$

$\boxed{Q.11}$ $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$

Substitution $x = \tan\theta$ $\quad \theta = \tan^{-1}x$

$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \cos^{-1}(\cos 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}x$

$\frac{dy}{dx} = 2 \times \frac{1}{1+x^2}$

$= \frac{2}{1+x^2}$

$\boxed{Q.12}$ $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$; $0 < x < 1$

Substitution $x = \tan\theta$ $\quad \theta = \tan^{-1}x$

$y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}(\cos 2\theta)$ $\qquad \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$y = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right)$

$y = \frac{\pi}{2} - 2\theta$

$y = \frac{\pi}{2} - 2\tan^{-1}x$

$\frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2} = \frac{-2}{1+x^2}$

$\boxed{Q.13}$ $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, $-1 < x < 1$

Substitution $x = \tan\theta$ $\quad \tan^{-1}x = \theta$

$y = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$y = \cos^{-1}(\sin 2\theta)$

$y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

$y = \frac{\pi}{2} - 2\theta$

$\boxed{y = \frac{\pi}{2} - 2\tan^{-1}x}$

Diff.

$\frac{dy}{dx} = 0 - \frac{2}{1+x^2}$

$= \frac{-2}{1+x^2}$

$\boxed{Q.14}$ $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$; $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

put $x = \sin\theta$ $\quad \sin^{-1}x = \theta$

$y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$y = \sin^{-1}(2\sin\theta\cos\theta)$

$y = \sin^{-1}(\sin 2\theta)$

$y = 2\theta$

$y = 2\sin^{-1}x$

Diff

$\frac{dy}{dx} = 2 \times \frac{1}{\sqrt{1-x^2}}$

$\boxed{Q.15}$ $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$, $0 < x < \frac{1}{\sqrt{2}}$

Put $x = \cos\theta$ $\cos^{-1}x = \theta$

$$y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$$

$$y = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta)$$

$$y = 2\theta = 2\cos^{-1}x$$

$$y = 2\cos^{-1}x$$

Diff.

$$\frac{dy}{dx} = 2 \times \left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-2}{\sqrt{1-x^2}}$$

Class 12 Maths [Continuity & Differentiability]

Differentiation of logarithmic function & Exponential Fn.

$\begin{pmatrix} a>0 \\ a\neq 1 \end{pmatrix}$ $\boxed{y = \log_a x}$

(Common logarithm) $\left(y = \log_{10} x\right)$ P & C

(Natural Logarithm) $\boxed{y = \log_e(x)} = \ln(x)$ M

~~log~~ $\log x$

$\left(\begin{matrix} e = \text{euler's no.} \\ e = 2.718\ldots \end{matrix}\right)$

$\boxed{y = a^x}$ $y = 2^x, 10^x$ $y = e^x$

e

$y = e^x \rightarrow \dfrac{dy}{dx} = e^x$

$y = \log_e x \rightarrow \dfrac{dy}{dx} = \dfrac{1}{x}$

maths $\dfrac{d(\log x)}{dx}$

a

$\dfrac{d(a^x)}{dx} = a^x \cdot \log_e a$

$\dfrac{d(\log_a x)}{dx} = \dfrac{1}{x \cdot \log_e a}$

e.g. $y = 2^x \rightarrow \dfrac{dy}{dx} = 2^x \cdot \log_e 2$

$y = 10^x \rightarrow$ ~~log~~ $\dfrac{dy}{dx} = 10^x \cdot \log_e 10$

$y = e^x \rightarrow \dfrac{dy}{dx} = e^x$

e.g. $y = \log x \rightarrow \frac{dy}{dx} = \frac{1}{x}$

(maths log का Base $= e$)

e.g. $\boxed{y = \log_{10} x = \frac{1}{x \cdot \log_e 10}}$

Base change theorem

$$\boxed{\frac{\log_a x}{\log_a y} = \log_y x}$$

$\hookrightarrow y = \log_{10} x$

$$y = \frac{\log_e x}{\log_e 10}$$ (Base change theorem)

$\log_e 10 \} \rightarrow$ Constant.

Diff. —

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log_e 10} \cdot \frac{d(\log_e x)}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\log_e 10} \cdot \frac{1}{x}}$$

e.g. $y = \log(\log(x^2))$ $\quad \frac{d(\log X)}{dX} = \frac{1}{X}$

(Chain Rule)

Diff.

$$\frac{dy}{dx} = \frac{1}{\log x^2} \cdot \frac{1}{x^2} \cdot (2x)$$

$$= \frac{2}{(\log x^2) \cdot x}$$

Class 12 Maths [Continuity and Differentiability]

Exercise 5.4 Chapter 5

$$\frac{d(\log x)}{dx} = \frac{d(\log_e x)}{dx} = \frac{d(\ln x)}{dx} = \frac{1}{x}$$

Differentiate with respect to 'x'

$$\frac{d(e^x)}{dx} = e^x$$

Q.1 $\frac{e^x}{\sin x}$ $\left(\frac{u}{v}\right)' = \frac{u'.v - u.v'}{v^2}$

$$\frac{d\left(\frac{e^x}{\sin x}\right)}{dx} = \frac{e^x.\sin x - e^x.\cos x}{\sin^2 x} = \frac{e^x(\sin x - \cos x)}{\sin^2 x}$$

Q.2 $e^{\sin^{-1} x}$ (fun^n. in fun^n) $\longrightarrow$ Chain Rule

$$\frac{d(e^{\sin^{-1}x})}{dx} = e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}$$

Q.3 $$\frac{d(e^{(x^3)})}{dx} = e^{x^3} . 3x^2$$

$$\frac{d(x^3)}{dx} = 3.x^{3-1}$$

Q.4 $$\frac{d\left\{\sin\left[\tan^{-1}(e^{-x})\right]\right\}}{dx} = \cos(\tan^{-1}(e^{-x})) \cdot \frac{1}{1+(e^{-x})^2} \cdot e^{-x} \cdot (-1)$$

(Chain)

Q.5 $\log(\cos e^x)$ (Chain Rule)

$$\frac{d(\log(\cos e^x))}{dx} = \frac{1}{\cos(e^x)} \cdot (-\sin e^x) \cdot e^x$$

$$= -e^x \cdot \tan e^x$$

Q.6 $e^x + e^{x^2} + \dots + e^{x^5}$

$$\frac{d(x^n)}{dx} \to n \cdot x^{n-1}$$

$$\frac{d(e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5})}{dx} = \frac{d(e^x)}{dx} + \frac{d(e^{x^2})}{dx} + \frac{d(e^{x^3})}{dx} + \frac{d(e^{x^4})}{dx} + \frac{d(e^{x^5})}{dx}$$

$$= e^x + e^{x^2} \cdot (2x) + e^{x^3} \cdot 3x^2 + e^{x^4} \cdot 4x^3 + e^{x^5} \cdot 5 \cdot x^4$$

Q.7 $\sqrt{e^{\sqrt{x}}}$ $\quad \left(e^{(x)^{1/2}}\right)^{1/2}$ $\quad x^{1/2} = \sqrt{x}$ $\quad \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

$$\frac{d\left(\sqrt{e^{\sqrt{x}}}\right)}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \times e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{4\sqrt{x e^{\sqrt{x}}}}$$

$\boxed{Q.8}$ $\log(\log x)$ $(x>1)$ $\dfrac{d(\log x)}{dx} = \dfrac{1}{x}$

$$\frac{d(\log(\log x))}{dx} = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$$

$\boxed{Q.9}$

$$\frac{d\left(\frac{\cos x}{\log x}\right)}{dx} = \frac{(-\sin x).\log x - \cos x \cdot \left(\frac{1}{x}\right)}{(\log x)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{u'.v - u.v'}{v^2}$$

$$= -\left(\frac{x \sin x . \log x + \cos x}{x(\log x)^2}\right)$$

$\boxed{Q.10}$ $\underset{\text{Chain}}{\cos(\log x + e^x)}$ $\dfrac{d(\cos x)}{dx} = -\sin x$

$$\frac{d(\cos(\log x + e^x))}{dx} = -\sin(\log x + e^x).\left\{\frac{1}{x} + e^x\right\}$$

$$= -\left(\frac{1}{x} + e^x\right). \sin(\log x + e^x)$$

Class 12 maths [Continuity & Differentiability]

Logarithmic Differentiation (लघुगणकीय अवकलन)

कब? (i) $(\text{Variable})^{\text{Variable}}$ (ii) $\dfrac{(f_1(x))^{n_1}\cdot(f_2(x))^{n_2}\cdots}{(g_1(x))^{k_1}\cdot(g_2(x))^{k_2}\cdots}$

$x^x,\ (\sin x)^x,\ x^{\tan x}$ $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)}}$

कैसे? $y = f(x)$

log $\longrightarrow$ Diff. $\longrightarrow$ $\dfrac{dy}{dx}$

$e = $ Base

log की Properties: ① $\log(m^n) = n\log m$

$\dfrac{d(\log x)}{dx} = \dfrac{1}{x}$

② $\log(m.n) = \log m + \log n$

③ $\log\left(\dfrac{m}{n}\right) = \log m - \log n$

e.g. Differentiate $(\tan x)^x$ with respect to x.

$(\text{Var.})^{\text{Var.}}$ $\dfrac{dy}{dx} = ?$

$y = (\tan x)^x$

$\Rightarrow \log y = \log(\tan x)^x$

$\Rightarrow \log y = x\cdot\log(\tan x)$

$(u\cdot v)' = u'\cdot v + u\cdot v'$

by Diff. w.r.t. 'x'

$\Rightarrow \dfrac{1}{y}\cdot\dfrac{dy}{dx} = 1\times\log\tan x + x\cdot\dfrac{\sec^2 x}{\tan x}$

$\Rightarrow \dfrac{dy}{dx} = (\tan x)^x\left(\log\tan x + \dfrac{x\cdot\sec^2 x}{\tan x}\right)$

e.g. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$ with respect to x.

Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}} = \left(\frac{- \quad -}{- \quad -}\right)^{1/2}$

log

$\Rightarrow \log y = \log\left(\frac{(x-1)(x-2)}{(x-3)(x-4)}\right)^{1/2} = \frac{1}{2}\log\left(\frac{(x-1)(x-2)}{(x-3)(x-4)}\right)$

$\Rightarrow \log(y) = \frac{1}{2}\left\{\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4)\right\}$

$\Rightarrow$ Diff. w.r.t. x $\rightarrow$

$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left\{\frac{1}{x-1}\times(1-0) + \frac{1}{(x-2)} - \frac{1}{x-3} - \frac{1}{(x-4)}\right\}$

$\Rightarrow \underline{\frac{dy}{dx} = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}\cdot\frac{1}{2}\cdot\left\{\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4}\right\}}$

Class 12 Maths [Continuity and Differentiability]

Exercise-5.5 Chapter-5

Logarithmic Differentiation

Ⅰ	Ⅱ
$(\text{Variable})^{\text{Variable}}$	$\dfrac{(f_1(x))^{n_1}(f_2(x))^{n_2}\ldots}{(g_1(x))^{k_1}(g_2(x))^{k_2}\ldots}$

① $y = \boxed{}$

② log

③ log की properties

- $\log(m^n) = n.\log m$
- $\log(m.n) = \log m + \log n$
- $\log\left(\frac{m}{n}\right) = \log m - \log n$

④ Diff. $\frac{dy}{dx}$

Exercise 5.5

$(u.v)' = u'v + u.v'$

Q.1 let $y = \cos x . \cos 2x . \cos 3x$

$\Rightarrow \log y = \log(\cos x . \cos 2x . \cos 3x)$

$\Rightarrow \log y = \log \cos x + \log \cos 2x + \log \cos 3x$

by Differentiating w.r.t. 'x'

$$\frac{d(\log x)}{dx} = \frac{1}{x}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}.(-\sin x) + \frac{1}{\cos 2x}.(-\sin 2x).(2) + \frac{1}{\cos 3x}.(-\sin 3x)(3)$$

$$\Rightarrow \frac{dy}{dx} = y.\left(-\tan x - 2\tan 2x - 3\tan 3x\right)$$

$$= -\cos x \cos 2x \cos 3x \left(\tan x + 2\tan 2x + 3\tan 3x\right)$$

Q.2 $y = \sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} = \left(\dfrac{(\ )(\ )}{(\ )(\ )(\ )}\right)^{1/2}$

$\Rightarrow \log y = \log\left[\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]^{1/2}$

$\Rightarrow \log y = \frac{1}{2}\cdot\log\left\{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right\}$

$\Rightarrow \log y = \frac{1}{2}\left\{\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)\right\}$

Diff. w.r.t. 'x'

$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left\{\frac{1}{x-1}\cdot(1-0) + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right\}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\cdot\left\{\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right\}$

Q.3 $y = (\log x)^{\cos x}$

$\Rightarrow \log y = \log\left[\log x^{\cos x}\right]$

$\Rightarrow \log y = \cos x\cdot\log(\log x)$

$(u\cdot v)' = u'\cdot v + u\cdot v'$

Diff. w.r.t. 'x'

$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = -\sin x\cdot(\log\log x) + \cos x\cdot\frac{1}{\log x}\cdot\frac{1}{x}$

$y = (\log x)^{\cos x}$

$\frac{dy}{dx} = (\log x)^{\cos x}\cdot\left\{-\sin x\cdot\log\log x + \frac{\cos x}{x\log x}\right\}$

Q.4 let $y = x^x - 2^{\sin x}$ $\Rightarrow y = u - v$

$u = x^x$, $v = 2^{\sin x}$

$\hookrightarrow \underbrace{\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}}$

$u = x^x$

$\Rightarrow \log u = \log(x^x)$

$\Rightarrow \log u = \underline{x} \cdot \underline{\log x}$

Diff. w.r.t. 'x'

