Linear Programming (Step by Step - Full Process)

- Inequalities ($\geq, \leq$)
- Objective Function ($z = ax+by$)
- Diagram
  - Feasible
  - No Feasible Region ✗
- Corner Points
- Table (M/m)
  - Bounded (M/m) ✓
  - Unbounded (M/m)
    - $ax+by > M$ → No Common ✓ / Common ✗
    - $ax+by < m$ → No Common ✓ / Common ✗
    - (Diagram)

(Table)

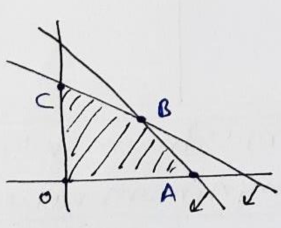
Corner Points	$z = ax+by$
A(,)	—
B(,)	—
C(,)	—

M = maximum value

m = minimum value

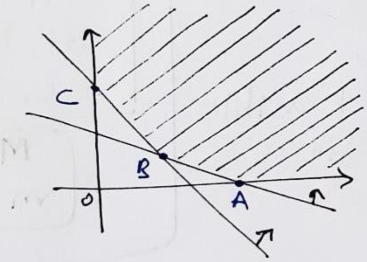
IOD.FCT.22

Objective Function $z = ax+by$ (जिसका minimum या maximum पता करना है)



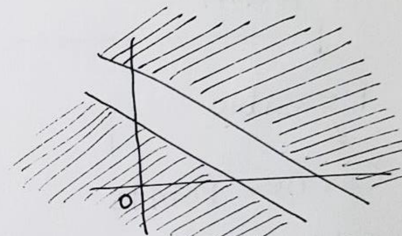
Feasible

(Bounded)



Feasible

(Unbounded)



No Feasible Region

(रैखिक प्रोग्रामन)

Practical Situation.

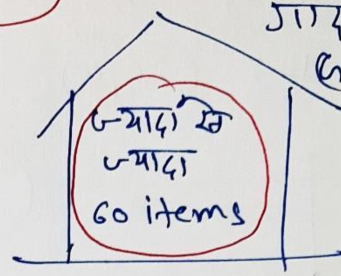
(Shop Keeper)

(Tables + Chairs)

Maximum Profit ?

₹ 50000

गोदाम (GoDown)



Cost of 1 table = ₹ 2500 | Profit on 1 table = ₹ 250

Cost of 1 Chair = ₹ 500 | Profit on 1 Chair = ₹ 75

Let No. of tables = x
No. of Chairs = y } → (Decision Variables)

Total Cost = $2500x + 500y \leq 50000$
Space = $x + y \leq 60$
} Constraints
(व्यवरोध)
(पाबंदी)

$\boxed{\text{Profit } (z) = 250x + 75y}$ → maximum

Objective function → M (maximum)
→ m (minimum)

0 table
y chairs } → $0 + \cancel{500}y \leq \cancel{50000}$ 100

$y \leq 100$

$x = 0$
$y = 60$

Profit (z)
$= 250 \times 0 + 75 \times 60$
$= 4500$

x tables
0 chair } → $\cancel{2500}x + 0 \leq \cancel{50000}$ 20

$x \leq 20$

$x + 0 \leq 60$

$x = 20$
$y = 0$

Profit (z)
$= 250x + 75y$
$= 250 \times 20 + 0$
$= 5000$

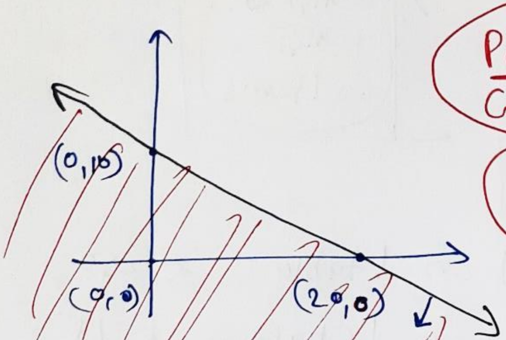
How to solve linear Inequalities (in 2 variables)?

