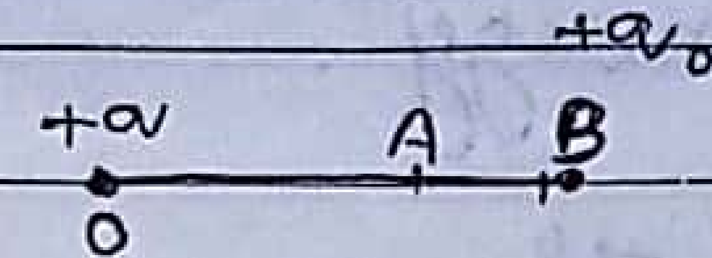
CHPTER – TWO — Electrostatic Potential and capacitance

Electrostatic Potential Difference



The potential difference b/w two points in any electric field will be equal to the ratio of doing work to carry the testing charge q_0 from one point to another at the testing charge.

Let us taking two points A & B a testing charge $+q_0$ is carry from point B to A the doing work is W. Therefore the potential difference b/w A & B –

$$\boxed{V_A - V_B = \frac{W}{q_0}}$$

The unit of potential difference is Joule/C. It is called volt.

$$1\text{ Volt} = \frac{1\text{ Joule}}{C}$$

The dimension formula of Electrostatic Potential difference is $[ML^2T^{-3}A^{-1}]$

Electrostatic Potential at a point

The electrostatic potential at any point in an electric field is equal to amount of work done in bringing the unit +ve test charge from infinity to that point.

$$\boxed{V_A = \frac{W}{q_0}}$$

The S.I. unit of Electrostatic potential is J/C i.e. called Volt

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Electron volt eV → This is the unit of energy.

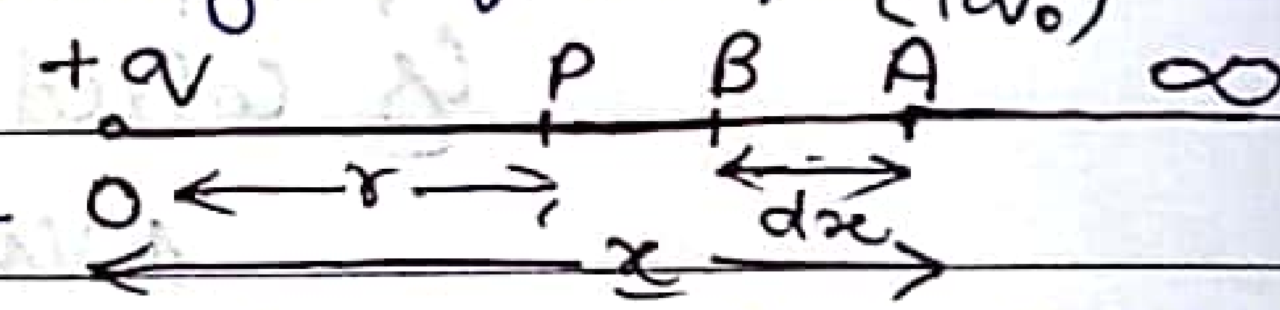
The energy require to move an electron at 1 volt potential difference.

$$\boxed{1\,eV = 1.6 \times 10^{-19}\,J}$$

✓☆

Electrostatic potential due to a point charge

Let P is the point at a distance r from the point O where a point charge +q is placed. Let A is a point at distance x from the point O. a testing the charge $+q_0$ is placed at point A.



The electrostatic force on charge q_0 by charge q

$$F = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{x^2}$$

Now the testing charge carrying a small distance dx then small work done –

$$dW = F \times dx$$

$$dW = \frac{-1}{4\pi\epsilon_0}\frac{q q_0}{x^2}\,dx$$

Therefore the workdone in carrying testing charge $+q_0$ from ∞ to P.

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_0}\frac{q q_0}{x^2}\,dx$$

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$$= \frac{-1}{4\pi\epsilon_0} q q_0 \int_{\infty}^{r} x^{-2} dx \qquad \left[\because \int x^{-2} dx = \frac{-1}{x}\right]$$

$$= \frac{-1}{4\pi\epsilon_0} q q_0 \left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$W = \frac{1}{4\pi\epsilon_0} q q_0 \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

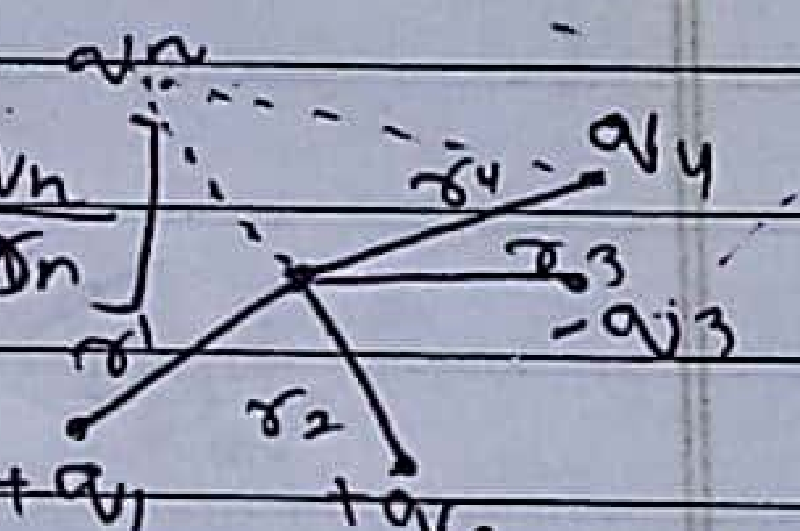
$$\left[W = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r}\right]$$

The potential at point P

$$\left[V = \frac{W}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}\right]$$

Electric Potential Due to a System of charge.

$$V = \frac{1}{4\pi\epsilon_0}\left[+\frac{q_1}{r_1} + \frac{q_2}{r_2} - \frac{q_3}{r_3} - \frac{q_4}{r_4} - - - - \frac{q_n}{r_n}\right]$$

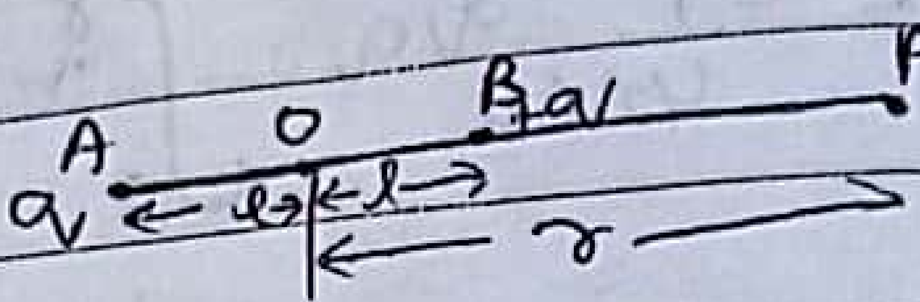


The electrostatic potential Due to a system of charges at any point P will be equal to algebraic sum potential due to an individual charge

Electrostatic Potential due to an an electric dipole.

1) At a point on Axial position:-

Electrostatic potential at



pole to +q charge.