$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = 1 \cdot \log x + \cancel{x} \cdot \frac{1}{\cancel{x}}$

$\Rightarrow \frac{du}{dx} = u(\log x + 1)$

$\Rightarrow \frac{du}{dx} = x^x(\log x + 1)$ ✓

$v = 2^{\sin x}$

$\Rightarrow \log v = \log(2^{\sin x})$

$\Rightarrow \log v = \sin x \cdot \underbrace{\log 2}_{}$ Constant

Diff. w.r.t. 'x'

$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \log 2 . \cos x$

$\Rightarrow \frac{dv}{dx} = 2^{\sin x} \cdot \log 2 . \cos x$ ✓

$\therefore \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = x^x(\log x + 1) - 2^{\sin x} \cdot \log 2 . \cos x$

Q.5 $y = (x+3)^2 . (x+4)^3 . (x+5)^4$

$\Rightarrow \log y = \log\left((x+3)^2 . (x+4)^3 . (x+5)^4\right)$

$\Rightarrow \log y = 2\log(x+3) + 3\log(x+4) + 4 . \log(x+5)$

Diff. w.r.t 'x'

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}$

$\Rightarrow \frac{dy}{dx} = (x+3)^2(x+4)^3(x+5)^4\left(\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right)$

$$\Rightarrow \frac{dy}{dx} = (x+3)^{1}(x+4)^{2}(x+5)^{3} \cdot \left\{ \frac{2}{\cancel{x+3}}\cancel{(x+3)}(x+4)(x+5) + \frac{3}{\cancel{x+4}}(x+3)\cancel{(x+4)}(x+5) + \frac{4}{\cancel{x+5}}(x+3)(x+4)\cancel{(x+5)} \right\}$$

$$\Rightarrow \frac{dy}{dx} = (x+3)(x+4)^{2}(x+5)^{3} \cdot \left\{ 2(x^2+9x+20) + 3(x^2+8x+15) + 4(x^2+7x+12) \right\}$$

$$\frac{dy}{dx} = (x+3)(x+4)^{2}(x+5)^{3} \cdot \left\{ 9x^2 + 70x + 133 \right\}$$

Class 12 Maths [Continuity and Differentiability]

Exercise 5.5 Chapter-5

- y
- $+$ log
- $+$ Prop.
- $-$ Diff $-\frac{dy}{dx}$

$\log(m^n)$
$\downarrow$
$n.\log m$

Q.6 $\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

$$y = \left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$$

$$\boxed{y = u + v}$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = \left(x+\frac{1}{x}\right)^x$$

$$\Rightarrow \log u = \log\left(x+\frac{1}{x}\right)^x$$

$$\Rightarrow \log u = x \cdot \log\left(x+\frac{1}{x}\right)$$

Diff. w.r.t. x

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = 1 \times \log\left(x+\frac{1}{x}\right) + x\cdot\frac{1}{x+\frac{1}{x}}\cdot\left(1-\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{du}{dx} = u\left[\log\left(x+\frac{1}{x}\right) + \cancel{x}\cdot\frac{\cancel{x}}{x^2+1}\cdot\left(\frac{x^2-1}{\cancel{x^2}}\right)\right]$$

$$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\cdot\left[\log\left(x+\frac{1}{x}\right) + \frac{x^2-1}{x^2+1}\right]$$

$$v = x^{\left(1+\frac{1}{x}\right)}$$

$$\log v = \left(1+\frac{1}{x}\right)\cdot\log x$$

Diff w.r.t. x –

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = \left(0-\frac{1}{x^2}\right)\cdot\log x + \left(1+\frac{1}{x}\right)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{-\log x}{x^2} + \frac{x+1}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{x^{\left(1+\frac{1}{x}\right)}}{x^2}\left[-\log x + x + 1\right]$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \underbrace{\left(x+\frac{1}{x}\right)^x \left[\log\left(x+\frac{1}{x}\right) + \frac{x^2-1}{x^2+1}\right]}$$

$$+ \underbrace{\frac{x^{\left(1+\frac{1}{x}\right)}}{x^2}\left[-\log x + x + 1\right]}$$

$\boxed{Q.7}$ $y = (\log x)^x + x^{\log x}$

$$y = u + v$$

$$\boxed{\frac{dy}{dx} = \underline{\frac{du}{dx}} + \underline{\frac{dv}{dx}}}$$

$$u = (\log x)^x$$

$$\Rightarrow \log u = \underline{x \cdot \log(\log x)}$$

Diff. w.r.t. x

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = 1\cdot\log(\log x) + \cancel{x}\cdot\frac{1}{\log x}\cdot\frac{1}{\cancel{x}}$$

$$\Rightarrow \underbrace{\frac{du}{dx} = (\log x)^x \cdot \left\{\log\log x + \frac{1}{\log x}\right\}}$$

$$v = x^{\log x}$$

$$\log v = \log\left(x^{\log x}\right)$$

$$\Rightarrow \log v = \log x \cdot \log x$$

$$\Rightarrow \log v = (\log x)^2$$

Diff. w.r.t. x

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = 2\log x \cdot \frac{1}{x}$$

$$\Rightarrow \underline{\frac{dv}{dx} = x^{\log x} \cdot \frac{2\log x}{x}}$$

$$\therefore \frac{dy}{dx} = (\log x)^x \cdot \left(\log\log x + \frac{1}{\log x}\right) + x^{\log x}\cdot\frac{2\log x}{x}$$

$\boxed{Q.8}$ $y = (\sin x)^x + \sin^{-1}(\sqrt{x})$

$y = u + v$

$$\boxed{\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}}$$

$u = (\sin x)^x$

$\Rightarrow \log u = x . \log(\sin x)$

Diff. w.r.t. (x)

$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = 1 . \log(\sin x) + x . \frac{1}{\sin x} . \cos x$

$\Rightarrow \frac{du}{dx} = (\sin x)^x . \left[\log \sin x + x . \cot x\right]$

$v = \sin^{-1}(\sqrt{x})$

Chain Rule

$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{(1-x).x}}$

$\frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}}$

$\therefore \frac{dy}{dx} = (\sin x)^x . \left[\log \sin x + x . \cot x\right] + \frac{1}{2\sqrt{x-x^2}}$

$\boxed{Q.9}$ $y = x^{\sin x} + (\sin x)^{\cos x}$

$\rightarrow y = u + v$

$\hookrightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$$u = x^{\sin x}$$

$$\Rightarrow \log u = \log\left(x^{\sin x}\right)$$

$$\Rightarrow \log u = \sin x \cdot \log x$$

Diff. w.r.t. x

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = \cos x \cdot \log x + \sin x \cdot \frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = \left(x^{\sin x}\right) \cdot \left[\cos x \cdot \log x + \frac{\sin x}{x}\right]$$

$$v = (\sin x)^{\cos x}$$

$$\log v = \cos x \cdot \log(\sin x)$$

Diff. x

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = (-\sin x) \cdot \log(\sin x) + \cos x \cdot \frac{1}{\sin x} \cdot \cos x$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} \cdot \left[-\sin x \cdot \log \sin x + \cos x \cdot \cot x\right]$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{dy}{dx} = x^{\sin x} \cdot \left(\cos x \cdot \log x + \frac{\sin x}{x}\right) + (\sin x)^{\cos x} \cdot \left[-\sin x \cdot \log \sin x + \cos x \cdot \cot x\right]$$

Class 12 maths [Continuity and Differentiability]

Exercise 5.5 Chapter 5

Q.10 Differentiate with respect to x.

$$y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$$

$y = (\text{Var.})^{\text{Var.}}$ → Log. Diff. → log → Diff.

$\log(m^n) = n\cdot\log m$

$$y = u + v$$

$$\boxed{\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}}$$

$(u\cdot v)' = u'\cdot v + u\cdot v'$

$(u\cdot v\cdot w)' = u'\cdot vw + u v' w + u\cdot v\cdot w'$

$$u = x^{x\cos x}$$

$$\Rightarrow \log u = \log\left(x^{x\cos x}\right)$$

$$\Rightarrow \log u = x\cos x\cdot\log x$$

Diff. w.r.t x

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = 1\cdot\cos x\cdot\log x + x(-\sin x)\log x + \cancel{x}\cos x\cdot\frac{1}{\cancel{x}}$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\cdot\left[\cos x\cdot\log x - x\sin x\cdot\log x + \cos x\right]$$

now $v = \dfrac{x^2+1}{x^2-1}$ $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - u\cdot v'}{v^2}$

$$\Rightarrow \frac{dv}{dx} = \frac{(2x+0)\cdot(x^2-1) - (x^2+1)\cdot(2x)}{(x^2-1)^2}$$

$$\frac{dv}{dx} = \frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{(x^2-1)^2}$$

$$\frac{dv}{dx} = \frac{-4x}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{dy}{dx} = \underbrace{x^{x\cos x} \cdot \left[\cos x . \log x - x \sin x \log x + \cos x\right]}_{} - \underbrace{\frac{4x}{(x^2-1)^2}}_{}$$

Q.11 $y = (x\cos x)^{x} + (x \sin x)^{1/x}$

$y = u + v \Rightarrow \boxed{\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}}$

$u = (x\cos x)^{x}$

$\Rightarrow \log u = \log (x \cos x)^{x}$

$\Rightarrow \log u = x . \log (x . \cos x)$

$\boxed{\text{Diff. w.r.t. } x}$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = 1 . \log(x \cos x) + \cancel{x} . \frac{1}{\cancel{x} \cos x} \cdot \left[1 . \cos x + x . (-\sin x)\right]$$

$$\Rightarrow \frac{du}{dx} = \underbrace{(x\cos x)^{x} . \left[\log(x \cos x) + 1 - x . \tan x\right]}_{} \quad \text{—(i)}$$

Now $V = (x \sin x)^{1/x}$

$\Rightarrow \log V = \log (x \sin x)^{1/x}$

$\Rightarrow \log V = \frac{1}{x} . \log (x . \sin x)$

by Diff. w.r.t. $(x) \longrightarrow$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \left(-\frac{1}{x^2}\right) \cdot \log(x \sin x) + \frac{1}{x} \cdot \frac{1}{x \sin x} \cdot (1 . \sin x + x . \cos x)$$

$$\Rightarrow \frac{dv}{dx} = (x \sin x)^{1/x} \cdot \left[-\frac{\log(x \sin x)}{x^2} + \frac{1}{x^2} \cdot (1 + x \cdot \cot x) \right]$$

$$\Rightarrow \frac{dv}{dx} = (x \sin x)^{1/x} \cdot \left[\frac{-\log(x \sin x) + 1 + x \cot x}{x^2} \right] \quad -(2)$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$ by eqⁿ (1) & (2)

$$\Rightarrow \boxed{\frac{dy}{dx} = (x \cos x)^x \cdot \left[\log(x \cos x) + 1 - x \tan x \right] + (x \sin x)^{1/x} \cdot \left[\frac{-\log(x \sin x) + 1 + x \cot x}{x^2} \right]}$$

✓

$$\Rightarrow \frac{dy}{dx} = \frac{-\left(x^y \cdot \frac{y}{x} + y^x \cdot \log y\right)}{\left(x^y \cdot \log x + y^x \cdot \frac{x}{y}\right)}$$

$$x^y \cdot \frac{y}{x}$$

$$x^{y-1} \cdot y$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\left(x^{y-1} \cdot y + y^x \cdot \log y\right)}{\left(x^y \cdot \log x + y^{x-1} \cdot x\right)}$$

Q.13 $y^x = x^y$

$\Rightarrow \log(y^x) = \log(x^y)$

$\Rightarrow x \cdot \log y = y \cdot \log x$

by Diff. w.r.t. x →

$$\Rightarrow 1 \cdot \log y + x \cdot \frac{1}{y} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \log x + y \cdot \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x}{y} - \log x\right) = \left(\frac{y}{x} - \log y\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{y}{x} - \log y}{\frac{x}{y} - \frac{\log x}{1}} = \frac{(y - x \log y)/x}{(x - y \cdot \log x)/y}$$

$$\frac{dy}{dx} = \frac{y}{x} \cdot \left(\frac{y - x \cdot \log y}{x - y \log x}\right)$$

Q.14 $(\cos x)^y = (\cos y)^x$

$\Rightarrow \log(\cos x)^y = \log(\cos y)^x$

$\Rightarrow y \cdot \log(\cos x) = x \cdot \log(\cos y)$

diff. w.r.t. x →

$\Rightarrow \frac{dy}{dx} \cdot \log\cos x + y \cdot \frac{1}{\cos x} \cdot (-\sin x) = 1 \cdot \log(\cos y) + x \cdot \frac{1}{\cos y}(-\sin y) \cdot \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(\log\cos x + x \cdot \tan y) = \log(\cos y) + y\tan x$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\log\cos y + y\tan x}{\log\cos x + x\tan y}}$$

Q.15 $xy = e^{(x-y)}$

$\Rightarrow \log(xy) = \log(e^{(x-y)})$

$\Rightarrow \log x + \log y = (x-y) \cdot \log_e e$

$\Rightarrow \log x + \log y = x - y$

Diff. w.r.t x

$\Rightarrow \frac{1}{x} + \frac{1}{y} \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y}+1\right) = 1 - \frac{1}{x}$

Maths → $\log x$

$\ln(x) \leftarrow \log_e x$

$\log_a a = 1$

$\frac{dy}{dx} = \frac{\left(1-\frac{1}{x}\right)}{\left(\frac{1}{y}+1\right)} = \frac{\left(\frac{x-1}{x}\right)}{\left(\frac{1+y}{y}\right)}$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}}$$

Class 12 maths [Continuity and Differentiability]

Exercise-5.5 Chapter-5

$\boxed{Q.16}$ Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

Solⁿ. $f(x) = (1+x).(1+x^2)(1+x^4).(1+x^8)$

$\Rightarrow \log(f(x)) = \log[(1+x)(1+x^2)(1+x^4)(1+x^8)]$

$\Rightarrow \log f(x) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$

by Diff. w.r.t. 'x'

$$\Rightarrow \frac{1}{f(x)} \cdot \frac{d(f(x))}{dx} = \frac{1.(0+1)}{1+x} + \frac{1.(0+2x)}{1+x^2} + \frac{1.(0+4x^3)}{1+x^4} + \frac{1(0+8.x^7)}{1+x^8}$$

$f'(x)$

$$\Rightarrow f'(x) = (1+x).(1+x^2).(1+x^4).(1+x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

Put $x=1$

$$f'(1) = (1+1)(1+1)(1+1)(1+1)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$$

$$f'(1) = \cancel{16}^{8} \times \frac{15}{\cancel{2}} = 120$$

Q.17

let $y = (x^2-5x+8).(x^3+7x+9)$

(i) Differentiate by using product rule.