(x) (y)

$\boxed{x+2y \le 20}$

$x+2y=20$

$(0,10)$ $(20,0)$



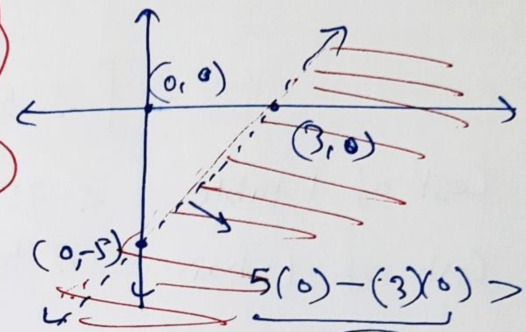
$(0,0)$ $\boxed{0+0 \le 20}$ True

Point
Cross check
(0,0)
Generally

$\boxed{5x-3y \ge 15}$

$5x-3y=15$

$(0,-5)$, $(3,0)$

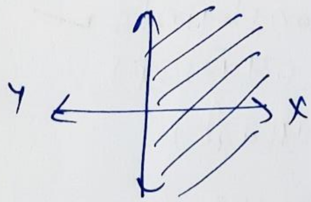


$5(0)-(3)(0) \ge 15$

$0 \ge 15$

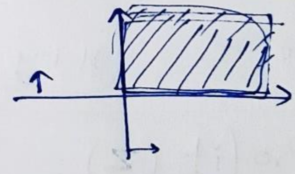
False

e.g. $x \ge 0$



$y \ge 0$

$\boxed{x \ge 0 \ \& \ y \ge 0}$ Common



How to Solve Questions of Linear programming?

Full Process - Step by Step

- Inequalities ($\geq, \leq$)
- Objective Function ($Z = ax+by$)
- Diagram
  - Feasible
  - No Feasible Region ✗
- Corner points (x, y)
- Table
  - Bounded (M/m) ✓
  - Unbounded (M/m)
    - $ax+by > M$
      - No Common ✓
      - Common ✗
    - $ax+by < m$
      - No Common ✓
      - Common ✗

Table

Corner Points	$Z = ax+by$
A (,) (x, y)	
B (,)	
C (,)	

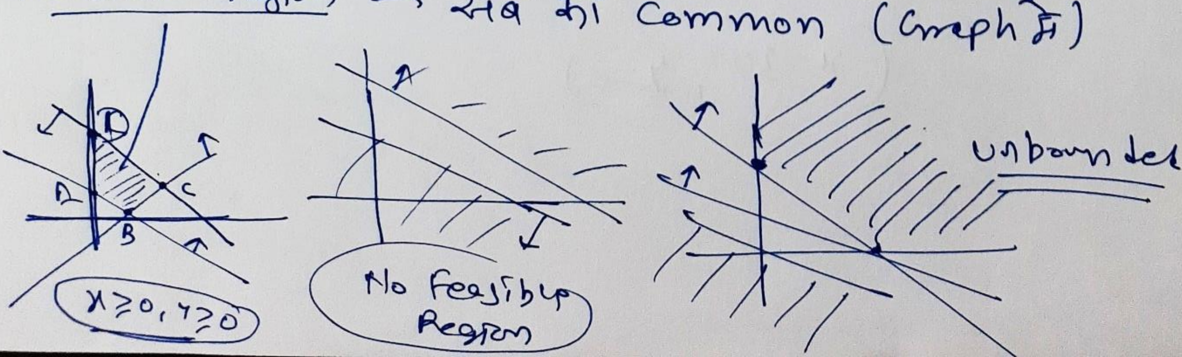
M = maximum value

m = minimum value

Remember $\Rightarrow$ $\boxed{\text{IOD. FCT. 22}}$

Objective function → जो Quantity जिसका maximum | minimum पता करना है।

Feasible Region → सब का Common (Graph में)



e.g. Solve the following linear programming problem graphically: $\boxed{\text{Maximise } Z = 4x+y}$ Objective Function.

Subject to the constraints: $x+y \le 50$ — (1)

$3x+y \le 90$ — (2)

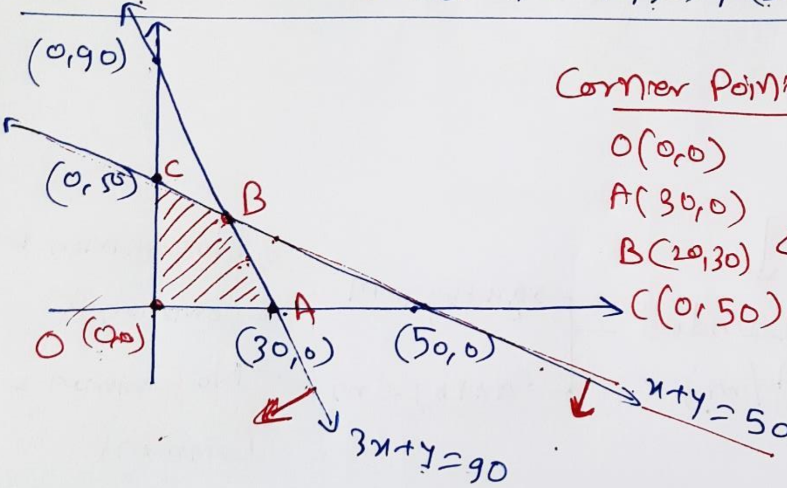
$x \ge 0,\ y \ge 0$ — (3)