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-l)}$$

Electrostatic potential at P due to -q charge

$$V_2 = \frac{-1}{4\pi\epsilon_0} \frac{q}{(r+l)}$$

Therefore the net potential at P due to Electric dipole.

$$V = V_1 + V_2$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-l)} - \frac{1}{4\pi\epsilon_0} \frac{q}{(r+l)}$$

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-l)} - \frac{1}{(r+l)} \right]$$

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{\cancel{r}+l-\cancel{r}+l}{r^2-l^2} \right]$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{(2ql)}{r^2}$$

$\because l << r$

$$V = \frac{1}{4\pi\epsilon_0} \frac{(2ql)}{r^2}$$ $l^2 \rightarrow$ Negligible

$\because (2ql = p)$

$$\left[V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} \right]$$

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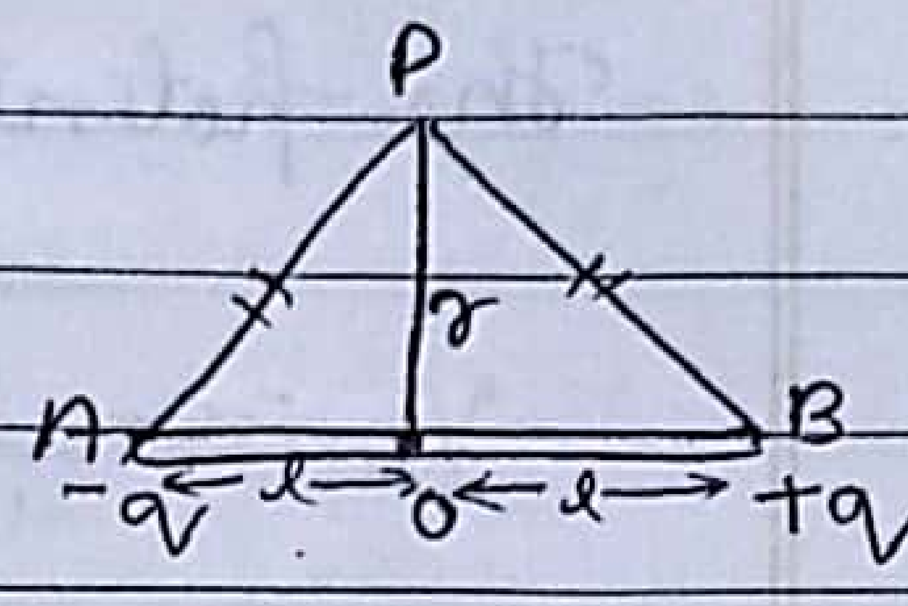
Q At a point on equitorial position

$AP = BP = \sqrt{r^2+l^2}$



The potential at P due to +q charge

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{BP}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{r^2+l^2}}$$

The potential at P due to -q charge

$$V_2 = -\frac{1}{4\pi\epsilon_0}\frac{q}{AP}$$

$$V_2 = -\frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{r^2+l^2}}$$

The net potential at ~~point~~ P due to dipole

$$V = V_1 + V_2$$

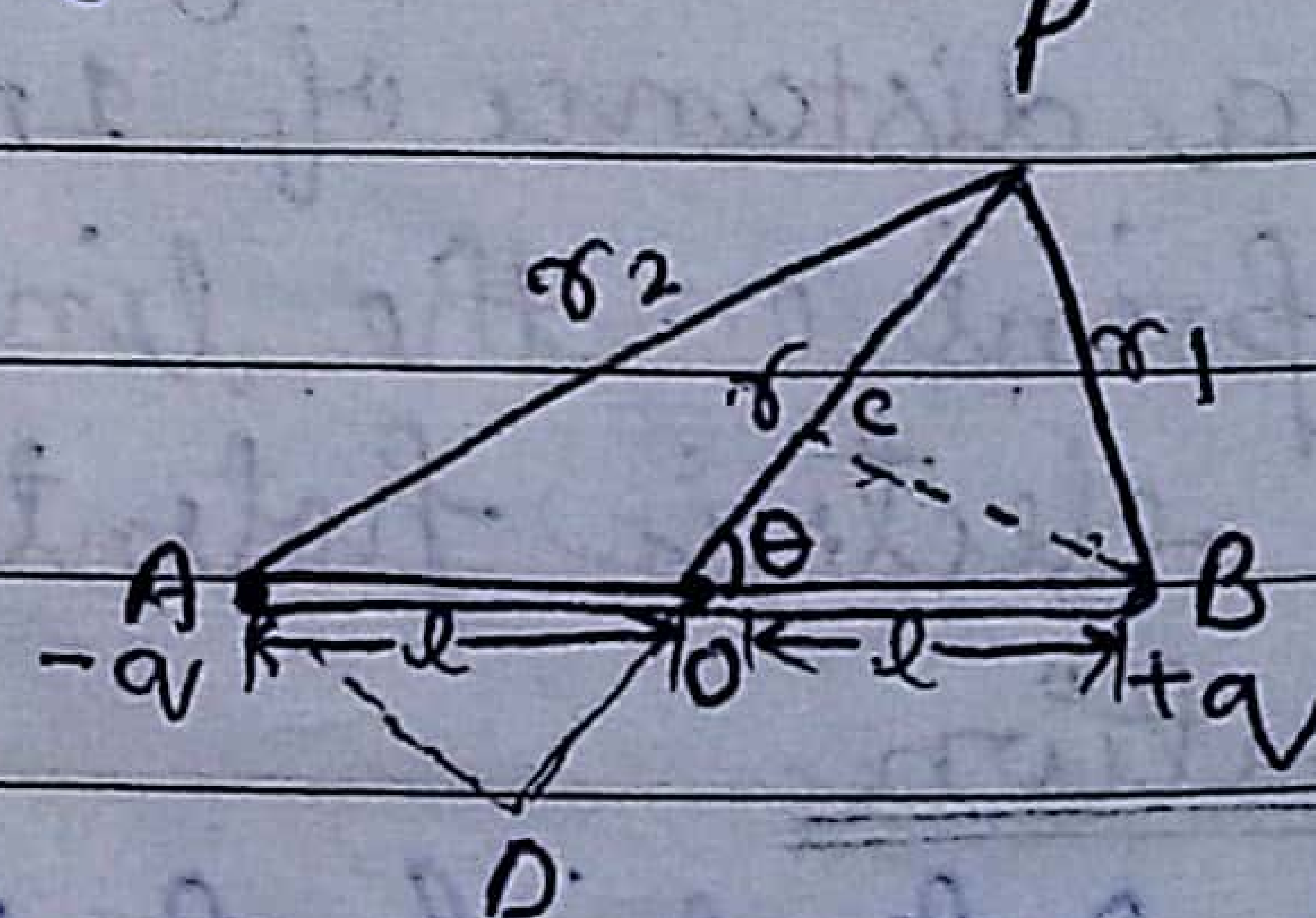
$$V = \frac{1}{4\pi\epsilon_0}\cancel{\frac{q}{\sqrt{r^2+l^2}}} - \cancel{\frac{1}{4\pi\epsilon_0}}\cancel{\frac{q}{\sqrt{r^2+l^2}}}$$

$$\boxed{V = 0}$$

Electrostatic Potential due to an dipole at any point P (r, θ)

Potential at p due to +q charge

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1}$$



Potential at ~~poi~~ p due to -q charge

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{(-q)}{r_2} = \frac{-1}{4\pi\epsilon_0}\frac{q}{r_2}$$

Net Potential at P-

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad \text{— (1)}$$

In Δ OBC

$OC = OB\cos\theta = l\cos\theta$

$PB = (r_1) = PC = OP - OC = r - l\cos\theta$

$AP = r_2 = PD = PO + OD = r + l\cos\theta \quad \because OD = l\cos\theta$

from eq — (1)

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{\cancel{r} + l\cos\theta - \cancel{r} + l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$\because l << r$

$\therefore l^2\cos^2\theta =$ Negligible

$$V = \frac{1}{4\pi\epsilon_0}\frac{(2ql)\cos\theta}{r^2}$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}} \quad \text{— (2)}$$

(1) For Axial

$\theta = 0$

from eq — (1)