$(u.v)' = u'.v + u.v'$

$$y = \overset{u}{(x^2-5x+8)}.\overset{v}{(x^3+7x+9)}$$

$\frac{du}{dx}$

diff. w.r.t. (x)

$$\frac{dy}{dx} = (2x-5).(x^3+7x+9) + (x^2-5x+8).(3x^2+7)$$

$$= 2x^4 + 14x^2 + 18x - 5x^3 - 35x - 45$$
$$+ 3x^4 + 7x^2 - 15x^3 - 35x + 24x^2 + 56$$

$$\frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$$

(ii) by expanding the product to obtain a single polynomial

$y = (x^2-5x+8).(x^3+7x+9)$

$$\Rightarrow y = x^5 + 7x^3 + 9x^2 - 5x^4 - 35x^2 - 45x$$
$$+ 8x^3 + 56x + 72$$

$$\Rightarrow y = x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72$$

$$\boxed{\frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11 + 0}$$

(iii) by logarithmic differentiation.

$$y = (x^2 - 5x + 8)(x^3 + 7x + 9)$$

$$\Rightarrow \log y = \log\left[(x^2 - 5x + 8)\cdot(x^3 + 7x + 9)\right]$$

$$\log(m \cdot n) = \log m + \log n$$

$$\Rightarrow \log y = \log(x^2 - 5x + 8) + \log(x^3 + 7x + 9)$$

by Diff. w.r.t. x →

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \left[\frac{1}{x^2 - 5x + 8} \times (2x - 5) + \frac{1}{x^3 + 7x + 9}\cdot(3x^2 + 7)\right]$$

$$\Rightarrow \frac{dy}{dx} = \cancel{(x^2 - 5x + 8)}\cdot\cancel{(x^3 + 7x + 9)}\left[\frac{(2x-5)(x^3+7x+9) + (3x^2+7)(x^2-5x+8)}{\cancel{(x^2-5x+8)}\cdot\cancel{(x^3+7x+9)}}\right]$$

$$\Rightarrow \frac{dy}{dx} = (2x-5)(x^3+7x+9) + (3x^2+7)(x^2-5x+8)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11}$$

Yes.

Q.18 $\frac{d}{dx}(u.v.w) = \frac{du}{dx}.v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$

(i) Prove by repeated application of Product rule.

$$LHS = \frac{d}{dx}(u.v.w) = \frac{d[u.(v.w)]}{dx}$$

$$= \frac{du}{dx}.(v.w) + u.\frac{d(v.w)}{dx}$$

$$= \frac{du}{dx}.v.w + u.\left(\frac{dv}{dx}.w + v.\frac{dw}{dx}\right)$$

$$= \underbrace{\frac{du}{dx}.v.w}_{} + \underbrace{u.\frac{dv}{dx}.w}_{} + \underbrace{u.v.\frac{dw}{dx}}_{} = RHS.$$

(ii) Prove by logarithmic differentiation.

$$\frac{d(u.v.w)}{dx}$$

Let $y = u.v.w$

$\Rightarrow \log y = \log(u.v.w)$

$\Rightarrow \log y = \log u + \log v + \log w$

diff. w.r.t x

$$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = \underbrace{\frac{1}{u}.\frac{du}{dx} + \frac{1}{v}.\frac{dv}{dx} + \frac{1}{w}.\frac{dw}{dx}}_{}$$

$$\Rightarrow \frac{dy}{dx} = (u.v.w)\left(\underbrace{\frac{1}{u}.\frac{du}{dx}}_{} + \underbrace{\frac{1}{v}.\frac{dv}{dx}}_{} + \frac{1}{w}.\frac{dw}{dx}\right)$$

$$\Rightarrow \frac{d(u.v.w)}{dx} = \frac{du}{dx}.v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$$

Class 12 maths [Continuity and Differentiability]

Derivatives of Functions in Parametric Form

(प्राचलिक रूप)

Functions

$y \longleftrightarrow x$

Explicit Function	Implicit Function
$y = x^2 + 3x + 1$	$xy^3 = y + \sin x$
$y = \sin x$	$y + \tan^{-1} x = \sec(xy)$
$y = e^{\tan^{-1}(x^2)}$	

Parametric Form

$x \not\longleftrightarrow y$

Parameter

New variable

t, r, θ etc.

e.g. $\begin{pmatrix} y = \cancel{2at}\ 4t \\ x = 7t^2 \end{pmatrix} \to t = \text{Parameter}$

e.g. $\begin{pmatrix} y = 2\sin\theta \\ x = 5\cos\theta \end{pmatrix} \to \theta = \text{Parameter}$

How to find $\dfrac{dy}{dx}$ if $x = f(t)$, $y = g(t)$?

$x \longleftrightarrow t$

$y \longleftrightarrow t$

Diff. w.r.to 't'

$y = g(t) \longrightarrow \dfrac{dy}{dt} = g'(t)$

$x = f(t) \longrightarrow \dfrac{dx}{dt} = f'(t)$

$$\boxed{\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}}$$

e.g. Find $\frac{dy}{dx}$, if $\underbrace{x = a\cos\theta}$, $\underbrace{y = a\sin\theta}$.

$a =$ Constant (अचर)

$\theta \rightarrow$ Parameter (प्राचल)

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{a\cos\theta}{-a\sin\theta}$$

$$\boxed{\frac{dy}{dx} = -\cot\theta}$$

$y = a\sin\theta$

diff. w.r.t 'θ'

$$\frac{dy}{d\theta} = a\cos\theta \quad \text{—} (1)$$

$x = a\cos\theta$

diff. w.r.t. θ

$$\frac{dx}{d\theta} = -a\sin\theta \quad \text{—} (2)$$

↳ Attached Question.

Find $\frac{dy}{dx}$ in terms of x & y.

$$\frac{dy}{dx} = -\cot\theta = \frac{a\cos\theta}{-a\sin\theta}$$

$x = a\cos\theta$

$y = a\sin\theta$

Given

Substitution

$$\Rightarrow \frac{dy}{dx} = \frac{x}{-y}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

Alternate way of solving attached Question

$$\left\{ \begin{aligned} x &= a\cos\theta \\ y &= a\sin\theta \end{aligned} \right\}$$

find $\frac{dy}{dx}$ in terms of x & y.

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

$x^2 = a^2\cos^2\theta$ — ①

$y^2 = a^2\sin^2\theta$ — ②

$+$

$x^2 + y^2 = a^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow \boxed{x^2 + y^2 = a^2}$

$\rightarrow$ by Diff. w.r.t 'x'

$\Rightarrow 2x + 2y\left(\frac{dy}{dx}\right) = 0$

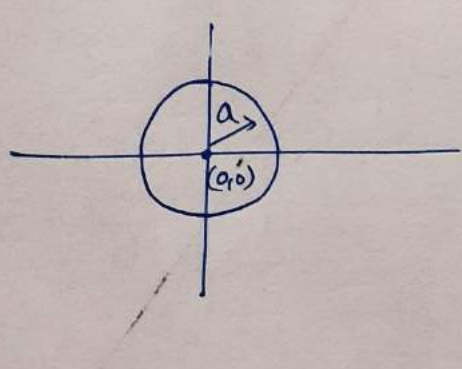
$\Rightarrow \frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

Note: $\boxed{x^2 + y^2 = a^2}$

Circle at Cartesian form

Centre $(0,0)$

Radius a



$$\left\{ \begin{aligned} x &= a\cos\theta \\ y &= a\sin\theta \end{aligned} \right\}$$

$\downarrow$

Circle at

Parametric form

Class 12 Maths [Continuity and Differentiability]

Exercise 5.6 Chapter-5

Parametric Form ☆ $\left(\dfrac{dy}{dx} = \dfrac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}\right)$

$x \cancel{\&} y$

Parameter $(r, t, \theta \ldots)$
(t)

$\boxed{Q.1}$ $x = 2at^2$, $y = at^4$

Diff. w.r.to 't'

$\dfrac{dx}{dt} = 2a(2t)$, $= 4at$ | $\dfrac{dy}{dt} = 4at^3$

$\dfrac{dy}{dx} = \dfrac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \dfrac{\cancel{4a}t^3}{\cancel{4a}t} = t^2$

$\boxed{Q.2}$ $x = a\cos\theta$, $y = b\cos\theta$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\frac{d(b\cos\theta)}{d\theta}}{\frac{d(a\cos\theta)}{d\theta}} = \frac{b(-\cancel{\sin\theta})}{a(-\cancel{\sin\theta})} = \frac{b}{a}$$

$\boxed{Q.3}$ $x = \sin t$, $y = \cos 2t$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{d(\cos 2t)}{dt}\right)}{\left(\frac{d(\sin t)}{dt}\right)} = \frac{-\sin 2t\,.(2)}{\cos t}$$

$$= \frac{-2\sin t\cdot\cancel{\cos t}\times 2}{\cancel{\cos t}}$$

$$= -4\sin t$$

$\boxed{Q.4}$ $x = 4t$, $y = \frac{4}{t}$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\frac{d\left(\frac{4}{t}\right)}{dt}}{\frac{d(4t)}{dt}} = \frac{\cancel{4}\left(-\frac{1}{t^2}\right)}{\cancel{4} \times 1} = -\frac{1}{t^2}$$

$\boxed{Q.5}$ $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\frac{d(\sin\theta - \sin 2\theta)}{d\theta}}{\frac{d(\cos\theta - \cos 2\theta)}{d\theta}} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + \sin 2\theta \cdot 2}$$

✓

$\boxed{Q.6}$ $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\frac{d[a(1+\cos\theta)]}{d\theta}}{\frac{d[a(\theta - \sin\theta)]}{d\theta}} = \frac{\cancel{a}(0 - \sin\theta)}{\cancel{a}(1 - \cos\theta)}$$

$$\frac{dy}{dx} = -\left(\frac{\sin\theta}{1-\cos\theta}\right) = -\left(\frac{\cancel{2}\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}}{\cancel{2}\sin^2 \theta/2}\right)$$

$\cos\theta = 1 - 2\sin^2\theta/2$

$$= -\frac{\cos\theta/2}{\sin\theta/2} = -\cot\frac{\theta}{2}$$

Q.7 $\quad x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}} \qquad y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} =$$

$$y = \frac{\cos^3 t}{\sqrt{\cos 2t}} \qquad \left(\frac{u}{v}\right)' = \frac{u'.v - u.v'}{v^2}$$

by diff. w.r. to 't'

$$\frac{dy}{dt} = \frac{3\cos^2 t \cdot (-\sin t) \cdot \sqrt{\cos 2t} - \cos^3 t \cdot \frac{1}{\cancel{2}\sqrt{\cos 2t}} \cdot (-\sin 2t) \cdot \cancel{2}}{\left(\sqrt{\cos 2t}\right)^2}$$

$$\frac{dy}{dt} = \frac{\left(\frac{-3\cos^2 t \cdot \sin t \cdot \cos 2t + \cos^3 t \cdot \sin 2t}{\sqrt{\cos 2t}}\right)}{\cos 2t}$$

$$\frac{dy}{dt} = \frac{-3\cos^2 t \cdot \sin t \cdot \cos 2t + \cos^3 t \cdot \sin 2t}{\cos 2t \cdot \sqrt{\cos 2t}} \quad \text{--- (1)}$$

$$x = \frac{\sin^3 t}{\sqrt{\cos 2t}} \qquad \left(\frac{u}{v}\right)' = \frac{u'.v - u.v'}{v^2}$$

by diff. w.r. to 't'

$$\frac{dx}{dt} = \frac{3\sin^2 t.\cos t.\sqrt{\cos 2t} - \sin^3 t.\frac{1}{\cancel{2}\sqrt{\cos 2t}}(-\sin 2t).\cancel{(2)}}{\cos 2t}$$

$$\frac{dx}{dt} = \frac{\frac{3\sin^2 t.\cos t.\cos 2t + \sin^3 t.\sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$$

$$\Rightarrow \frac{dx}{dt} = \frac{3\sin^2 t.\cos t.\cos 2t + \sin^3 t.\sin 2t}{\cos 2t.\sqrt{\cos 2t}} \quad \text{—(2)}$$

$$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$$

(by putting the values of $\frac{dy}{dt}$ & $\frac{dx}{dt}$ ~~in~~ from eq^n (1) & (2))