I-quadrant

Ans. Diagram,

$x+y \le 50 \rightarrow x+y=50 \quad (0,50),\ (50,0)$

$3x+y \le 90 \rightarrow 3x+y=90 \quad (0,90),\ (30,0)$



Corner Points

O(0,0)

A(30,0)

B(20,30)

C(0,50)

For Point (B)

$$\begin{aligned} 3x+\cancel{y} &= 90 \\ -\ \ x+\cancel{y} &= 50 \\ \hline 2x &= 40 \\ x &= 20 \\ y &= 30 \end{aligned}$$

Table

Corner Points	$Z = 4x+y$
O (0,0)	0
A (30,0)	120 ← M=120 maximum
B (20,30)	80+30 = 110
C (0,50)	50

$\therefore$ maximum value of $(Z = 4x+y) = 120$

$(x=30,\ y=0)$

e.g. A Furniture dealer deals in only two items – tables & chairs. He has ₹ 50000 to invest and has storage space of at most 60 pieces. A table costs ₹ 2500 and a chair ₹ 500. He estimates that from the sale of one table, he can make a profit of ₹ 250 and that from the sale of one chair a profit of ₹ 75. How many tables and chairs should be bought so as to maximise his profit? Also find maximum profit.

Let no. of tables $= x \geq 0$
no. of chairs $= y \geq 0$

No.	Cost	Profit
x	2500	250
y	500	75

Constraints (Inequalities)

$$2500x + 500y \leq 50000 \quad \text{(Investment Constraint)}$$

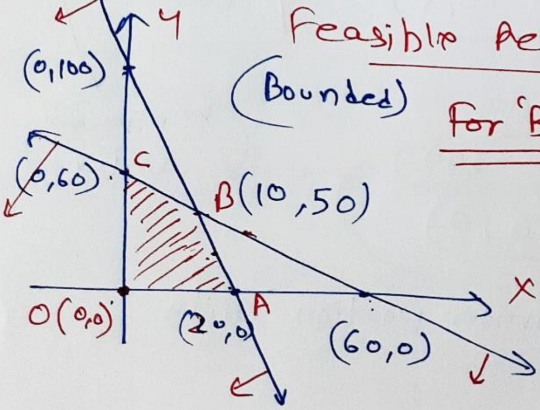
$$x + y \leq 60 \quad \text{(Space Constraint)}$$

Objective fn: $\boxed{\text{Profit } (Z) = 250x + 75y}$

Diagram:

$2500x + 500y \leq 50000 \Rightarrow 2500x + 500y = 50000$ $(0,100)$ $(20,0)$

$x + y \leq 60 \Rightarrow x + y = 60$ $(0,60)$, $(60,0)$



Feasible Region OABC (Bounded)

For 'B'

$$\begin{aligned} x + y &= 60 \\ -\;\; 5x + y &= 100 \\ \hline -4x &= -40 \\ x &= 10 \end{aligned}$$

$2500x + 500y = 50000$

$\Rightarrow \boxed{5x + y = 100}$

Table

Corner Points	Profit $Z = 250x + 75y$
O (0,0)	0
A (20,0)	5000
B (10,50)	6250 → M
C (0,60)	4500