$\left[V = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2}\right]$

(2) For equitorial

$\theta = 90°$

$[V = 0]$

Equipotential Surface

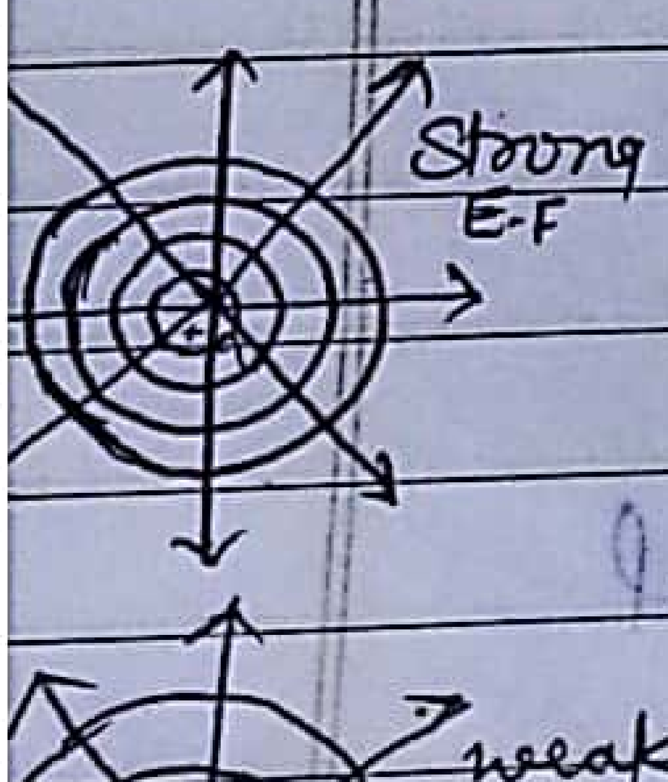
Any surface which has same electrostatic potential at every point on it is called equipotential surface.
The different properties of equipotential surface are following-

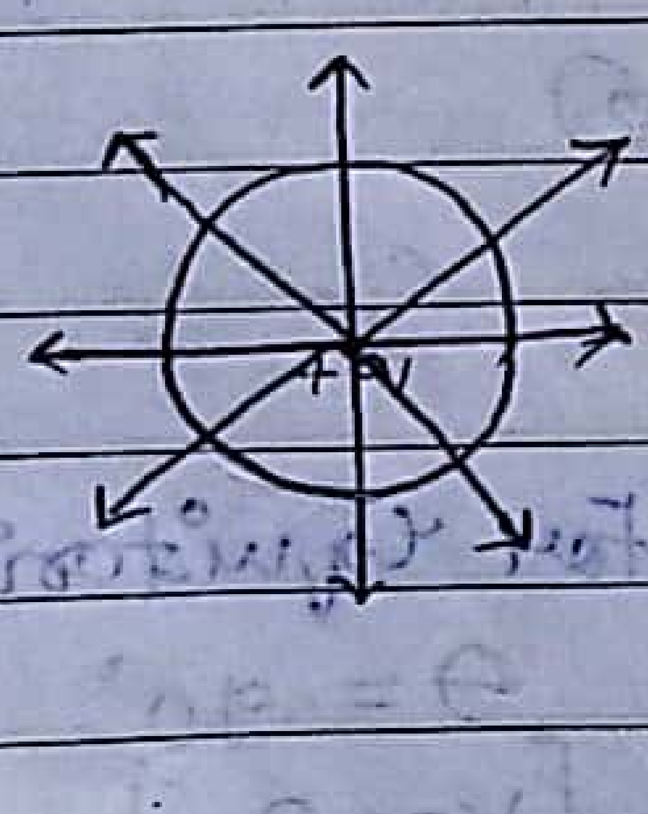
i) Equipotential surface donot bisect each other because if they bisect then there will be two direction of E.F. at bisect point which is not possible.

ii Equipotential surfaces are closely spaced in the region of strong electric field & widley space in the region of weak electric field.

iii For any charge equipotential surface through a point is normal to the electric field at that point.

iv No work is required to move a test charge on an equipotential surface.

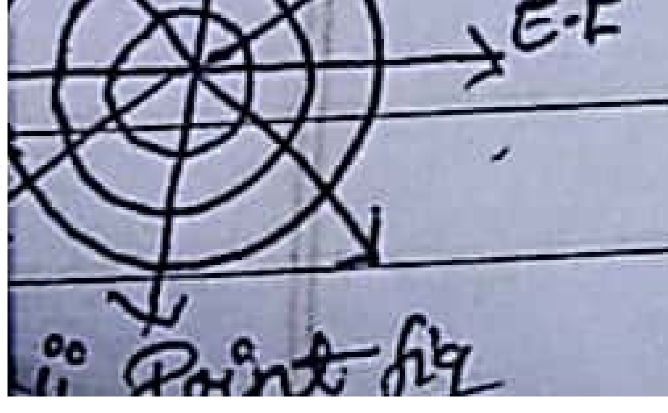




$$\left[W = q \times \Delta V\right]$$

$$\because \Delta V = 0$$

$$[W = 0]$$



ii Point fig

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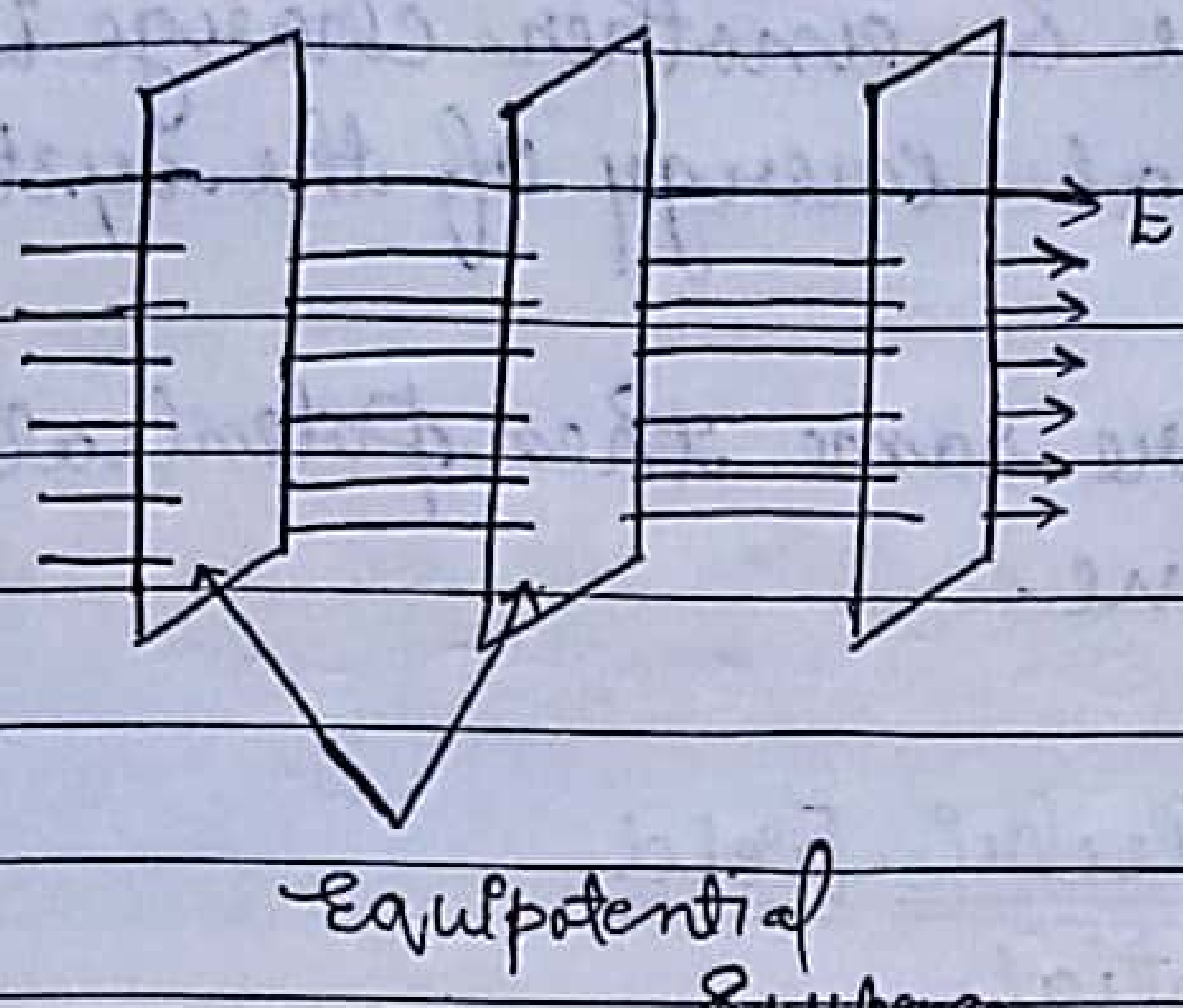


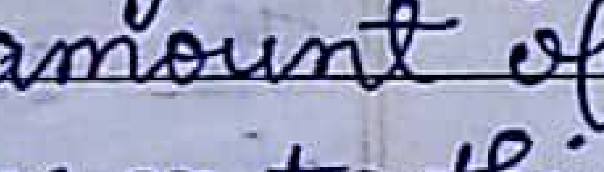
Electrostatic Potential Energy of a System of charges

The electrostatic potential energy of a system of point charge is equal to the total amount of work done in Bringing the different charges to their respective position from infinite.