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{-3\cos^2 t.\sin t.\boxed{\cos 2t} + \cos^3 t.\boxed{\sin 2t}}{\cancel{\cos 2t.\sqrt{\cos 2t}}}}{\frac{3\sin^2 t.\cos t.\boxed{\cos 2t} + \sin^3 t.\boxed{\sin 2t}}{\cancel{\cos 2t.\sqrt{\cos 2t}}}}$$

put $\boxed{\sin 2t = 2\sin t.\cos t}$

$$\Rightarrow \frac{dy}{dx} = \frac{-3\cos^2 t \cdot \sin t \cdot \boxed{\cos 2t} + \cos^3 t \cdot (2\sin t \cdot \cos t)}{3\sin^2 t \cdot \cos t \cdot \boxed{\cos 2t} + \sin^3 t \cdot (2\sin t \cdot \cos t)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cancel{\sin t} \cdot \cos^{\cancel{2}} t \cdot \left[-3 \cdot \cos 2t + 2\cos^2 t\right]}{\sin^{\cancel{2}} t \cdot \cancel{\cos t}\left[3\cos 2t + 2\sin^2 t\right]}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos t\left[-3(2\cos^2 t - 1) + 2\cos^2 t\right]}{\sin t\left[3(1 - 2\sin^2 t) + 2\sin^2 t\right]}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos t \cdot \left[-4\cos^2 t + 3\right]}{\sin t \cdot \left[3 - 4\sin^2 t\right]}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-4\cos^3 t + 3\cos t}{3\sin t - 4\sin^3 t} = \frac{-\cos 3t}{\sin 3t}$$

$$\boxed{\frac{dy}{dx} = -\cot 3t}$$

Q.8 $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a\sin t$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\frac{d(a\sin t)}{dt}}{\frac{d\left(a\left(\cos t + \log\tan\frac{t}{2}\right)\right)}{dt}}$$

$$= \frac{\cancel{a}\cos t}{\cancel{a}\left(-\sin t + \frac{1}{\tan t/2}\cdot\sec^2\frac{t}{2}\cdot\frac{1}{2}\right)}$$

$$= \frac{\cos t}{\left(-\sin t + \frac{\cancel{\cos t/2}}{2\sin t/2}\cdot\frac{1}{\cos^2 t/2}\right)} = \frac{\cos t}{\left(-\sin t + \frac{1}{\sin t}\right)}$$

$$= \frac{\cos t}{\left(\frac{-\sin^2 t + 1}{\sin t}\right)} = \frac{\cancel{\cos t}\cdot\sin t}{\cos^2 t}$$

$$= \tan t$$

$$\boxed{\frac{dy}{dx} = \tan t}$$

$\boxed{Q.9}$ $x = a\sec\theta$, $y = b\tan\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\left(\frac{d(b\tan\theta)}{d\theta}\right)}{\left(\frac{d(a\sec\theta)}{d\theta}\right)} = \frac{b\sec^2\theta}{a\cancel{\sec\theta}\cdot\tan\theta}$$

$$\frac{dy}{dx} = \frac{b}{a}\frac{\left(\frac{1}{\cancel{\cos\theta}}\right)}{\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)} = \frac{b}{a}\frac{1}{\sin\theta} = \frac{b}{a}\operatorname{cosec}\theta$$

$\boxed{Q.10}$ $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\frac{d(a(\sin\theta - \underline{\theta.\cos\theta}))}{d\theta}}{\frac{d(a(\cos\theta + \underline{\theta\cdot\sin\theta}))}{d\theta}}$$

$$= \frac{\cancel{a}[\cancel{\cos\theta} - 1.\cancel{\cos\theta} + \theta.\sin\theta]}{\cancel{a}[-\cancel{\sin\theta} + 1.\cancel{\sin\theta} + \theta.\cos\theta]}$$

$$= \frac{\cancel{\theta}.\sin\theta}{\cancel{\theta}.\cos\theta} = \tan\theta$$

Q.11 If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, then show that $\boxed{\dfrac{dy}{dx} = -\dfrac{y}{x}}$

$\boxed{x = a^{\frac{\sin^{-1}t}{2}}}$, $\boxed{y = a^{\frac{\cos^{-1}t}{2}}}$

I- method:

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\dfrac{d\left(a^{\frac{\cos^{-1}t}{2}}\right)}{dt}}{\dfrac{d\left(a^{\frac{\sin^{-1}t}{2}}\right)}{dt}} = \frac{a^{\frac{\cos^{-1}t}{2}} \cdot \cancel{\log a} \cdot \cancel{\frac{1}{2}} \cdot \cancel{\frac{-1}{\sqrt{1-t^2}}}}{a^{\frac{\sin^{-1}t}{2}} \cdot \cancel{\log a} \cdot \cancel{\frac{1}{2}} \cdot \cancel{\frac{1}{\sqrt{1-t^2}}}}$$

$$\boxed{\frac{d(a^x)}{dx} = a^x \cdot \log a}$$

$$= \frac{-y}{x}$$

II - method

$x = a^{\frac{\sin^{-1}t}{2}}$, $y = a^{\frac{\cos^{-1}t}{2}}$ (logarithmic Diff.)

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$= \frac{\dfrac{-y}{\cancel{\sqrt{1-t^2}}} \cdot \dfrac{\cancel{\log a}}{\cancel{2}}}{\dfrac{x}{\cancel{\sqrt{1-t^2}}} \cdot \dfrac{\cancel{\log a}}{\cancel{2}}}$$

$$= \frac{-y}{x}$$

$x = a^{\frac{\sin^{-1}t}{2}}$

$\Rightarrow \log x = \dfrac{\sin^{-1}t}{2} \cdot \log a$

diff. w.r.to t

$\Rightarrow \dfrac{1}{x} \cdot \dfrac{dx}{dt} = \dfrac{1}{\sqrt{1-t^2}} \cdot \dfrac{\log a}{2}$

$\Rightarrow \boxed{\dfrac{dx}{dt} = \dfrac{x}{\sqrt{1-t^2}} \cdot \dfrac{\log a}{2}}$

Similarly

$$\frac{dy}{dt} = \frac{-y}{\sqrt{1-t^2}} \cdot \frac{\log a}{2}$$

III - method

$$\underbrace{x = a^{\frac{\sin^{-1}t}{2}}}_{(1)}, \quad \underbrace{y = a^{\frac{\cos^{-1}t}{2}}}_{(2)}$$

To prove,

$$\frac{dy}{dx} = \frac{-y}{x}$$

Property $\boxed{\sin^{-1}t + \cos^{-1}t = \frac{\pi}{2}}$

By eq^n (1) × eq^n (2) :→

$$xy = a^{\frac{\sin^{-1}t}{2}} \cdot a^{\frac{\cos^{-1}t}{2}}$$

$$\Rightarrow xy = a^{\frac{1}{2}\left(\sin^{-1}t + \cos^{-1}t\right)} \rightarrow \frac{\pi}{2}$$

$$\Rightarrow \boxed{xy = a^{\frac{\pi}{4}}}$$

by diff. w.r.t. 'x'

$$\Rightarrow 1 \cdot y + x \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-y}{x}}$$

Second Order Derivative (द्वितीय कोटि अवकलज)

$y = f(x) \longrightarrow$ Function

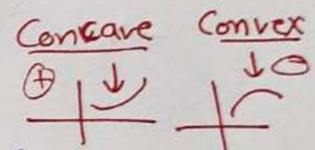
$D^1 y = y' = y_1 = \dfrac{dy}{dx} = \dfrac{d(f(x))}{dx} = f'(x) \longrightarrow$ First order Derivative

$D^2 y = y'' = y_2 = \dfrac{d^2 y}{dx^2} = \dfrac{d^2(f(x))}{dx^2} = \dfrac{d\left(\dfrac{d(f(x))}{dx}\right)}{dx} = f''(x)$

$\hookrightarrow$ Second order Derivative

$y = f(x) \longrightarrow$ Function

$\dfrac{dy}{dx} = y' = f'(x) \longrightarrow$ Slope of function



$\dfrac{d^2y}{dx^2} = y'' = f''(x) \longrightarrow$ Concavity of the function.

Distance (S)	Velocity (V)	Acceleration (a)
	$V = \dfrac{d(S)}{dt}$	$a = \dfrac{dv}{dt} = \dfrac{d\left(\dfrac{ds}{dt}\right)}{dt} = \dfrac{d^2(S)}{dt^2}$

e.g. If $y = \sin^{-1} x$, then prove that $\boxed{(1-x^2)\, y'' - x \cdot y' = 0}$

$y'' \to \dfrac{d^2y}{dx^2}$, $\quad y' \to \dfrac{dy}{dx}$

$y = \sin^{-1} x$

Diff. w.r.t. x $\rightarrow$

$y' = \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

$y = \sin^{-1} x$

$y' = \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

Again diff. w.r.t. $x \rightarrow$

$\left(\frac{u}{v}\right)' = \frac{u'.v - u.v'}{v^2}$

$y'' = \frac{d^2y}{dx^2} = \frac{d\left(\frac{1}{\sqrt{1-x^2}}\right)}{dx} = \frac{\cancel{(0.\sqrt{1-x^2})} - 1.\left(\frac{1.(0-\cancel{2}x)}{\cancel{2}\sqrt{1-x^2}}\right)}{(1-x^2)}$

$y'' = +\frac{x}{(1-x^2)\sqrt{1-x^2}}$

$\Rightarrow (1-x^2).y'' = \frac{x}{\sqrt{1-x^2}}$

$\Rightarrow (1-x^2).y'' = x.y'$

$\Rightarrow \boxed{(1-x^2).y'' - x.y' = 0}$ H.P.

Fast.

$y = \sin^{-1} x$

Diff. $\Rightarrow y' = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow (\sqrt{1-x^2}).y' = 1$

~~Diff~~

Square

$\Rightarrow \underbrace{(1-x^2).(y')^2}_{(u.v)'} = 1$

Diff.

$(-2x).(y')^2 + (1-x^2).2(y').(y'') = 0$

$\Rightarrow 2y'(-xy' + (1-x^2).y'') = 0$

$\Rightarrow \boxed{(1-x^2).y'' - xy' = 0}$

e.g. If $x = a\cos\theta$, $y = b\sin\theta$, Find $\dfrac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

$y = b\sin\theta$

$\dfrac{dy}{d\theta} = b\cos\theta$

$x = a\cos\theta$

$\dfrac{dx}{d\theta} = -a\sin\theta$

$$\frac{dy}{dx} = \frac{b\cos\theta}{-a\sin\theta}$$

$$\boxed{\frac{dy}{dx} = -\frac{b}{a}\cdot\cot\theta}$$

$\dfrac{d^2y}{dx^2} = ?$

x, y, θ

by diff. w.r.t. x $\rightarrow$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a}\cdot(-\text{cosec}^2\theta)\cdot\frac{d\theta}{dx}$$

$$\boxed{\frac{dx}{d\theta} = -a\sin\theta}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a}(\text{cosec}^2\theta)\cdot\left(\frac{1}{-a\sin\theta}\right)$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{b}{a^2}\cdot\text{cosec}^3\theta}$$

Class 12 Maths [Continuity and Differentiability]

Exercise-5.7 Chapter-5

Find the second order derivatives.

$\boxed{y} \to \boxed{\frac{dy}{dx}} \to \boxed{\frac{d^2y}{dx^2}}$

Q.1

let $y = x^2+3x+2$

$\Rightarrow \frac{dy}{dx} = 2x+3$

$\Rightarrow \frac{d^2y}{dx^2} = 2$

Q.2 $y = x^{20}$

$\Rightarrow \frac{dy}{dx} = 20.x^{19}$ $\Rightarrow \frac{d^2y}{dx^2} = 20.19.x^{18} = 380x^{18}$

Q.3 $y = x\cos x$

$(u.v)'$

$\Rightarrow \frac{dy}{dx} = 1.\cos x + x(-\sin x) = \cos x - x\sin x$

$(u.v)'$

$\Rightarrow \frac{d^2y}{dx^2} = -\sin x - (1.\sin x + x.\cos x)$

$\Rightarrow \frac{d^2y}{dx^2} = -2\sin x - x\cos x.$

Q.4 $y = \log x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$ $\Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{-1}{x^2}}$

$\frac{1}{x} = x^{-1}$ $x^n \to n.x^{n-1}$

$\boxed{Q.5}$ $y = \underbrace{x^3}\cdot\underbrace{\log x}$ $\underline{(u.v)'}$

$\Rightarrow \frac{dy}{dx} = 3x^2.\log x + x^3.\frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \underbrace{3x^2}.\underbrace{\log x} + x^2$

$\hookrightarrow \frac{d^2y}{dx^2} = \underbrace{6x.\log x + 3x^2.\frac{1}{\cancel{x}}} + 2x$

$\hookrightarrow = 6x.\log x + 5x$

$\boxed{Q.6}$ $y = \underbrace{e^x}_{(u.v)'}.\underbrace{\sin 5x}$ $\frac{d(e^x)}{dx} = e^x$

$\frac{dy}{dx} = e^x.\sin 5x + e^x.\cos 5x.5$

$\Rightarrow \frac{dy}{dx} = e^x.\sin 5x + 5e^x.\cos 5x$

$\Rightarrow \frac{dy}{dx} = \underbrace{e^x}.\underbrace{(\sin 5x + 5\cos 5x)}$

$\hookrightarrow \frac{d^2y}{dx^2} = \underbrace{e^x.(\sin 5x + 5\cos 5x)} + \underbrace{e^x(5\cos 5x + 5\times 5\times(-\sin 5x))}$

$\frac{d^2y}{dx^2} = e^x(-24\sin 5x + 10.\cos 5x)$

$\boxed{Q.7}$ $y = \underbrace{e^{6x}} \cdot \underbrace{\cos 3x}$

$$\frac{dy}{dx} = \underbrace{e^{6x} \cdot 6} \cdot \cos 3x + e^{6x} \cdot \underbrace{(-3\sin 3x)}$$

$$\frac{dy}{dx} = 3\underbrace{e^{6x}} \cdot \underbrace{(2\cos 3x - \sin 3x)}$$

$$\frac{d^2y}{dx^2} = 3 \cdot e^{6x} \cdot 6 \, (2\cos 3x - \sin 3x)$$
$$+ 3(e^{6x}) \cdot (-2\sin 3x \cdot 3 - \cos 3x \cdot 3)$$

$$\frac{d^2y}{dx^2} = 3e^{6x} \cdot [9\cos 3x - 12\sin 3x]$$
$$= 9e^{6x}(3\cos 3x - 4\sin 3x)$$

$\boxed{Q.8}$ $y = \tan^{-1} x$

$$\frac{d\left(\frac{1}{x}\right)}{dx} = -\frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(1+x^2)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{(1+x^2)^2} \times (0+2x) = \frac{-2x}{(1+x^2)^2}$$

Q.9 $y = \log(\log x)$ (Chain Rule)

$\frac{d(\log x)}{dx} = \frac{1}{x}$

$\frac{d\left(\frac{1}{x}\right)}{dx} = -\frac{1}{x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$ $u.v$

$\frac{d^2y}{dx^2} = -\frac{1}{(x \log x)^2} \cdot \left(1.\log x + \cancel{x}.\frac{1}{\cancel{x}}\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{(\log x + 1)}{(x \log x)^2}$

Q.10 $y = \sin(\log x)$ Chain

$\Rightarrow \frac{dy}{dx} = \cos(\log x) \cdot \frac{1}{x}$ $(u.v)'$

$\Rightarrow \frac{d^2y}{dx^2} = -\sin(\log x) \cdot \frac{1}{x} \cdot \frac{1}{x} + \cos(\log x) \cdot \left(-\frac{1}{x^2}\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{\sin(\log x) + \cos(\log x)}{x^2}\right)$

Q.11 If $y = 5\cos x - 3\sin x$, Prove that $\frac{d^2y}{dx^2} + y = 0$

$y = 5\cos x - 3\sin x$

$\frac{dy}{dx} = -5\sin x - 3\cos x$

$\Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -(5\cos x - 3\sin x)$

$\Rightarrow \frac{d^2y}{dx^2} = -y$

$\Rightarrow \boxed{\frac{d^2y}{dx^2} + y = 0}$

Q.12 If $y = \cos^{-1}x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone.