No. of table $= x = 10$
No. of chairs $= y = 50$

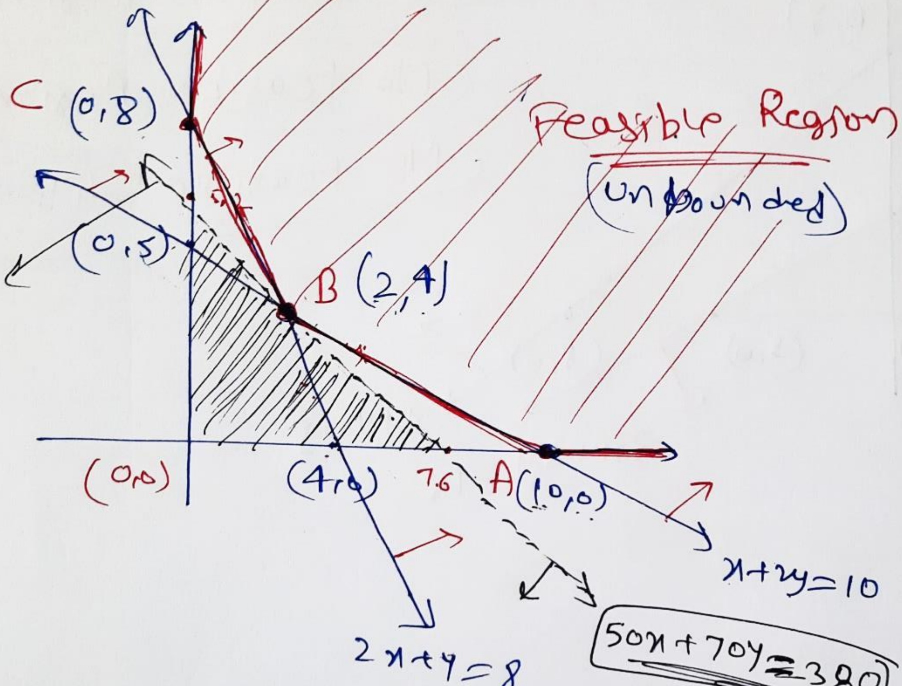
maximum Profit

e.g. Determine graphically the minimum value of the objective function $\boxed{Z = 50x + 70y}$ subject to constraints :

$2x + y \geq 8 \rightarrow 2x + y = 8 \;\; (0, 8), (4, 0)$

$x + 2y \geq 10 \rightarrow x + 2y = 10 \;\; (0, 5), (10, 0)$

$\boxed{x \geq 0, y \geq 0} \rightarrow$ I.



For (B)

$x + 2y = 10$

$2x + y = 8$

$2x + 4y = 20$

$-3y = -12$

$y = 4$

$x = 2$

Table

Corner Point	$Z = 50x + 70y$
A (10,0)	500
B (2,4)	380 ← m = minimum value.
C (0,8)	560

∵ Our feasible region is unbounded.

New Inequality. $\boxed{50x + 70y < 380}$ Dotted line

Diagram.

$50x + 70y = 380$

(0, 5.2)

(7.6, 0)

(2,4)

$100 + 280 = 380$

$380 = 380$

∴ $50x + 70y < 380$ & feasible Region has no common portion.

∴ minimum value of $Z = 380$

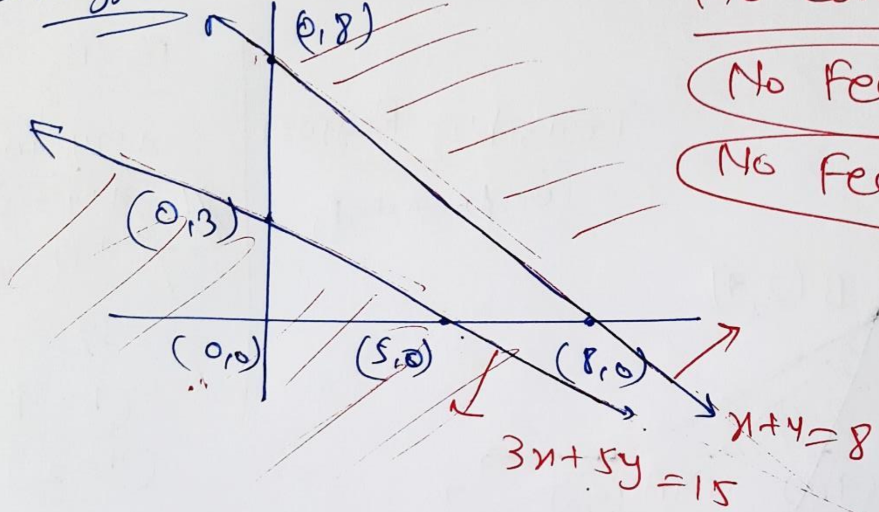
e.g. Minimize $\boxed{Z = 3x+2y}$ subject to constraints

$x+y \geq 8 \longrightarrow x+y=8 \rightarrow (0,8),(8,0)$

$3x+5y \leq 15 \longrightarrow 3x+5y=15 \quad (0,3),(5,0)$

$x \geq 0,\ y \geq 0 \rightarrow$ I Quad

Diagram



No Common Region

No Feasible Region

No Feasible Solution

Exercise 12.1

Solve the following Linear Programming Problems graphically:

Q.1 Maximise $\boxed{Z = 3x + 4y}$

Subject to the constraints : $x + y \le 4$, $x \ge 0$, $y \ge 0$.