Electrostatic potential of a System of two point charge

If two charges q_1 & q_2 are separated at distance small r then the electrostatic potential energy of the system-

Potential at B $V = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r}$



$$W = q \times V$$

$$= q_1 \times \frac{1}{4\pi\epsilon_0}\frac{q_2}{r}$$

$$\left[U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \right]$$

* If one charge is +ve & another charge is -ve then potential energy of the system will be +ve.
* If both charges are same then potential energy will be +ve.

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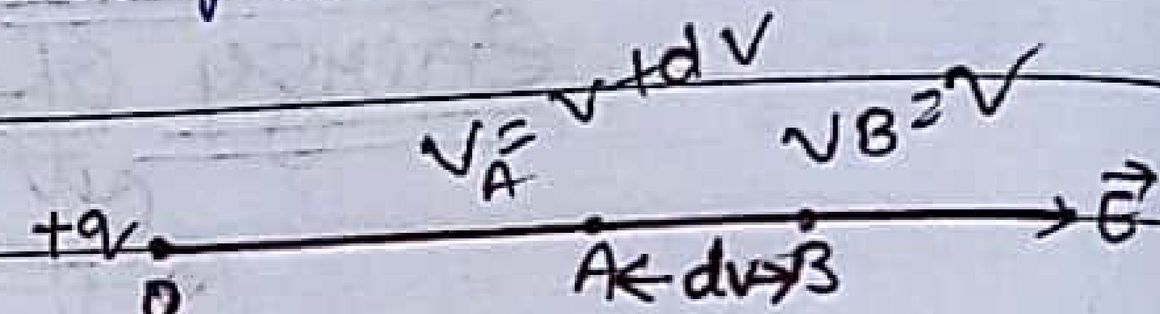
Relation b/w Electric Field And Potential

Let a +q charge located at point O. Let A & B are two point separated by distance dr



Let Potential at B is V
& potential at A is V+dV
The external force required to move the testing charge $+q_0$ from B to A. —

$$F = -q_0 E$$

Therefore the work done to move the testing charge —

$$W = F \times dr = -q_0 E\, dr \quad \text{—} (1)$$

Because the potential difference b/w A & B is dV therefore the work done to move testing charge from A to B.

$$W = q_0 \times dV \quad \text{—} (2)$$

From eq (1) & (2)

$$-\cancel{q_0} E\, dr = \cancel{q_0}\, dV$$

$$\left[E = -\frac{dV}{dr} \right] \text{ (Potential Gradient)}$$

$\frac{dV}{dr}$ is the rate of change of potential with respect to distance and it is called potential gradient

"The electric field at any point is equal to the negative of the potential gradient at that point."

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Conductors and Insulators

Those substances in which a large scale of physical movement of electric charges allow through them when an external E.F applied is called Conductors
for ex- All metals, Human body, Acids, Alkali etc.

Those substances which do not allow the physical movement of charges through them when an electric field is applied

is called Insulators or dielectric
// for eg. Wood, Glass, Mica, wax etc...

The rubbed insulators where able to retain charges placed on them therefore they were called dielectric

Free electrons and Bound charges

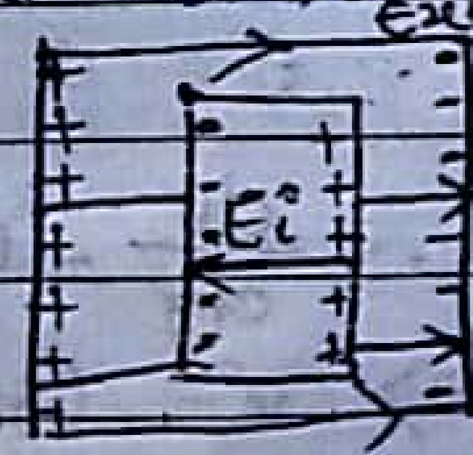
Every substances are made by atoms and every atom consist a positive nucleus and a negative electron moving various orbitals around nucleus.
The electrons of outermost orbit ar lostely bounded to the nucleus they detach from the atom and move freely these are called Free Electrons.

The positive ions which consist of nuclei & electrons of inner shells remain in fixed positions these charges are called bounded charges

Bhaviour of Conductors in Electrostatic field.

when a conductor is placed in electrostatic fied the conductor shows the following property-

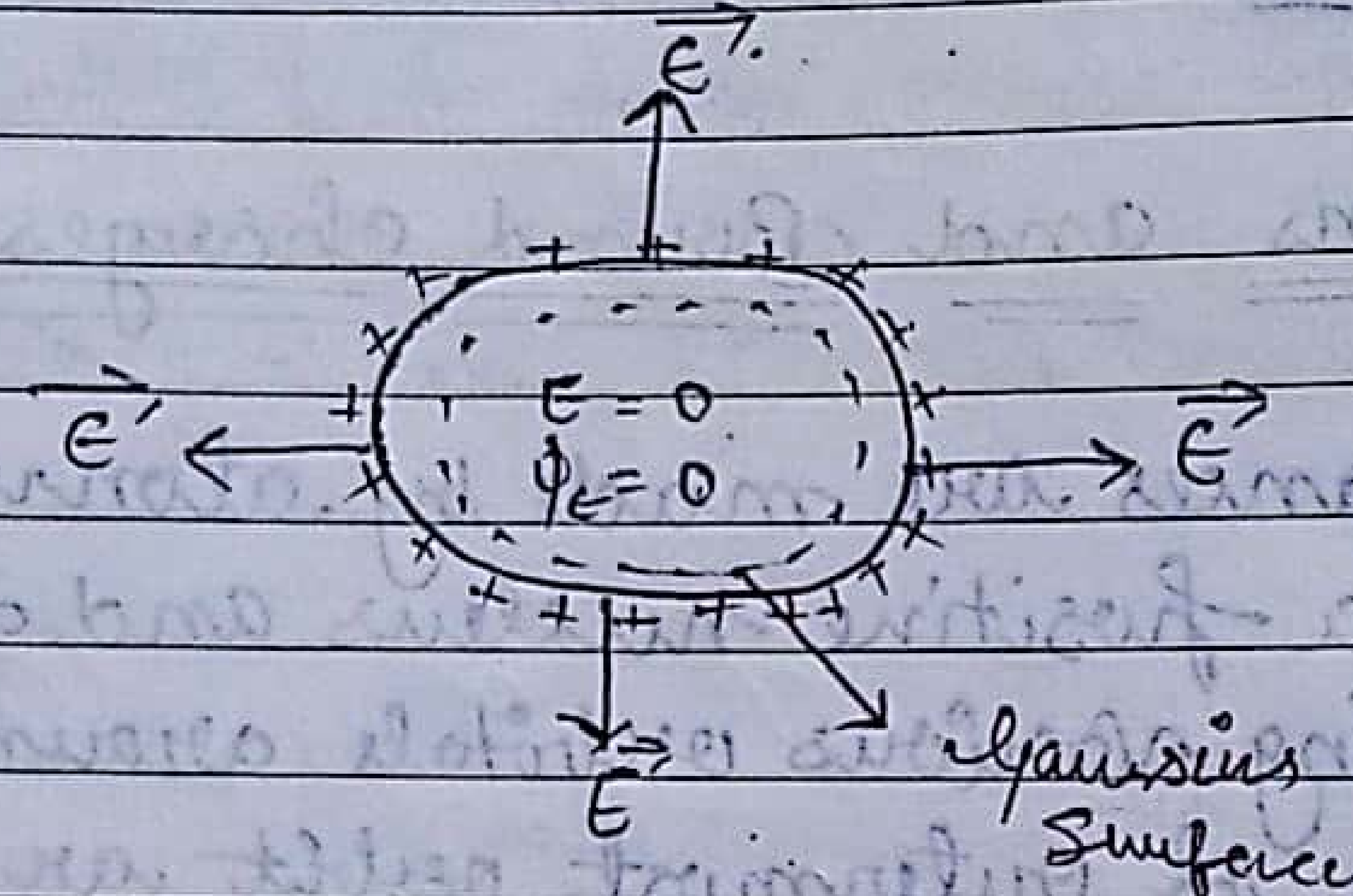
(1) Net electrostatic field will be zero inside the conductor.