Ans. $y = \cos^{-1}x$

$\Rightarrow \cos y = x$

Diff. w.r.t. 'x'

$\Rightarrow -\sin y \frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sin y}$ —— (1)

$\frac{d\left(\frac{1}{x}\right)}{dx} = \frac{-1}{x^2}$

Diff. w.r.t. x

$\Rightarrow + \frac{d^2y}{dx^2} = -\left(-\frac{1}{\sin^2 y}\right) \cdot (\cos y) \cdot \frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{\cos y}{\sin^2 y} \cdot \left(\frac{-1}{\sin y}\right)$

$$\boxed{\frac{d^2y}{dx^2} = -\cot y \cdot \operatorname{cosec}^2 y}$$

Class 12 maths [Continuity and Differentiability]

Exercise 5.7 Chapter (5)

Q.13 If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + x y_1 + y = 0$. $\left(y_2 = \frac{d^2y}{dx^2},\ y_1 = \frac{dy}{dx}\right)$

Ans

$$y = 3\cos(\log x) + 4\sin(\log x)$$

Diff. w.r.t. x —

$$\Rightarrow \underbrace{\frac{dy}{dx}}_{y_1} = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$$

$$\Rightarrow x.y_1 = -3\sin(\log x) + 4\cos(\log x)$$

Diff. w.r.t. x

$$\Rightarrow 1.y_1 + x.y_2 = \frac{-3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$$

$$\Rightarrow x y_1 + x^2 y_2 = -(3\cos\log x + 4\sin\log x)$$

$$\Rightarrow x^2 y_2 + x y_1 = -(y)$$

$$\Rightarrow \boxed{x^2 y_2 + x y_1 + y = 0}$$

$\boxed{Q.14}$ If $y = Ae^{mx} + Be^{nx}$, show that

$$\underbrace{\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny}_{\text{LHS}} = 0$$

Ans.

$$y = Ae^{mx} + Be^{nx}$$

$$\frac{dy}{dx} = m.A.e^{mx} + n.B.e^{nx}$$

$$\frac{d^2y}{dx^2} = m^2A.e^{mx} + n^2Be^{nx}$$

$$\text{LHS} = \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mn.y$$

$$= \underbrace{m^2Ae^{mx} + n^2Be^{nx}}\; - (m+n)(mAe^{mx} + nBe^{nx})$$
$$+ mn(Ae^{mx} + Be^{nx})$$

$$= \cancel{m^2Ae^{mx}} + \cancel{n^2Be^{nx}} - \cancel{m^2Ae^{mx}} - \cancel{mnBe^{nx}}$$
$$- \cancel{mnAe^{mx}} - \cancel{n^2Be^{nx}} + \cancel{mnAe^{mx}} + \cancel{mnBe^{nx}}$$

$$= 0 = \text{RHS}.$$

$\boxed{Q.15}$ If $y = 500.e^{7x} + 600.e^{-7x}$, show that $\boxed{\frac{d^2y}{dx^2} = 49y}$

$$\frac{dy}{dx} = \underbrace{500.e^{7x}.7} + \underbrace{600.e^{-7x}.(-7)}$$

$$\frac{d^2y}{dx^2} = 500.e^{7x}.(7)^2 + 600.e^{-7x}.(-7)^2$$

$$\frac{d^2y}{dx^2} = 49\left(\underbrace{500e^{7x} + 600.e^{-7x}}_{y}\right) = 49.y$$

Q.16 If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$.

Sol: $\underbrace{e^y}\cdot\underbrace{(x+1)}=1$ $\qquad (u.v)'=u'.v+u.v'$

Diff. w.r.t. x →

$\Rightarrow e^y\cdot\frac{dy}{dx}\cdot(x+1)+e^y\cdot(1+0)=0$

$\Rightarrow e^y\left(\frac{dy}{dx}\cdot(x+1)+1\right)=0$

$\Rightarrow \frac{dy}{dx}(x+1)+1=0 \Rightarrow \boxed{\frac{dy}{dx}=-\frac{1}{(x+1)}}$

By Diff. w.r.t. x → $\qquad -\frac{dy}{dx}=\frac{1}{x+1}$

$\frac{d^2y}{dx^2}=-\left(\frac{-1}{(x+1)^2}\right). \Rightarrow \frac{d^2y}{dx^2}=\left(\frac{1}{x+1}\right)^2$

$\Rightarrow \boxed{\frac{d^2y}{dx^2}=\left(-\frac{dy}{dx}\right)^2=\left(\frac{dy}{dx}\right)^2}$

Q.17. If $y=(\tan^{-1}x)^2$, show that

$$\boxed{(x^2+1)^2 \cdot y_2 + 2x(x^2+1) \cdot y_1 = 2}$$

$\frac{d^2y}{dx^2}$ $\frac{dy}{dx}$

Solution.

$$y = (\tan^{-1}x)^2$$

$$\Rightarrow \frac{dy}{dx} = y_1 = \frac{2(\tan^{-1}x)}{1+x^2}.$$

$$\Rightarrow (x^2+1) \cdot y_1 = 2\tan^{-1}x$$

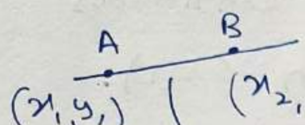
Diff. w.r.t. x →

$$\Rightarrow 2x \cdot y_1 + (x^2+1) \cdot y_2 = \frac{2}{1+x^2}$$

$$\Rightarrow \boxed{2x(1+x^2) \cdot y_1 + (x^2+1)^2 \cdot y_2 = 2}$$

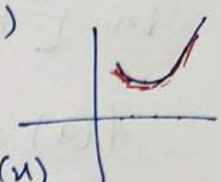
Class 12 maths [Continuity and Differentiability]

Today → Rolle's Theorem
→ Mean Value Theorem (मध्यमान प्रमेय)



Slope (प्रवणता) $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$y = f(x)$



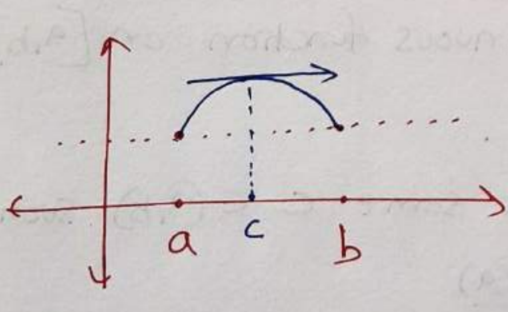
$\dfrac{dy}{dx} = f'(x) \to$ Slope

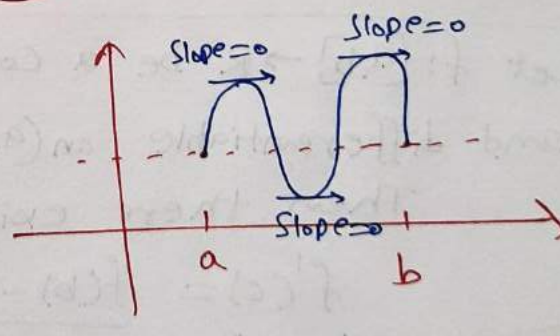
Rolle's Theorem:

~~Let f: [A,B] → R~~

Let $f: [a,b] \to R$ be continuous on $[a,b]$ and differentiable on (a,b), Such that $f(a) = f(b)$, where a, b are some real numbers. Then there exists some c in (a,b) such that $f'(c) = 0$.

$f'(c) \to$ slope at $x = c$





(Slope = 0) → graph → Horizontal

Rolle's

$f: [a,b] \to R$

① Continuous
② Differentiable
③ $f(a) = f(b)$
④ $f'(c) = 0, \; c \in (a,b)$

e.g. Verify Rolle's Theorem for the function $y = x^2+2$, $x \in [-2, 2]$

Ans.

$f(x) = x^2+2$

We know that $f(x) = x^2+2$ is continuous in $[-2,2]$ & differentiable in $(-2,2)$.

$f(a) = f(-2) = (-2)^2+2 = 6$

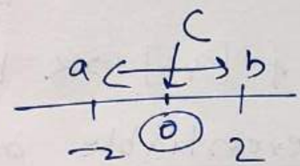
$f(b) = f(2) = (2)^2+2 = 6$

$\Rightarrow \underline{f(a) = f(b)}$

$f(x) = x^2+2 \longrightarrow \boxed{f'(x) = 2x}$

$f'(c) = 2c = 0$

$\Rightarrow \boxed{c=0} \in [-2,2]$



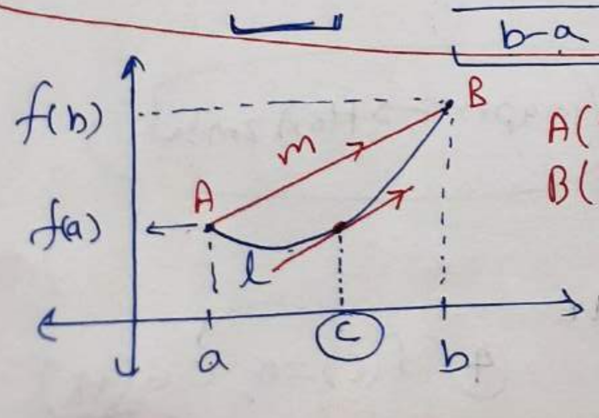
$\therefore$ Rolle's Theorem is verified.

Mean Value Theorem (MVT) (मध्यमान प्रमेय)

Let $f: [a,b] \to R$ be a continuous function on $[a,b]$ and differentiable on (a,b).

Then there exists some $c \in (a,b)$ such that

$$\underbrace{f'(c)} = \underbrace{\frac{f(b)-f(a)}{b-a}}$$



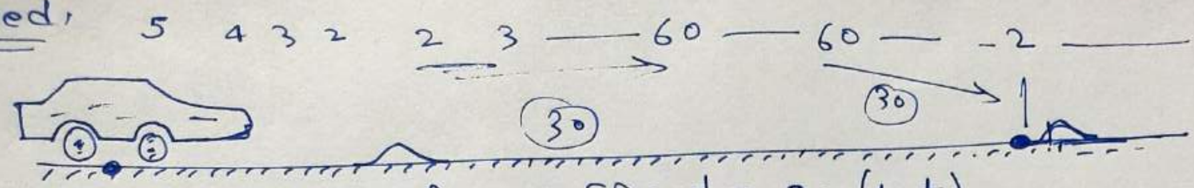
$A(a, f(a))$

$B(b, f(b))$

$m_{AB} = \frac{f(b)-f(a)}{b-a}$

Slope of 'f' at c $= f'(c)$

Speed,



Average Speed = 30 (let)

At some instant

$f'(c) = 30$ } speed $= \frac{f(b)-f(a)}{b-a}$

e.g. Verify mean value theorem for the function $f(x) = x^2$ in the interval $[2, 4]$

MVT → Continuous ✓ in $[2, 4]$

Differentiable ✓ in $(2, 4)$ (a, b)

$f'(c) = \frac{f(b)-f(a)}{b-a}$, $c \in (2, 4)$??