Ans:

Inequalities: $\boxed{x + y \le 4} \Rightarrow x + y = 4$ $(0,4)$, $(4,0)$

$\boxed{x \ge 0, y \ge 0}$ → I-quadrant.

I II III IV

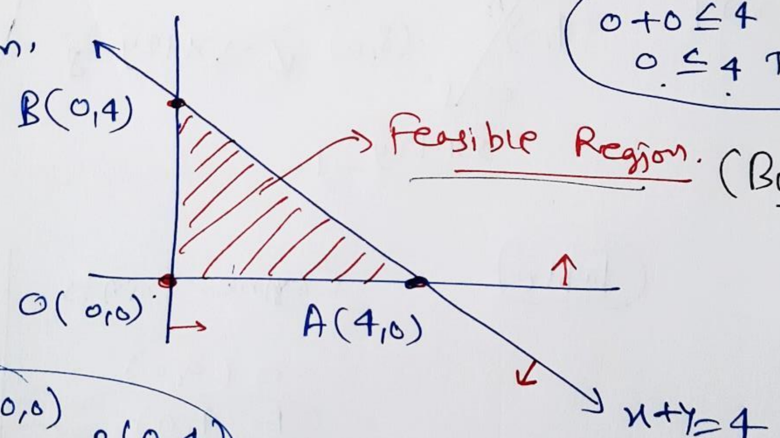
Objective Function

$(Z = 3x + 4y)$

$0 + 0 \le 4$

$0 \le 4$ True

Diagram:



Corner Points: O(0,0) A(4,0) B(0,4)

Table (M/m)

Corner Points	$Z = 3x + 4y$ (maximize)
O (0,0)	0
A (4,0)	12
$\boxed{\text{B (0,4)}}$	$\boxed{16}$ ← M = maximum

∵ Since Feasible region is bounded,

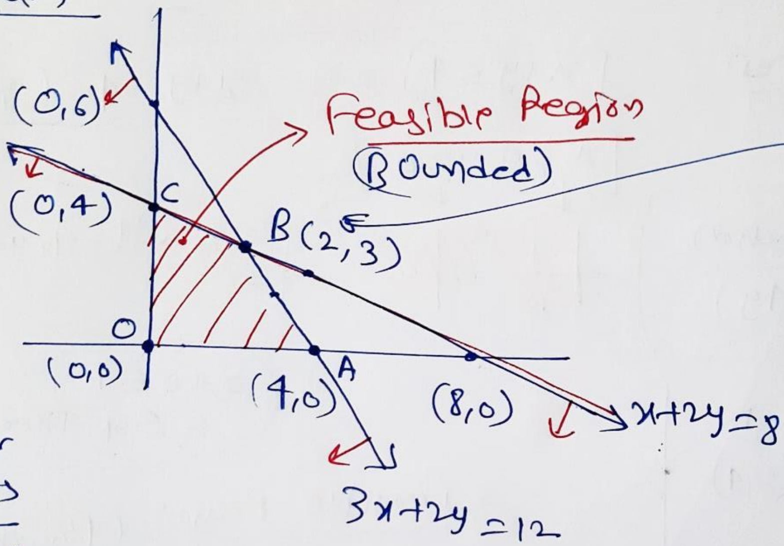
∴ maximum value of $(Z = 3x + 4y) = 16$ at $(0,4)$

Q.2 Minimize $\boxed{Z = -3x + 4y}$ ← objective fn.

Subject to constraints:

Inequalities
$$\begin{cases} x + 2y \leq 8 \rightsquigarrow x + 2y = 8 \quad (0,4),(8,0) \\ 3x + 2y \leq 12 \rightsquigarrow 3x + 2y = 12 \quad (0,6),(4,0) \\ x \geq 0, y \geq 0 \rightsquigarrow \text{I quadrant} \end{cases}$$

Diagram



For 'B'

$$3x + \cancel{2y} = 12$$
$$x + \cancel{2y} = 8$$
$$2x = 4$$
$$x = 2$$
$$y = 3$$

Corner Points

O (0,0)
A (4,0)
C (0,4)
B (2,3)