$$\left[\begin{array}{l} E_{ext} = E_{int} \\ E = 0 \end{array}\right]$$

② Just outside the surface of a charged conductor electric field is normal to the surface.

③ The net charge inside a conductor is zero.



By Gauss' Theorem

$$\phi_E = \frac{q}{\epsilon_0}$$

$$0 = \frac{q}{\epsilon_0}$$

$$\Rightarrow \boxed{q = 0}$$

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Electrostatic Shielding

The phenomenon of a making a reason free from any electric field is called electrostatic shielding it is based on the fact that electric field is zero in the cavity of a hollow charged conductor.

Potential is constant within and on the surface of conductor.

$$E = -\frac{dv}{dr}$$

$$\because E = 0 \text{ (Inside the conductor)}$$

$$\therefore \frac{dv}{dr} = 0 \Rightarrow dv = 0$$

$$\left[\Rightarrow V = \text{Constant} \right]$$

Electrical Capacitance of a Conductor

The electrical capacitance of a conductor is the measure of its ability to hold the electric charge. If we increase the charge on a conductor its potential is also increases it mean the given charge Q will be directly proportional to the voltage V.

$$Q \propto V$$

$$Q = CV$$

C → Capacitance

$$\boxed{C = \frac{Q}{V}}$$

C is called the capacitance of a conductor.

If V = 1 volt

Then C = Q

"The capacitance of a conductor is the charge require to increase the potential of conductor by unit amount.

The capacitance of conductor depends on following factors" -

i) The size of conductor.

ii) Nature of the surrounding medium.

iii) Presence of other conductor in its neighbourhood

The unit of capacitance is Farad

$$\left[1 \text{ Farad} = \frac{1 \text{ Coulumb}}{1 \text{ Volt}} \right]$$

"1 farad is defined as if we give 1 Coloumb charge to any conductor and its voltage increase by 1 Volt then the capacitance of conductor will be 1 farad"

Dimension of Capacitance

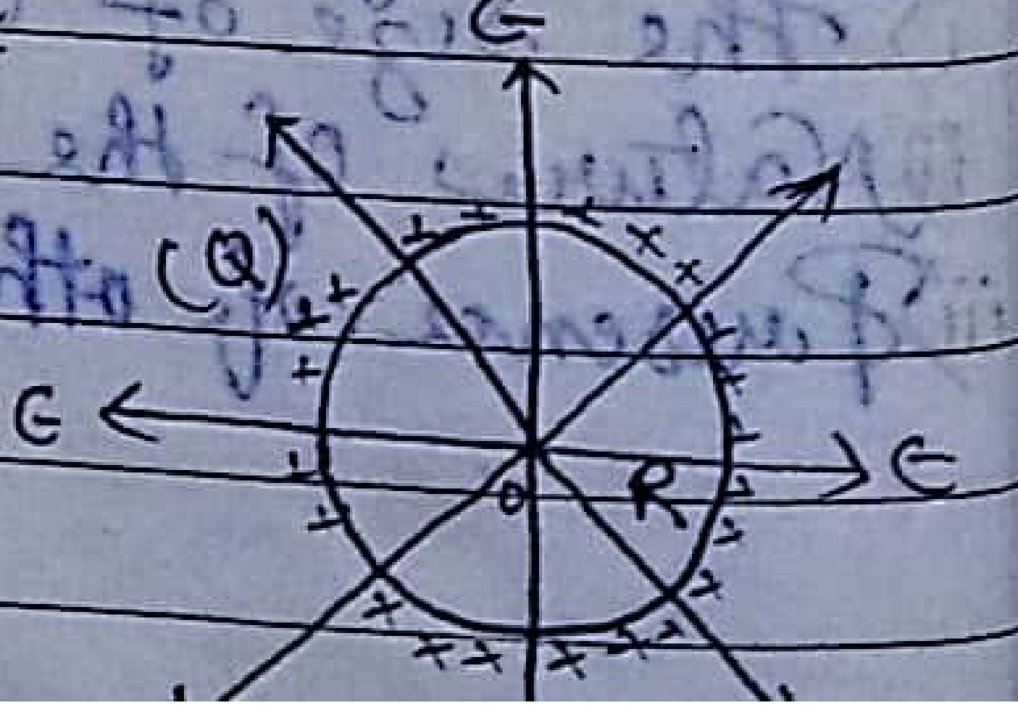
$$C = \frac{\text{Coulomb}}{\text{Joule}/\text{coulomb}}$$

$$C = \frac{AT}{\frac{ML^2T^{-2}}{AT}}$$

$$C = \frac{A^2T^2}{ML^2T^{-2}}$$

$$C = \left[M^{-1}L^{-2}T^4A^2 \right]$$

Capacitance of an Isolated Spherical Conductor



Let us Consider an isolated spherical conductor of radius

R. The charge Q is uniformly distributed over its surface.

The potential on the surface of conductor —

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

Therefore the capacitance of spherical conductor —

$$C = \frac{Q}{V}$$

$$C = \frac{\cancel{Q}}{\frac{1}{4\pi\epsilon_0}\frac{\cancel{Q}}{R}}$$

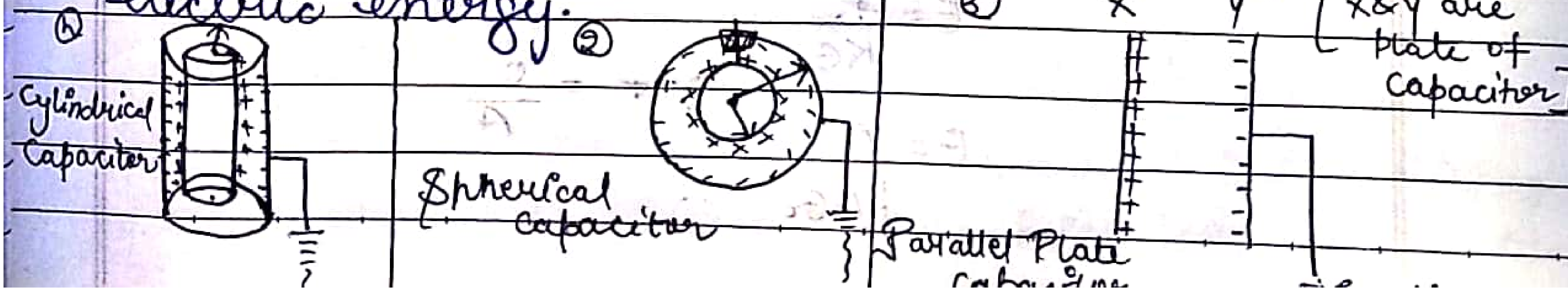
$$\left[C = 4\pi\epsilon_0 R \right]$$

Therefore the capacitance of spherical conductor is proportional to its radius.