$f(x) = x^2$

$f'(x) = 2x$

$f'(c) = 2c$

$\frac{f(4)-f(2)}{4-2} = \frac{16-4}{4-2} = \frac{12}{2} = 6$

$2c = 6$

$c = 3 \in (2, 4)$ ✓

∴ MVT ✓

Comparison,

$f: [a, b] \to R$

$c \in (a, b)$

Rolle's Theorem	MVT
① continuous	① continuous
② Differentiable	② Differentiable
③ $f(a) = f(b)$	③ $f'(c) = \frac{f(b)-f(a)}{b-a}$
④ $f'(c) = 0$	

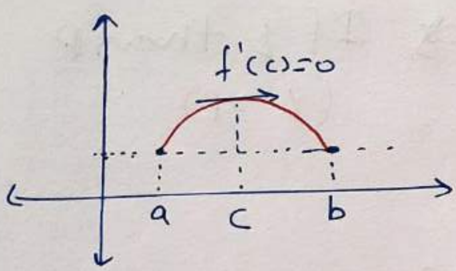
Class 12 Maths [Continuity and Differentiability]

Exercise-5.8 Chapter 5

Rolle's Theorem

$f:[a,b]\to R$

1. Continuous in $[a,b]$
2. differentiable in (a,b)
3. $f(a)=f(b)$
4. $f'(c)=0$, $\Rightarrow \boxed{c\in(a,b)}$

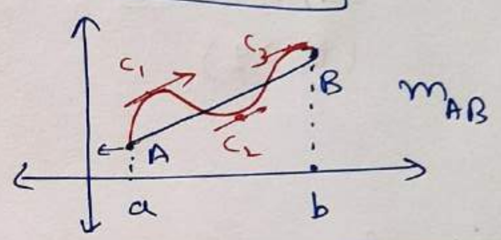


Mean Value Theorem (MVT)

$f:[a,b]\to R$

1. Continuous in $[a,b]$
2. differentiable in (a,b)
3. $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

$\Rightarrow \boxed{c\in(a,b)}$



Q.1 Verify Rolle's Theorem; $f(x)=x^2+2x-8,\ x\in[-4,2]$

$a=-4$
$b=2$

we know that $f(x)=x^2+2x-8$ is continuous in $[-4,2]$ and differentiable in $(-4,2)$.

$f(-4)=(-4)^2+2(-4)-8 = 16-8-8=0$

$f(2)=2^2+2(2)-8 = 8-8=0$

$\therefore f(-4)=f(2)$

$f(x)=x^2+2x-8$

$f'(x)=2x+2$

$f'(c)=0$

$\Rightarrow 2c+2=0$

$\Rightarrow \boxed{c=-1}\in(-4,2)$

Q.2 Rolle's Theorem? | Converse of Rolle's Theorem?

If $f:[a,b]\to R$ is continuous on $[a,b]$ and differentiable on (a,b) such that $f(a)=f(b)$, **then** there exists some c in (a,b) such that $f'(c)=0$.

If there exists some c in (a,b) such that $f'(c)=0$ **then** f is continuous on $[a,b]$ and differentiable on (a,b) such that $f(a)=f(b)$.

Class 11 Math. Reasoning.

If p, then q $\xrightarrow{\text{Converse}}$ If q then p

$(p\to q)$ $\qquad$ $(q\to p)$

(i) $f(x)=[x]$ for $x\in[5,9]$

(ii) $f(x)=[x]$ for $x\in[-2,2]$

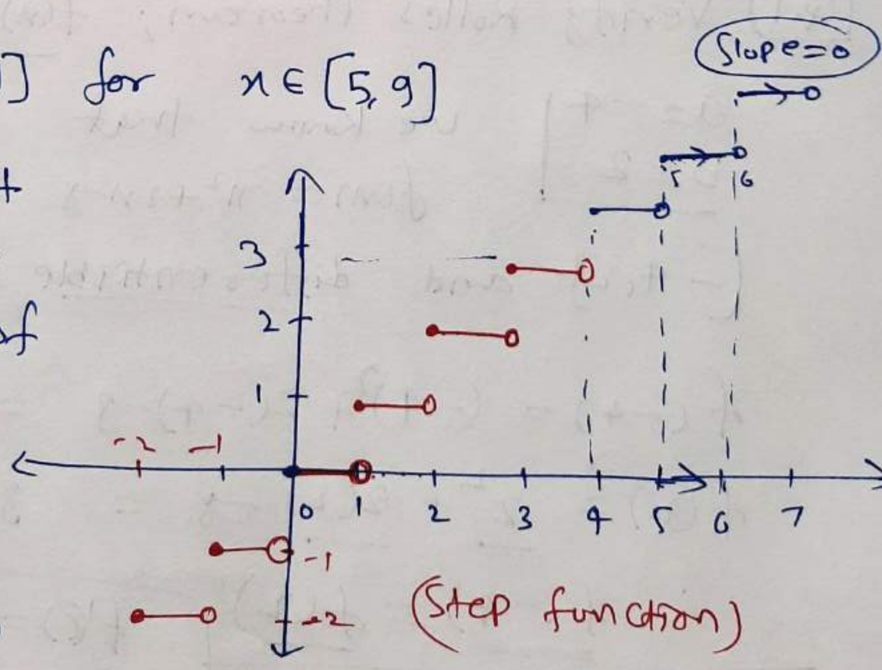
Similar

(i) $f(x)=[x]$ for $x\in[5,9]$

$f(x)=[x]$ is not continuous at integral values of x in $[5,9]$

6, 7, 8, 9

$\therefore$ Rolle's theorem is not applicable.



$f(x) = [x]$ for $[5,9]$ Converse of Rolle's Theorem

If $\underbrace{c \in (a,b),\ f'(c) = 0}$ then

- Continuous $[a,b]$ ✗
- Diff. (a,b) ✗
- $f(a) \neq f(b)$ ✗

$f(5) \neq f(9)$

$[5] \neq [9]$

$5 \neq 9$

$\underbrace{c \in (5,9),\ f'(c) = 0}$

$\downarrow$

Slope at $x = c$

$\downarrow$

0

Slope $= 0$ $\rightarrow$ (Graph Horizontal)

Here $f'(c) = 0$ for some $c \in (5,9)$

but f is not continuous in $[5,9]$

$\therefore$ Converse of Rolle's theorem is not applicable.

(iii) $f(x) = x^2 - 1$ for $x \in [1,2]$

Rolle's Theorem: Continuous in $[1,2]$

Diff. in $(1,2)$

$f(a) \neq f(b)$

$f(1) \neq f(2)$

$\downarrow \qquad \downarrow$

$1^2 - 1 = 0 \qquad 2^2 - 1 = 3$

Rolle's theorem not applicable

$f(x) = x^2 - 1$ for $x \in [1,2]$

Converse of Rolle's Theorem

If there exists some $c \in (a,b)$ such that $f'(c) = 0$ then

- Continuous $[a,b]$
- Diff. (a,b)
- $f(a) = f(b)$

$f(x) = x^2 - 1$

$f'(x) = 2x$

$f'(c) = 2c = 0$

$\Rightarrow \boxed{c=0} \notin (1,2)$

'f' does not satisfy the converse of Rolle's Theorem

$\boxed{Q.3}$ If $f: [-5,5] \to R$ is a differentiable function and $f'(x)$ does not vanish anywhere, ~~where~~ Prove that $f(-5) \neq f(5)$.

Vanish $= 0$ Zero

Solution

f is a diff. function then it is continous.

by MVT

$c \in (-5, 5)$, $a = -5$, $b = 5$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$f'(x) \neq 0$

$f'(c) \neq 0$

$$f'(c) = \frac{f(5) - f(-5)}{5 - (-5)} \neq 0$$

$\Rightarrow f(5) - f(-5) \neq 0$

$\Rightarrow \boxed{f(5) \neq f(-5)}$

Q.4 Verify MVT, if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $\underline{a = 1}$ and $\underline{b = 4}$.

Ans. $f(x) = x^2 - 4x - 3$ is continuous in $[1, 4]$ and differentiable in $(1, 4)$

Let $c \in (1, 4)$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 2c - 4 = \frac{(-3) - (-6)}{4 - 1}$$

$$\Rightarrow 2c - 4 = \frac{3}{3} = 1$$

$$\Rightarrow c = 5/2 \in (1, 4)$$

$$5/2 = 2.5$$

$\therefore$ MVT $\longrightarrow$ Verified

$f(x) = x^2 - 4x - 3$

$f'(x) = 2x - 4$

$f'(c) = 2c - 4$

$f(b) = f(4)$

$= \cancel{4^2} - \cancel{16} - 3$

$= -3$

$f(a) = f(1)$

$= 1^2 - 4 - 3$

$= -6$

Q. 5 Verify MVT, if $f(x) = x^3 - 5x^2 - 3x$ in the interval $[a, b]$, where $a = 1$, $b = 3$. Find all $c \in (1,3)$ for which $f'(c) = 0$.

Sol^n: f is continuous in $[1,3]$ and differentiable in $(1,3)$.

$a = 1, b = 3$

$f(x) = x^3 - 5x^2 - 3x$

$f'(x) = 3x^2 - 10x - 3$

$f'(c) = 3c^2 - 10c - 3$

$f(b) = f(3)$
$= 27 - 45 - 9$
$= -29$ wrong

Correct -27

$f(a) = f(1) = 1^3 - 5 - 3$
$= -7$

$c \in (1,3)$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 3c^2 - 10c - 3 = \frac{(-29) - (-7)}{3 - 1}$$

$$\Rightarrow 3c^2 - 10c - 3 = \frac{-22}{2} = -11$$

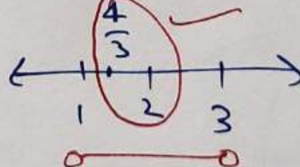
$$\Rightarrow 3c^2 - 10c + 8 = 0$$

$$\Rightarrow 3c^2 - 6c - 4c + 8 = 0$$

$$\Rightarrow 3c(c-2) - 4(c-2) = 0$$

$$\Rightarrow (c-2).(3c-4) = 0$$

wrong $c = 2, \frac{4}{3} \in (1,3)$



MVT verified

Correct $c = 1, 7/3$ $\quad c = \frac{7}{3}$

$f'(c) = 0 \quad c \in (1,3)$

$$\Rightarrow 3c^2 - 10c - 3 = 0 \Rightarrow c = \frac{10 \pm \sqrt{100 + 36}}{6} \approx \frac{10 \pm 11.6}{2}$$

~~$\Rightarrow 3c^2 - 9c - c - 3 = 0$~~

$(+)$ $10.8 \notin (1,3)$ $\qquad (-)$ $-0.8 \notin (1,3)$

Q6 Verify MVT for Q.2 (Parts)

(i)

$f(x) = [x], \; x \in [5, 9]$

not continuous

MVT ✗

(ii)

$f(x) = [x]$

$x \in [-2, 2]$

not continuous

MVT ✗

(iii) $f(x) = x^2 - 1$

$x \in [1, 2]$

→ Continuous ✓ $[1, 2]$

→ Differentiable ✓ $(1, 2)$

→ $c \in (1, 2)$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$f(x) = x^2 - 1$

$f'(x) = 2x$

$f'(c) = 2c$

$a = 1, \; b = 2$

$f(a) = f(1) = 0$ ✓

$f(b) = f(2) = 3$ ✓

$$\Rightarrow 2c = \frac{3 - 0}{2 - 1}$$

$$\Rightarrow 2c = 3$$

$$\Rightarrow \boxed{c = 3/2} \in (1, 2)$$

1.5 ✓

MVT ✓

Class 12 maths [Continuity and Differentiability]

Miscellaneous Exercise on Chapter-5

$\boxed{Q.1}$ $(3x^2 - 9x + 5)^9$

$$\frac{d(x^n)}{dx} = n \cdot x^{n-1} \quad \text{(Chain Rule)}$$

$$\frac{d(3x^2 - 9x + 5)^9}{dx} = 9(3x^2 - 9x + 5)^8 \cdot (6x - 9)$$

$$= 27(2x - 3) \cdot (3x^2 - 9x + 5)^8$$

$\boxed{Q.2}$

$$y = \sin^3 x + \cos^6 x = (\sin x)^3 + (\cos x)^6$$

$$\frac{dy}{dx} = \underline{3(\sin^2 x) \cdot \cos x} + 6 \cdot \underline{\cos^5 x \cdot (-\sin x)}$$

$$= 3 \sin x \cos x (\sin x - 2\cos^4 x)$$

$\boxed{Q.3}$

$$y = (5x)^{3\cos 2x}$$

(Let) (Variable) Variable

logarithmic Diff.

x^x x^n a^x

$$\Rightarrow \log y = \log (5x)^{3\cos 2x}$$

$$\Rightarrow \log y = \underbrace{3 \cdot \cos 2x}_{(u} \cdot \underbrace{\log 5x}_{v)'}$$

$$\frac{d(\log x)}{dx} = \frac{1}{x}$$

Diff. w.r.t. (x) $\rightarrow$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \left[[-3\sin 2x \cdot 2] \cdot \log 5x + 3\cos 2x \cdot \frac{1}{\cancel{5}x} \cdot \cancel{5}\right]$$

$$\Rightarrow \frac{dy}{dx} = (5x)^{3\cos 2x} \cdot \left[-6\sin 2x \cdot \log 5x + \frac{3\cos 2x}{x}\right]$$

Q.4 $y = \sin^{-1}(x\sqrt{x})$; $0 \le x \le 1$

$$\underbrace{x\sqrt{x}} = x^1 \cdot x^{\frac{1}{2}} = \underbrace{x^{3/2}}$$

$$y = \sin^{-1}\left(x^{3/2}\right)$$

diff. w.r.t. x → (chain) $\frac{d\,x^n}{dx} = n \cdot x^{n-1}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(x^{3/2}\right)^2}} \cdot \frac{3}{2} \cdot x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{3\sqrt{x}}{2\sqrt{1-x^3}} = \frac{3}{2}\sqrt{\frac{x}{1-x^3}}$$