Table

Corner Points	minimize $Z = -3x + 4y$
O (0,0)	0
A (4,0)	-12 = m
B (2,3)	6
C (0,4)	16

minimum value

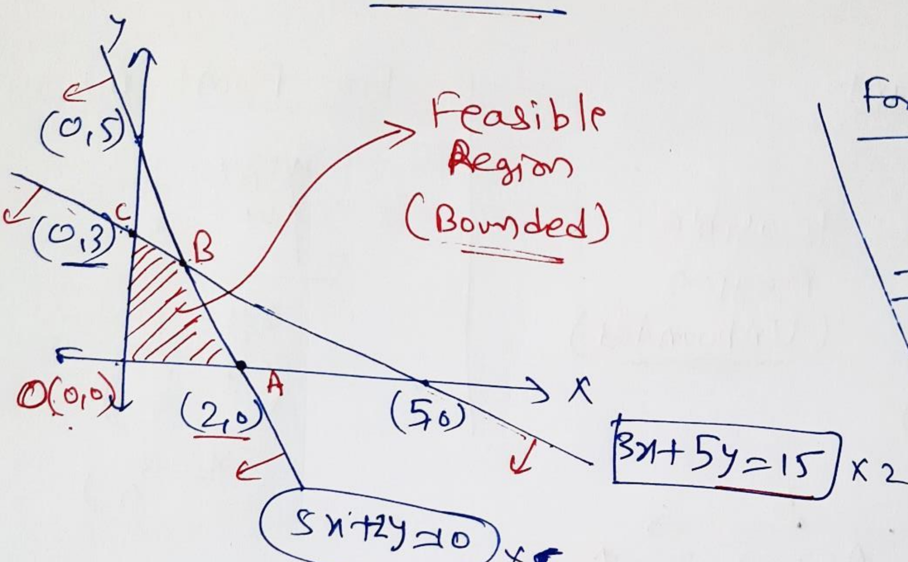
$\boxed{\text{minimum value of } (Z = -3x + 4y) = -12 \text{ at } (4,0)}$

Q.3 maximize $\boxed{Z = 5x + 3y}$ subject to → objective function

$3x + 5y \le 15$, → $3x + 5y = 15$ (0,3), (5,0)

$5x + 2y \le 10$, → $5x + 2y = 10$ (0,5), (2,0)

$\boxed{x \ge 0,\ y \ge 0}$ → I-quadrant



For Point B

$$6x + 10y = 30$$
$$25x + 10y = 50$$
$$\overline{-19x = -20}$$

$x = \frac{20}{19}$

$y = \frac{45}{19}$ B

Table

Corner Points	$Z = 5x + 3y$
O (0,0)	0
A (2,0)	10
B $\left(\frac{20}{19}, \frac{45}{19}\right)$	$235/19$ = M
C (0,3)	9

(maximum value)

- Since feasible region is bounded

∴ maximum value of $(Z = 5x + 3y) = \frac{235}{19}$

at $\left(\frac{20}{19}, \frac{45}{19}\right)$

Q.4 Minimise $\boxed{Z = 3x + 5y}$ such that

$\to$ objective Function

Ineq.

$x + 3y \geq 3 \longrightarrow x + 3y = 3 \quad (0,1), (3,0)$

$x + y \geq 2 \longrightarrow x + y = 2 \quad (0,2), (2,0)$

$x, y \geq 0$

$\downarrow$

I-quadrant.

For Point (B)

$x + 3y = 3$

$x + y = 2$

$\overline{2y = 1}$

$y = \frac{1}{2}$

$(x = 3/2)$

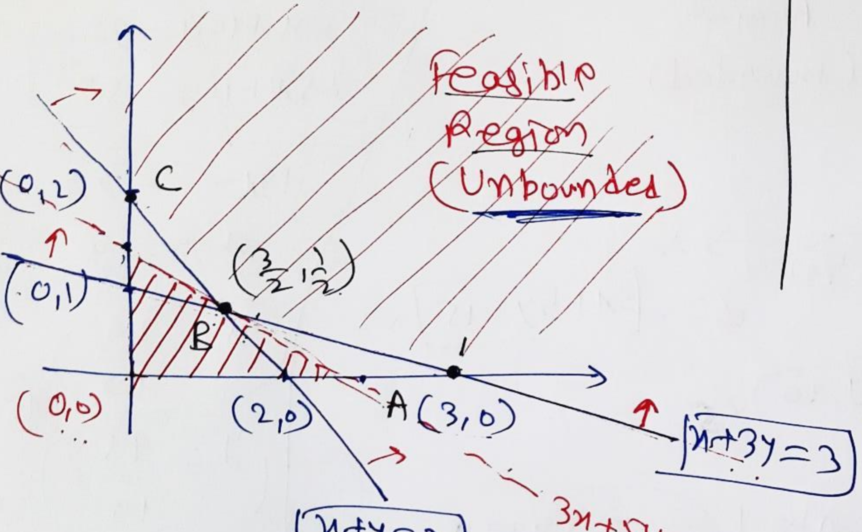


Table.