[Capacitor]

The capacitance of an insulated conductor is considerably increase by putting an earth connected conductor near it.

A capacitor is an arrangement of two conductors separated by an insulating medium that is used to store to electric charge and electric energy.



If the charge on any plate of capacitor is Q and potential b/w the plates is V then Capacitance of capacitor :-

$$\left[C = \frac{Q}{V} \right]$$

unit of Capacitance is Coloum/volt i.e. called Farad.

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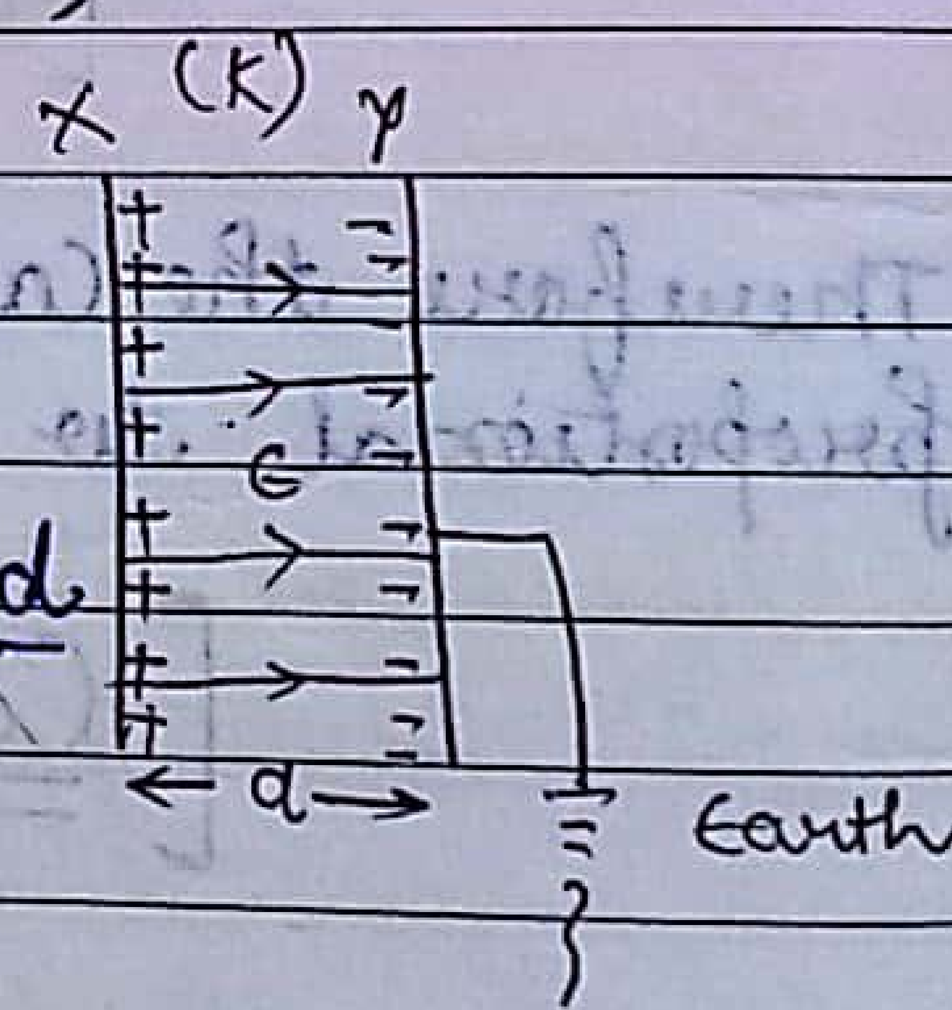
Capacitance of Parallel Plate Capacitor

Let X, Y are the plates of a parallel plate capacitor the area of plates are $\underline{A}$ and distance b/w the plates is $\underline{d}$. The charge density of plates is $\underline{\sigma}$



$$\sigma = \frac{Q}{A}$$

A dielectric media of dielectric Constant $\underline{K}$ is fully placed b/w the plates. The potential between the plates :-

$$V = Ed.$$

The intensity of electric field between the plates

$$E = \frac{\sigma}{K\epsilon_0}$$

$$\because \sigma = \frac{Q}{A}$$

$$E = \frac{Q}{KA\epsilon_0}$$

Therefore $V = Ed$

$$V = \frac{Qd}{KA\epsilon_0}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\cancel{Q}}{\frac{\cancel{Q}d}{KA\epsilon_0}}$$

$$\boxed{C = \frac{KA\epsilon_0}{d}}$$

① $C \propto K$

② $C \propto A$

③ $C \propto \frac{1}{d}$

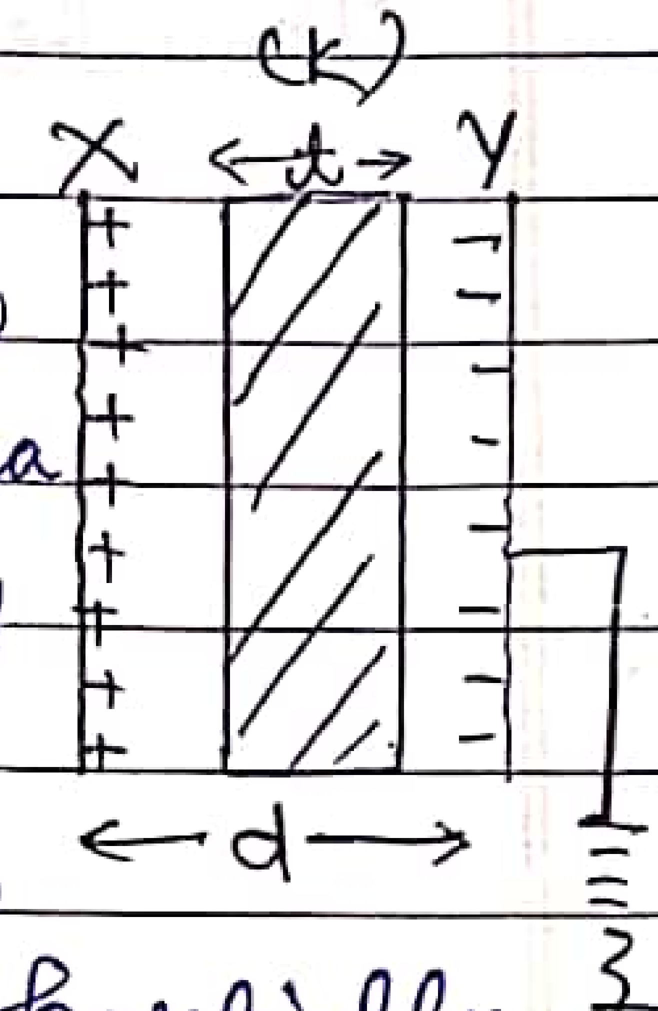
If there are air or vaccum b/w the plates then $K = 1$

$$\left[C_0 = \frac{A\epsilon_0}{d}\right]$$

Therefore $\frac{C}{C_0} = \frac{KA\epsilon_0/\cancel{d}}{\cancel{A\epsilon_0}/\cancel{d}}$

$$\boxed{K = \frac{C}{C_0}}$$

Capacitance of Capacitor when a dielectric media Partially filled b/w Plates



Let there is a parallel plate capacitor the distance b/w plates is d the area of plates is A and charge on every plates is Q

A dielectric media of thickness t and dielectric constant K is partially filled b/w the plates.