Q.5 $y = \frac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}$, $-2 < x < 2$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

$$\Rightarrow y = \cos^{-1}\frac{x}{2} \cdot \frac{1}{\sqrt{2x+7}}$$

$$\frac{d(x^n)}{dx} = n \cdot x^{n-1}$$

$$\Rightarrow y = \underbrace{\cos^{-1}\frac{x}{2}} \cdot \underbrace{(2x+7)^{-\frac{1}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \underbrace{-\frac{1}{\sqrt{\frac{1}{1}-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}} \cdot (2x+7)^{-\frac{1}{2}} + \cos^{-1}\frac{x}{2} \cdot \underbrace{\left[-\frac{1}{\cancel{2}}(2x+7)^{-\frac{1}{2}-1} \cdot \cancel{2}\right]}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{\cancel{2}\sqrt{\frac{4-x^2}{\cancel{4}}} \cdot \sqrt{2x+7}} - \frac{\cos^{-1}\left(\frac{x}{2}\right)}{(2x+7)^{3/2}}$$

$$\frac{dy}{dx} = -\left[\frac{1}{\sqrt{4-x^2} \cdot \sqrt{2x+7}} + \frac{\cos^{-1}x/2}{(2x+7)^{3/2}}\right]$$

Q.6.

$$y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right], \quad 0 < x < \frac{\pi}{2}$$

$$1+\sin x = \underbrace{\sin^2\frac{x}{2} + \cos^2\frac{x}{2}} + 2\sin\frac{x}{2}\cdot\cos\frac{x}{2} = \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2$$

$$1-\sin x = \sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cdot\cos\frac{x}{2} = \left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2$$

$$\boxed{\sqrt{x^2} = |x|}$$

$$\boxed{(\sqrt{x})^2 = x}$$

$$\boxed{|x| = \begin{cases} x, & x \geq 0 \quad + \\ -x, & x < 0 \quad - \end{cases}}$$

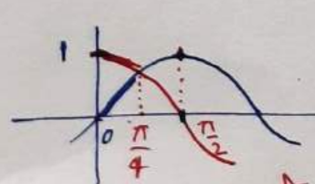
$$y = \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right), \quad 0 < x < \frac{\pi}{2}$$

$$y = \cot^{-1}\left(\frac{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2} + \sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2} - \sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}\right)$$

$$y = \cot^{-1}\left(\frac{\left|\sin\frac{x}{2} + \cos\frac{x}{2}\right| + \overset{-}{\left|\sin\frac{x}{2} - \cos\frac{x}{2}\right|}}{\left|\sin\frac{x}{2} + \cos\frac{x}{2}\right| - \underset{-}{\left|\sin\frac{x}{2} - \cos\frac{x}{2}\right|}}\right)$$

$0 < x < \frac{\pi}{2}$

$0 < \frac{x}{2} < \frac{\pi}{4}$



$$\cos\frac{x}{2} > \sin\frac{x}{2}$$

$$\bigstar \; \left|\sin\frac{x}{2} - \cos\frac{x}{2}\right| = -\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{\left(\cancel{\sin\frac{x}{2}} + \cos\frac{x}{2}\right) - \left(\cancel{\sin\frac{x}{2}} - \cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2} + \cancel{\cos\frac{x}{2}}\right) + \left(\sin\frac{x}{2} - \cancel{\cos\frac{x}{2}}\right)}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{\cancel{2}\cos x/2}{\cancel{2}\sin x/2}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \frac{x}{2}$$

Diff. w.r.t. x $\rightarrow$

$$\frac{dy}{dx} = \frac{1}{2}$$

Class 12 Maths [Continuity and Differentiability]

Miscellaneous Exercise on Chapter-5

Q.7 $y = (\log x)^{\log x}, \quad x > 1$

Logarithmic Differentiation

V.V. → log, Diff

$\Rightarrow \log y = \log (\log x)^{\log x}$

$\Rightarrow \log y = \log x \cdot \log(\log x)$

$\frac{d(\log x)}{dx} = \frac{1}{x}$

by Diff. w.r.t. x →

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \left[\frac{1}{x} \cdot \log(\log x) + \cancel{\log x} \cdot \left(\frac{1}{\cancel{\log x}} \cdot \frac{1}{x}\right)\right]$

$\Rightarrow \frac{dy}{dx} = (\log x)^{\log x} \cdot \left[\frac{\log(\log x)}{x} + \frac{1}{x}\right]$

Q.8 $y = \cos(a\cos x + b\sin x)$ a, b → constants.

(Chain Rule)

$\frac{dy}{dx} = -\sin(a\cos x + b\sin x) \cdot (-a\sin x + b\cos x)$

$= (a\sin x - b\cos x) \cdot \sin(a\cos x + b\sin x)$

$\boxed{Q.9}$ $y = (\sin x - \cos x)^{(\sin x - \cos x)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$

$\Rightarrow \log y = \log \left[(\sin x - \cos x)^{(\sin x - \cos x)} \right]$ $\quad \log m^n = n . \log m$

$\Rightarrow \log y = (\sin x - \cos x) . \log (\sin x - \cos x)$

diff. w.r.t. x $\longrightarrow$

$\Rightarrow \frac{1}{y} . \frac{dy}{dx} = \left[(\cos x + \sin x) . \log (\sin x - \cos x) + \cancel{(\sin x - \cos x)} \left(\frac{1}{\cancel{(\sin x - \cos x)}} . (\cos x + \sin x) \right) \right]$

$\Rightarrow \frac{dy}{dx} = (\sin x - \cos x)^{(\sin x - \cos x)} . (\cos x + \sin x) .$

$\left[\log (\sin x - \cos x) + 1 \right]$

$\boxed{Q.10}$ $y = x^x + x^a + a^x + a^a$ $\quad$ (Fixed) $a>0$, $x>0$

$$\frac{dy}{dx} = \frac{d(x^x)}{dx} + \frac{d(x^a)}{dx} + \frac{d(a^x)}{dx} + \frac{d(a^a)}{dx}$$

$$\frac{dy}{dx} = \underbrace{\frac{d(x^x)}{dx}} + a.x^{a-1} + \underbrace{\frac{d(a^x)}{dx}} + 0 \quad \text{—— (1)}$$

$a^a = \text{const.}$

$x^a \rightarrow x^n \rightarrow n.x^{n-1}$

$$\frac{d(a^x)}{dx} = a^x . \log a$$

$u = x^x$ (let)

$\Rightarrow \log u = \log x^x$

$\Rightarrow \log u = \underbrace{x}.\underbrace{\log x}$

diff. w.r.t. x $\rightarrow$

$\Rightarrow \frac{1}{u}.\frac{du}{dx} = \left(1.\log x + \cancel{x}.\frac{1}{\cancel{x}}\right)$

$\Rightarrow \frac{du}{dx} = x^x(\log x + 1)$

$\Rightarrow \frac{d(x^x)}{dx} = \underbrace{x^x(\log x+1)}$

$v = a^x$

$\Rightarrow \log v = x.\log a$

$\Rightarrow \frac{1}{v}.\frac{dv}{dx} = 1.\log a$

$\Rightarrow \frac{dv}{dx} = \underbrace{a^x.\log a}$

By eq^n (1) $\rightarrow$

$$\frac{dy}{dx} = \underbrace{x^x.(\log x+1)} + \underbrace{a.x^{a-1}} + \underbrace{a^x.\log a} + \underbrace{0}$$

Q.11 $y = x^{x^2-3} + (x-3)^{x^2}$, for $x>3$

$$\boxed{y = u + v} \Rightarrow \boxed{\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}}$$

?

$u = x^{x^2-3}$

$\Rightarrow \log u = \log\left(x^{x^2-3}\right)$

$\Rightarrow \log u = (x^2-3).\log x$

diff. w.r.t. x $\rightarrow$

$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = \left[(2x).\log x + (x^2-3).\left(\frac{1}{x}\right)\right]$

$\Rightarrow \frac{du}{dx} = x^{x^2-3}.\left[2x\log x + \frac{x^2-3}{x}\right]$

$V = (x-3)^{x^2}$

$\Rightarrow \log V = \log (x-3)^{x^2}$

$\Rightarrow \log V = x^2.\log(x-3)$

Diff. w.r.t. x $\rightarrow$

$\Rightarrow \frac{1}{V}\cdot\frac{dv}{dx} = \left[(2x).\log(x-3) + x^2.\frac{1}{x-3}\right]$

$\Rightarrow \frac{dv}{dx} = (x-3)^{x^2}.\left[2x.\log(x-3) + \frac{x2}{x-3}\right]$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$$\Rightarrow \frac{dy}{dx} = \underbrace{x^{x^2-3}.\left[2x.\log x + \frac{x^2-3}{x}\right]} + \underbrace{(x-3)^{x^2}\left[2x.\log(x-3) + \frac{x^2}{x-3}\right]}$$

Class 12 Maths [Continuity and Differentiability]

Miscellaneous Exercise on Chapter 5

Q.12 Find $\frac{dy}{dx}$, if $y = 12(1-\cos t)$, $x = 10(t - \sin t)$; $-\frac{\pi}{2} < t < \frac{\pi}{2}$

(Parametric Form)

$$\frac{dy}{dx} = \frac{\left(\frac{d(y)}{dt}\right)}{\left(\frac{d(x)}{dt}\right)} = \frac{\left(\frac{d(12(1-\cos t))}{dt}\right)}{\left(\frac{d(10(t-\sin t))}{dt}\right)}$$

$$= \frac{12(0+\sin t)}{10(1-\cos t)} = \frac{6(\sin t)}{5(1-\cos t)}$$

$$\left(\begin{aligned} \sin t &= 2\sin t/_2 \cdot \cos t/_2 \\ 1-\cos t &= 2\sin^2 t/_2 \end{aligned}\right) \quad = \frac{6 \cdot \cancel{2}\cancel{\sin t/_2} \cdot \cos t/_2}{5(\cancel{2}\sin^2 t/_2)}$$

$$\frac{dy}{dx} = \frac{6}{5}\cot\left(\frac{t}{2}\right)$$

Q.13 Find $\frac{dy}{dx}$, if $y = \sin^{-1}x + \sin^{-1}\left(\sqrt{1-x^2}\right)$, $0 < x < 1$

I-method: $y = \sin^{-1}x + \sin^{-1}\left(\sqrt{1-x^2}\right)$ (Chain Rule)

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}} \times \frac{1}{\cancel{2}\sqrt{1-x^2}}(0-\cancel{2}x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-(1-x^2)} \cdot \sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{\cancel{1}-\cancel{1}+x^2} \cdot \sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = +\cancel{\frac{1}{\sqrt{1-x^2}}} - \cancel{\frac{\cancel{x}\,1}{\cancel{x} \cdot \sqrt{1-x^2}}}$$

$$\frac{dy}{dx} = 0$$

II - method. $y = \sin^{-1}(x) + \sin^{-1}\sqrt{1-x^2}$

Inverse trigonometry

Substitution. $x = \sin\theta$

$$y = \sin^{-1}(\sin\theta) + \sin^{-1}\sqrt{1-\sin^2\theta}$$

$$\Rightarrow y = \theta + \sin^{-1}\sqrt{\cos^2\theta}$$

$$\Rightarrow y = \theta + \sin^{-1}(\cos\theta)$$

$\sin(90^\circ-\theta) = \cos\theta$

$$\Rightarrow y = \theta + \cancel{\sin^{-1}}\left(\cancel{\sin}\left(\frac{\pi}{2}-\theta\right)\right)$$

$$\Rightarrow y = \cancel{\theta} + \frac{\pi}{2} - \cancel{\theta}$$

$$\Rightarrow y = \frac{\pi}{2} \leftarrow \text{Constant}$$

$$\boxed{\frac{dy}{dx} = 0}$$

Q.14 If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$,

Prove that $\boxed{\dfrac{dy}{dx} = -\dfrac{1}{(1+x)^2}}$

Ans.

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

Square both sides.

(original equation)^2 → extra roots. Satisfy करना

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 + x^2y = y^2 + xy^2$

$\Rightarrow (x^2 - y^2) + (x^2y - xy^2) = 0$

$\Rightarrow (x+y)(x-y) + xy(x-y) = 0$

$\Rightarrow (x-y)\{x+y+xy\} = 0$

$x = y$, $x + y + xy = 0$

$y = x$ (I)

$\Rightarrow y(1+x) = -x$

$\Rightarrow \boxed{y = \dfrac{-x}{1+x}}$ (II)

original eqn. Cross check

$y = x \to 0$, -1; $y=0$, $y=-1$

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

(I) $y = x$ $\Rightarrow x\sqrt{1+x} + x\sqrt{1+x} = 0$

$\Rightarrow 2x\cdot\sqrt{1+x} = 0 \to \boxed{x = 0,\ x = -1}$

Constants. → Reject.

Now we will check (II) $\boxed{y = \frac{-x}{1+x}}$ in the original equation $\rightarrow$ $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$$\Rightarrow x\sqrt{1 - \frac{x}{(1+x)}} - \frac{x}{(1+x)}\sqrt{1+x} = 0$$

$$\Rightarrow x\sqrt{\frac{1+\cancel{x}-\cancel{x}}{1+x}} - \frac{x}{\sqrt{1+x}} = 0$$

$$\Rightarrow x\sqrt{\frac{1}{1+x}} - \frac{x}{\sqrt{1+x}} = 0$$

$$\Rightarrow \frac{\cancel{x}}{\cancel{\sqrt{1+x}}} - \frac{\cancel{x}}{\cancel{\sqrt{1+x}}} = 0 \Rightarrow \boxed{0 = 0}$$

$y = \frac{-x}{1+x}$ Accepted

by Diff. w.r.t. x $\rightarrow$

$$\left(\frac{u}{v}\right)' = \frac{u'\cdot v - u\cdot v'}{v^2}$$

$$\frac{dy}{dx} = \frac{(-1)\cdot(1+x) - (-x)\cdot(1)}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1-\cancel{x}+\cancel{x}}{(1+x)^2} = \frac{-1}{(1+x)^2}$$

Class 12 Maths [Continuity and Differentiability]

Miscellaneous Exercise on Chapter 5

Q.15 If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$,

Prove that $\dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$ is a constant independent of 'a' and 'b'.