Corner Points.	$Z = (3x + 5y)$ (Minimise)
A (3, 0)	9
B $(3/2, \frac{1}{2})$	7 $\leftarrow$ m = minimum value.
C (0, 2)	10

Since Feasible region is unbounded.

$\Rightarrow \boxed{3x + 5y < 7}$ Diagram $3x + 5y = 7$ (0, 1.4)

$(3/2, \frac{1}{2})$ will lie on (2.3, 0)

$\Rightarrow \because$ There is no Common Part of $3x + 5y < 7$ and Feasible region $\rightarrow \therefore \text{min.}(Z) = 7$ at $(\frac{3}{2}, \frac{1}{2})$

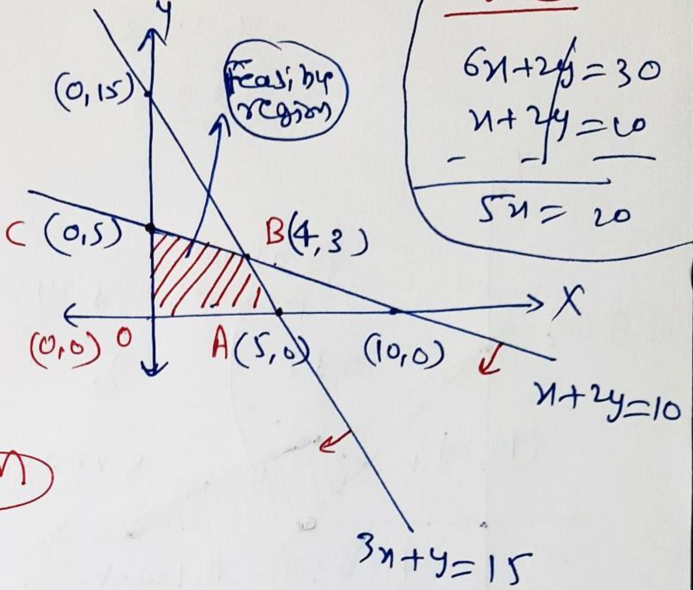
Q5 Maximize $Z = 3x+2y$ subject to

$\underbrace{x+2y \le 10}_{\downarrow}$, $\underbrace{3x+y \le 15}_{\downarrow}$, $\underbrace{x, y \ge 0}$.

$x+2y=10$	$3x+y=15$
$(0,5), (10,0)$	$(0,15), (5,0)$

For (B)

$6x+2y=30$
$x+2y=10$
$-\quad -\quad\; -$
$5x = 20$



Table

Corner Points	maximise $Z = 3x+2y$
O (0,0)	0
A (5,0)	15
B (4,3)	18 = M
C (0,5)	10

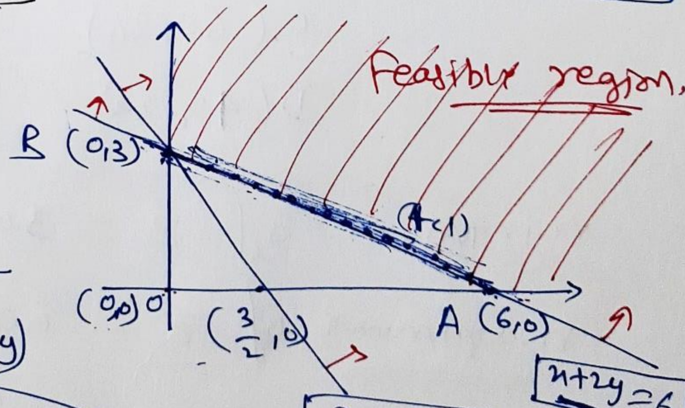
maximum value of $Z = 18$ at $(4,3)$ (Bounded)

Q.6 Minimize $Z = x+2y$ subject to $2x+y \ge 3$, $x+2y \ge 6$, $x \ge 0, y \ge 0$.

Show that the minimum of Z occurs at more than two points.