Potential b/w Plates of Capacitor.

$$V = V_{air} + V_{medium}$$

$$V = E_{air} \times (d-t) + E_{medium} \times t$$

$$V = \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma}{K\epsilon_0} t$$

$$V = \frac{\sigma}{\epsilon_0}\left[d - t + \frac{t}{K}\right] \qquad \because \sigma = \frac{Q}{A}$$

$$V = \frac{Q}{A\epsilon_0}\left[d - t + \frac{t}{K}\right]$$

Therefore the capacitance of capacitor

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\epsilon_0}\left[d - t + \frac{t}{K}\right]}$$

$$\boxed{C = \frac{A\epsilon_0}{\left[d - t + \frac{t}{K}\right]}}$$

If the dielectric medium placed fully b/w plates then-

$$t = d$$

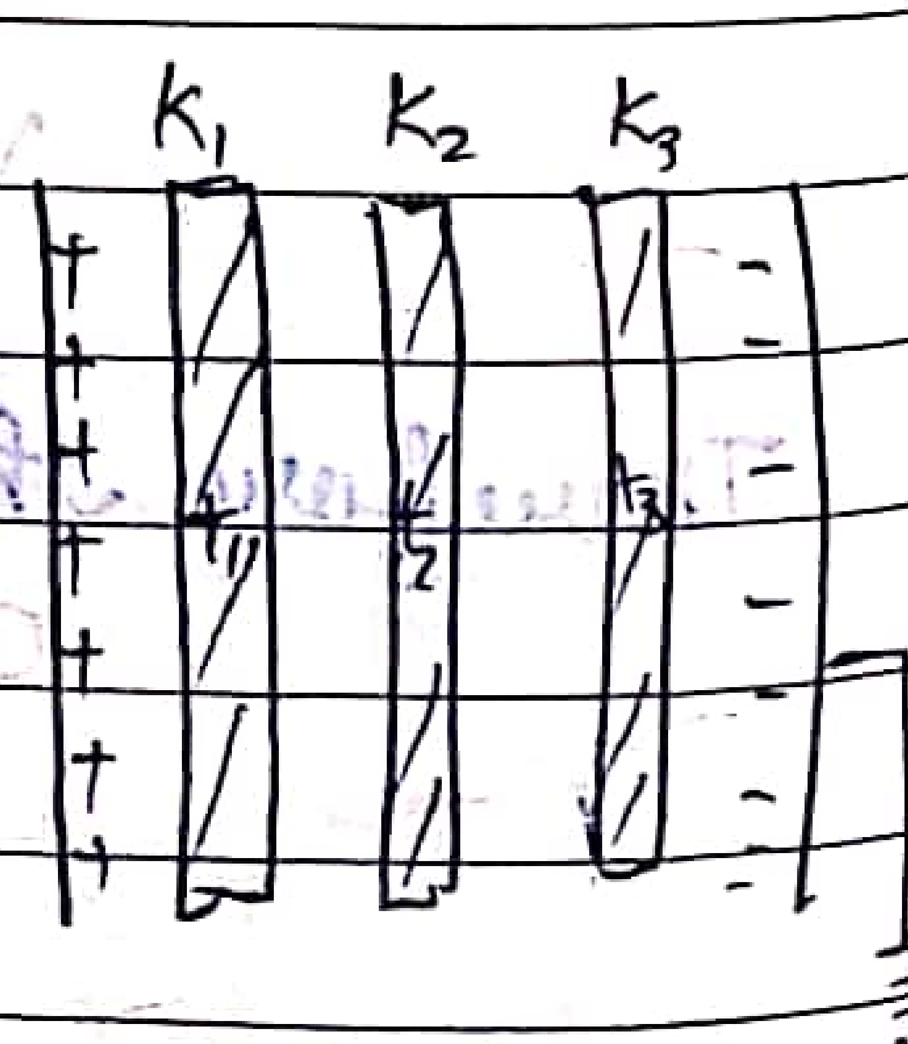
Therefore, Capacitance

$$C = \frac{A\epsilon_0}{\cancel{d} - \cancel{d} + \frac{d}{K}}$$

$$\boxed{C = \frac{KA\epsilon_0}{d}}$$

* If there are many dielectric medium placed b/w plates having thickness t_1, t_2, t_3 —— and dielectric constant K_1, K_2, K_3 —— then the capacitance of capacitor will be.—

$$\left\{ C = \frac{A\epsilon_0}{\left[d - (t_1 + t_2 + t_3) + \frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} - - - \right]} \right\}$$



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Combination of capacitor

Capacitor can be combine in two series form

① **In series** – In this form capacitors are combine end to end i.e. the second plate of first capacitor combine with first plate of second and the second plate of second capacitor combine with first plate of third capacitor.

In same way we combine each capacitor the last plate of last capacitance is connecting with earth and first plate of first capacitance is connect with source.

In series combination the charge on every plate are same but potential difference will be different.

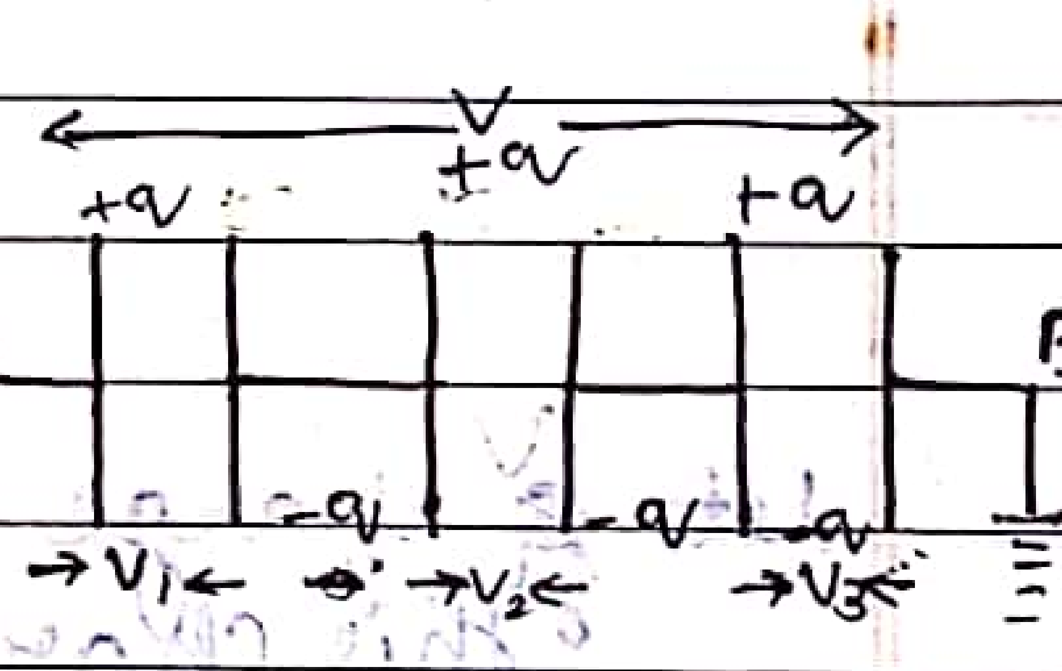
Let C_1, C_2 & C_3 capacitance are combine in series b/w A and B.

Let P.d. b/w A and B is V

Therefore → $V = V_1 + V_2 + V_3$ — ①

If the equivalance capacitance b/w A & B is C

then –



$$V = \frac{q}{C},\ V_1 = \frac{q}{C_1},\ V_2 = \frac{q}{C_2} \text{ and } V_3 = \frac{q}{C_3}$$