Sol^n: $(x-a)^2+(y-b)^2=c^2$ $a, b, c \rightarrow$ Constant

by diff. w.r.t. 'x' $\rightarrow$

$$\Rightarrow 2(x-a) + 2(y-b)\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow (x-a) + \underbrace{(y-b)\cdot\frac{dy}{dx}}_{u.v} = 0 \qquad \boxed{\frac{dy}{dx} = \frac{-(x-a)}{(y-b)}}$$

by diff. w.r.t. 'x' $\rightarrow$

$$\Rightarrow 1 + \left(\frac{dy}{dx}-0\right)\cdot\frac{dy}{dx} + (y-b)\cdot\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow 1+\left(\frac{dy}{dx}\right)^2 + (y-b)\cdot\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow (y-b)\cdot\frac{d^2y}{dx^2} = -\left\{1+\left(\frac{dy}{dx}\right)^2\right\}$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{-\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}{y-b}}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\left[-\left(\frac{x-a}{y-b}\right)\right]^2 = 1+\frac{(x-a)^2}{(y-b)^2}$$

$$= \frac{(y-b)^2+(x-a)^2}{(y-b)^2} = \frac{c^2}{(y-b)^2}$$

Given. $\boxed{(x-a)^2+(y-b)^2=c^2}$

$$\Rightarrow \boxed{1+\left(\frac{dy}{dx}\right)^2 = \frac{c^2}{(y-b)^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}{(y-b)} = -\frac{\left\{\frac{c^2}{(y-b)^2}\right\}}{(y-b)}$$

$$\boxed{\frac{d^2y}{dx^2} = -\frac{c^2}{(y-b)^3}}$$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[\frac{c^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}} = \frac{\frac{\cancel{c^3}\,c}{\cancel{(y-b)^3}}}{-\frac{\cancel{c^2}}{\cancel{(y-b)^3}}} = -c$$

Constant, independent of a & b

$\boxed{Q.16}$ If $\cos y = x \cos(a+y)$, with $\cos a \neq \pm 1$,

Prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a+y)}{\sin a}$.

Ans. $\cos y = x . \cos(a+y)$ $\quad (u.v)' = u'.v + u.v'$

diff. w.r.t. x $\rightarrow$

$$\Rightarrow -\sin y . \frac{dy}{dx} = 1 . \cos(a+y) - x \sin(a+y) . \left(\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} \left[x . \sin(a+y) - \sin y \right] = \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{x . \sin(a+y) - \sin y}$$

Given $\cos y = x . \cos(a+y)$

$x = \dfrac{\cos y}{\cos(a+y)}$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{\frac{\cos y}{\cos(a+y)} . \sin(a+y) - \sin y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{\left\{ \frac{\cos y . \sin(a+y) - \sin y . \cos(a+y)}{\cos(a+y)} \right.}$$

$\sin A . \cos B - \sin B \cos A = \sin(A-B) = \sin(a+y-\cancel{y})$

$A \to a+y$, $B \to y$

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin[(a+\cancel{y}) - \cancel{y}]} \leftarrow \text{By } \sin(A-B)$$

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

Hence Proved.

$\boxed{Q.17}$ If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t.\cos t)$, find $\frac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

$$\frac{dy}{dt} = a(\cancel{\cos t} - 1.\cancel{\cos t} + t.\sin t)$$

$$\frac{dy}{dt} = at\sin t$$

$$\frac{dx}{dt} = a(-\cancel{\sin t} + 1.\cancel{\sin t} + t.\cos t)$$

$$\frac{dx}{dt} = at.\cos t$$

$$\frac{dy}{dx} = \frac{\cancel{at}\sin t}{\cancel{at}\cos t} = \tan t$$

$$\boxed{\frac{dy}{dx} = \tan t}$$

diff. w.r.t. 'x' (again)

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2 t . \left(\frac{dt}{dx}\right)$$

Reciprocal,

$$\frac{dt}{dx} = \frac{1}{at\cos t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2 t . \frac{1}{at\cos t}$$

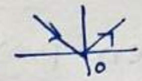
$$\boxed{\frac{d^2y}{dx^2} = \frac{\sec^3 t}{at}}$$

Class 12 Maths

Continuity and Differentiability

Miscellaneous Exercise on chapter (5)

Q.18 If $f(x) = |x|^3$, show that $f''(x)$ exists for all real x and find it.

Sol^n. $|x|$ changes its nature at $x=0$ 

f Differentiable $\Rightarrow$ f' exists

f' differentiable $\Rightarrow$ f'' exists

Continuity

yes

LHD = RHD

$f'(0^-) = f'(0^+)$

LHD = RHD.

$f''(0^-) = f''(0^+)$

$|x|^3$ is continuous everywhere.
we have to check diff. at $x=0$.

$f(x) = |x|^3$

$$f(x) = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2, & x \geq 0 \rightarrow \text{RHD of } f = f'(0^+) = 3(0)^2 = 0 \\ -3x^2, & x < 0 \rightarrow \text{LHD of } f = f'(0^-) = -3(0)^2 = 0 \end{cases}$$

$$f''(x) = \begin{cases} 6x, & x \geq 0 \rightarrow \text{RHD of } f' = f''(0^+) = 6(0) = 0 \\ -6x, & x < 0 \rightarrow \text{LHD of } f' = f''(0^-) = -6(0) = 0 \end{cases}$$

$f'' \rightarrow$ exists

$f''(x) = |6x|$

Q.19 Prove by PMI $\boxed{\frac{d(x^n)}{dx} = n \cdot x^{n-1}}$ for all positive integers n.

P(n): Statement

(1) P(1) true (Prove)

(2) let P(k) is true

then prove P(k+1) is also true

$n \in \{1, 2, 3 \ldots\}$

$$P(n): \boxed{\frac{d(x^n)}{dx} = n \cdot x^{n-1}}$$

Step-(1) we have to prove that P(1) is true.

$$P(1): \frac{d(x^1)}{dx} = 1 \cdot x^{1-1}$$

$$1 = 1 \cdot x^0$$

$$1 = 1 \quad \text{true}$$

Step-(2) Let P(k) is true

$$P(k): \boxed{\frac{d(x^k)}{dx} = k \cdot x^{k-1}} \quad \text{---(1)}$$

Now We have to prove that P(k+1) is true.

i.e. $P(k+1): \frac{d(x^{k+1})}{dx} = (k+1) \cdot x^k$

LHS of $P(K+1) = \frac{d(x^{K+1})}{dx} = \frac{d(x^K \cdot x)}{dx}$

$(u.v)' = u'.v + u.v'$

$= \frac{d(x^K)}{dx} \cdot x + x^K \cdot \frac{d(x)}{dx}$

$\downarrow$ eqⁿ ①

$= (K \cdot x^{K-1}) \cdot x' + x^K \cdot (1)$

$= \underbrace{K \cdot x^K + x^K}$

$= (K+1)\, x^K$ = RHS of $P(K+1)$

$\therefore$ $P(K+1)$ is also true.

$\Rightarrow$ $P(n): \frac{d(x^n)}{dx} = n \cdot x^{n-1}$ is true. $n \in N$

$\boxed{Q.20}$ Using the fact that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and the differentiation, obtain the sum formula for cosines. $\longrightarrow$ $\cos(A+B) = ?$

Ans.

$\sin(A+B) = \sin A . \cos B + \cos A \cdot \sin B$

A & B $\to$ variable

by Differentiating with respect to 'x' $\to$

$$\Rightarrow \cos(A+B) \cdot \left(\frac{dA}{dx} + \frac{dB}{dx}\right) = \cos A \cdot \frac{dA}{dx} \cdot \cos B + \sin A \cdot \sin B \cdot \frac{dB}{dx}$$
$$- \sin A \cdot \frac{dA}{dx} \cdot \sin B + \cos A \cdot \cos B \cdot \frac{dB}{dx}$$

$$\Rightarrow \cos(A+B)\cdot\left(\frac{dA}{dx}+\frac{dB}{dx}\right) = \cos A\cdot\frac{dA}{dx}\cdot\cos B - \sin A\sin B\cdot\frac{dB}{dx}$$
$$- \sin A\cdot\frac{dA}{dx}\cdot\sin B + \cos A\cdot\cos B\cdot\frac{dB}{dx}$$

$$= \frac{dA}{dx}\left(\underline{\cos A\cos B - \sin A\sin B}\right)$$
$$+ \frac{dB}{dx}\left(\underline{\cos A\cos B - \sin A\sin B}\right)$$

$$\Rightarrow \frac{\cos(A+B)\cdot}{\left(\cancel{\frac{dA}{dx}+\frac{dB}{dx}}\right)} = \left(\cancel{\frac{dA}{dx}+\frac{dB}{dx}}\right)(\cos A\cos B - \sin A\sin B)$$

$$\Rightarrow \boxed{\cos(A+B) = \cos A\cos B - \sin A\sin B}$$

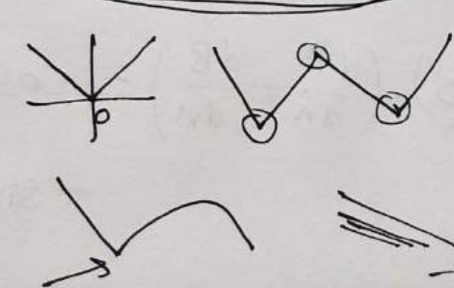
Q.21 Does there exist a function which is continuous every where but not differentiable at exactly two points? Justify.

Ans.

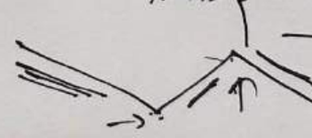
Graphically

Differentiable → Graph Smooth

Non Differentiable → Sharp corners



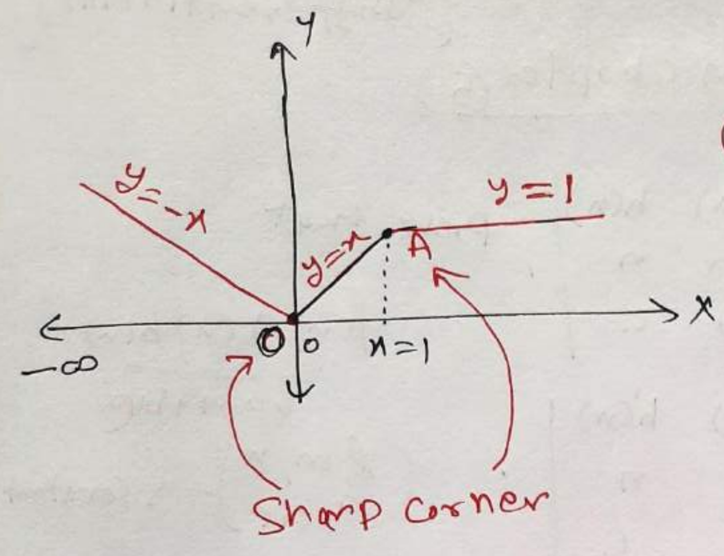
$$f(x) = \begin{cases} — , & x< \\ — , & x. \\ — , & x> \end{cases}$$



Continuous everywhere

(Non differentiable at exactly 2 points.)

(exactly 2 sharp corner)



$y = mx + c$

↑

(slope)

$y = -x \quad x \in (-\infty, 0)$

$y = x \quad x \in [0, 1]$

$y = 1 \quad x \in (1, \infty)$

$$\boxed{f(x) = \begin{cases} -x, & x < 0 \\ x, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}}$$

Continuous everywhere

but not differentiable at exactly 2 points. $\boxed{x = 0, 1}$

Class 12 maths [Continuity and Differentiability]

Miscellaneous Exercise on Chapter 5

Q.22. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that

$f(x), g(x), h(x)$ → variable

$\begin{pmatrix} l, m, n \\ a, b, c \end{pmatrix}$ → Constant

$$\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}.$$

Solution.

$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix} \rightarrow R_1 \text{ along expand}$$

$$y = f(x)\underbrace{\begin{vmatrix} m & n \\ b & c \end{vmatrix}}_{\text{Const.}} - g(x)\underbrace{\begin{vmatrix} l & n \\ a & c \end{vmatrix}}_{\text{Const.}} + h(x)\underbrace{\begin{vmatrix} l & m \\ a & b \end{vmatrix}}_{\text{Const.}}$$

diff. w.r.t. (x) →

$$\frac{dy}{dx} = f'(x).\begin{vmatrix} m & n \\ b & c \end{vmatrix} - g'(x).\begin{vmatrix} l & n \\ a & c \end{vmatrix} + h'(x).\begin{vmatrix} l & m \\ a & b \end{vmatrix}$$

$$\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

Note :
$$\begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix}' = \begin{vmatrix} f_1' & f_2' & f_3' \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1' & g_2' & g_3' \\ h_1 & h_2 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1' & h_2' & h_3' \end{vmatrix}$$

information for Q.22

Q.23 If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, show that

$$\boxed{(1-x^2)\cdot\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0}$$

Ans. $y = e^{a\cos^{-1}x}$

Diff. w.r.t. x → Chain Rule

$$\Rightarrow \frac{dy}{dx} = e^{a\cos^{-1}x}\cdot a\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \sqrt{1-x^2}\cdot\frac{dy}{dx} = -a\,y$$

by squaring →

$$\Rightarrow (1-x^2)\cdot\left(\frac{dy}{dx}\right)^2 = a^2y^2$$

by diff. w.r.t. x →

$$\Rightarrow (-\cancel{2}x)\cdot\left(\frac{dy}{dx}\right)^{\cancel{2}} + (1-x^2)\cdot\cancel{2}\cdot\cancel{\left(\frac{dy}{dx}\right)}\cdot\frac{d^2y}{dx^2} = a^2\cdot\cancel{2}y\cdot\cancel{\frac{dy}{dx}}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

Hence proved