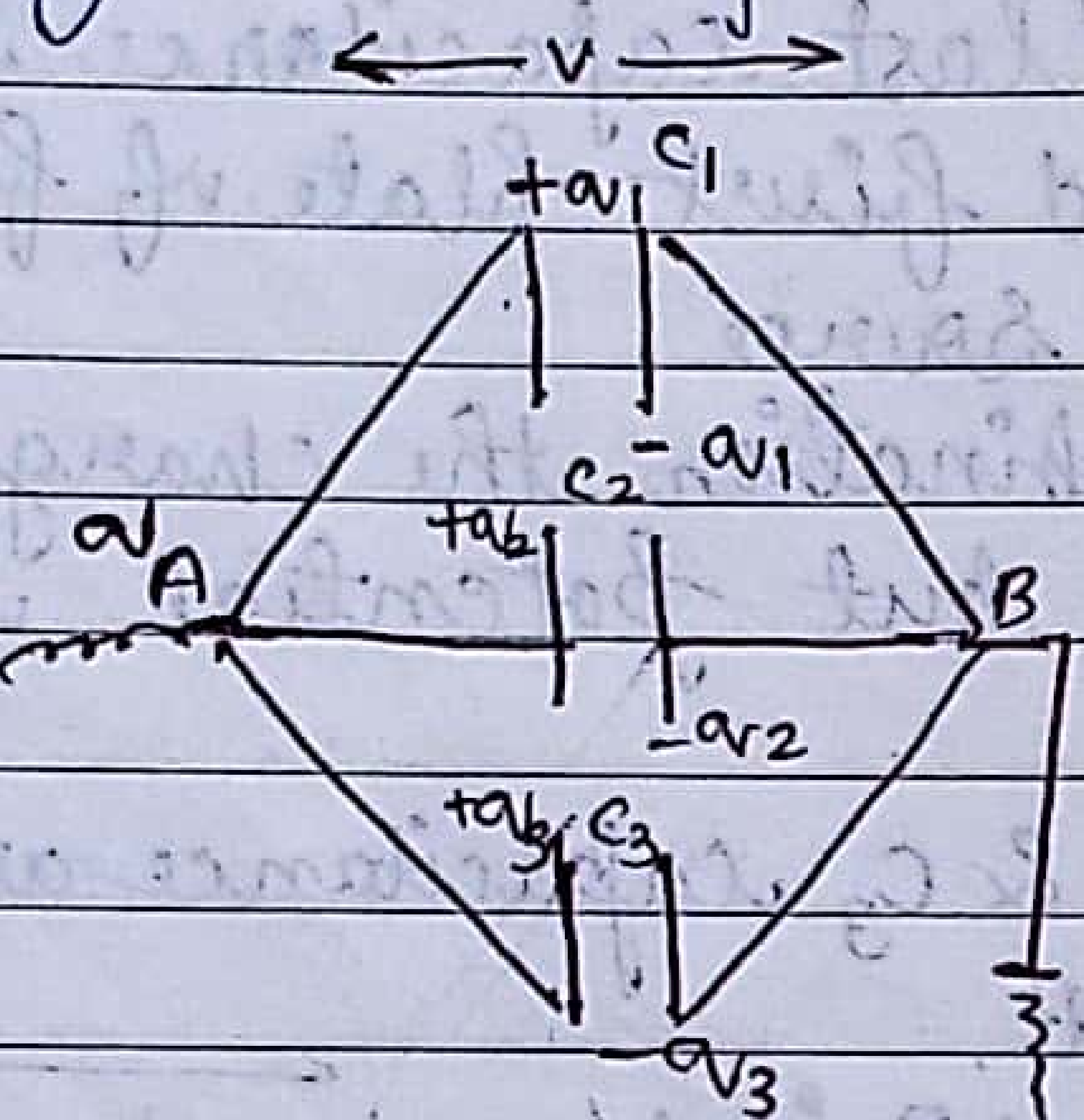
From eqn ①

$$\frac{\cancel{q}}{C} = \frac{\cancel{q}}{C_1} + \frac{\cancel{q}}{C_2} + \frac{q}{C_3}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

② **In parallel** - If first plate of every capacitor are combine at one point and second plates are combine at another point then this combination is called parallel combination.

In parallel combination the potential b/w the plates of every capacitor will be same and that will be equal to the p.d b/w A and B and charge will be distributed according to the capacitance of capacitor.



Let q charge is given by source at point A
This charge distributed on the capacitor
Let q_1 is the charge of 1st capacitor plates, q_2 on second & q_3 on third, so-

$$q = q_1 + q_2 + q_3 \quad -①$$

If the potential difference b/w A&B is V an equivalance capacitance b/w A & B is C
Therefore,

$$q = CV, \; q_1 = C_1V, \; q_2 = C_2V$$

and $q_3 = C_3V$

Therefore from eq ①

$$C\cancel{V} = C_1\cancel{V} + C_2\cancel{V} + C_3\cancel{V}$$

$$\left[C = C_1 + C_2 + C_3 \right]$$

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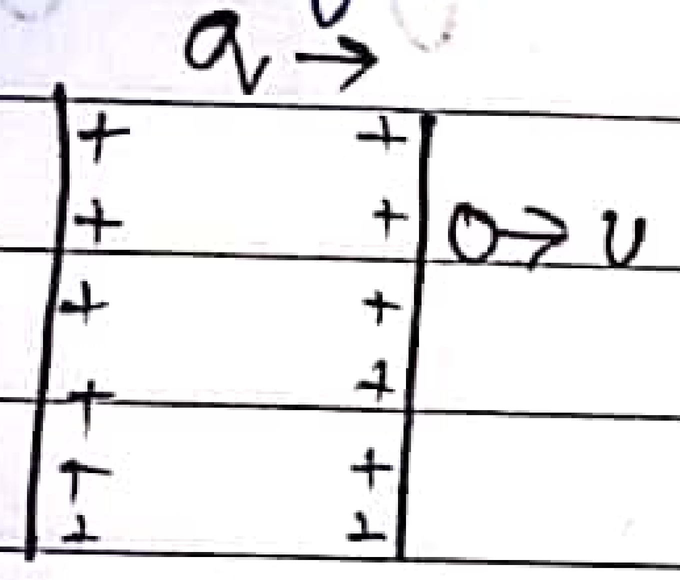
Energy Stored In Capacitor

The energy of a charge capacitor is measured by the total workdone in charging the Capacitor to a given capacitor.

Let a capacitor charge by q coulomb their increase from 0 to V.

Average potential increasement $= \frac{0+V}{2} = \frac{V}{2}$

therefore workdone –



$W =$ charge $\times$ Potential

$W = q \times \frac{V}{2}$

$W = \frac{1}{2} qV$

This workdone will be equal to stored energy.

Therefore

$$\boxed{U = \frac{1}{2} qV}$$

$\because q = CV$

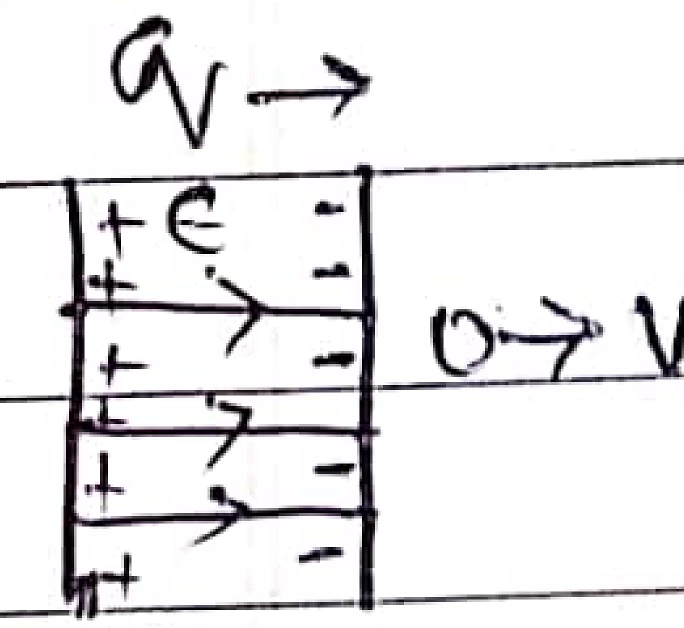
$$\boxed{U = \frac{1}{2} CV^2}$$

$\because V = \frac{q}{C}$

$$U = \frac{1}{2} \cancel{C} \frac{q^2}{C^{\cancel{2}}}$$

$$\boxed{U = \frac{1}{2} \frac{q^2}{C}}$$

If the intensity of EF b/w the plates of capacitor is E, then the energy stored per unit volume in capacitor,



$$\left[u = \frac{1}{2} \epsilon_0 E^2 \right]$$

This is called **energy density**.

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