$2x+y=3$	$x+2y=6$
$(0,3), (\frac{3}{2},0)$	$(0,3)$ $(6,0)$

Table	Corner Points	$Z = x+2y$
	A (6,0)	6
	B (0,3)	6



minimum value of $(Z = x+2y)$

$= 6$ at $A(6,0)$ & $B(0,3)$

↓

at all points lying on line segment $\overline{AB}$

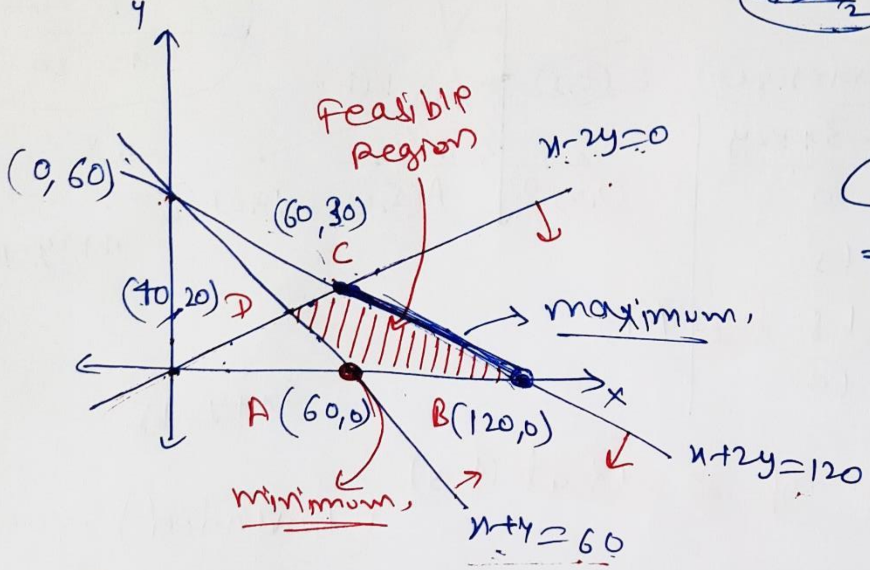
Q.7 Minimise and maximise $Z = 5x + 10y$

Subject to $x + 2y \le 120 \rightsquigarrow x + 2y = 120$ $(0, 60), (120, 0)$

$x + y \ge 60 \rightsquigarrow x + y = 60$ $(0, 60), (60, 0)$

$x - 2y \ge 0 \rightsquigarrow x - 2y = 0$ $(0, 0) \cdot (1, \frac{1}{2})$

$\boxed{x, y \ge 0}$ $x = 2y$ $y = \frac{x}{2}$



For (C)

$x = 2y$

$x + 2y = 120$

$\Rightarrow 4y = 120$

$y = 30$

For (D)

$x = 2y$

$x + y = 60$

$3y = 60$

$y = 20$

Table,

Corner Points	$Z = 5x + 10y$
A (60, 0)	300 ← m = minimum
B (120, 0)	600 } → M = maximum
C (60, 30)	600
D (40, 20)	400

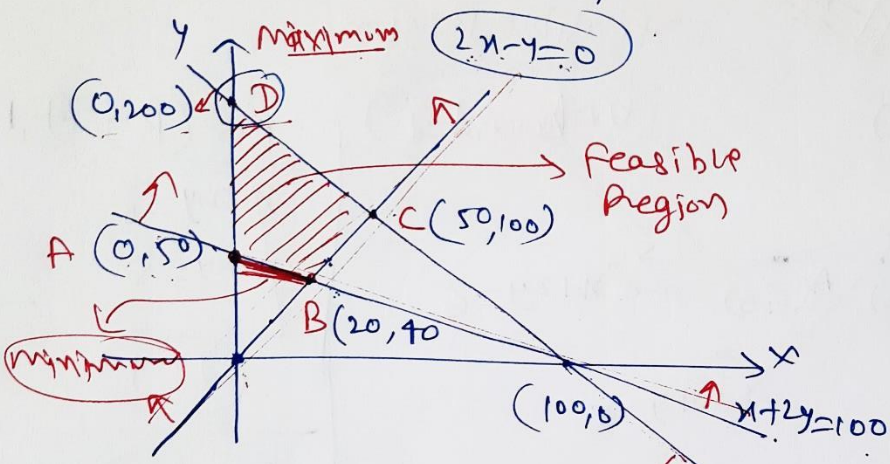
minimum of $Z = 300$ at $(60, 0)$

maximum of $Z = 600$ at all points lying on the line segment joining $(120, 0)$ & $(60, 30)$.

Q.8 Minimise and maximise $\boxed{Z = x+2y}$ subject to

$x+2y \geq 100$, $2x-y \leq 0$, $2x+y \leq 200$, $x, y \geq 0$.

$x+2y=100$	$2x-y=0$	$2x+y=200$
(0, 50), (100, 0)	(0,0), (1,2)	(0, 200), (100,0)



For (B)

$y=2x$

$x+2y=100$

$5x=100$

$x=20$

For C

$y=2x$

$2x+y=200$

$4x=200$

$x=50$

Table

Corner Points	$Z = x+2y$
A (0,50)	100
B (20,40)	100
C (50,100)	250
D (0,200)	400

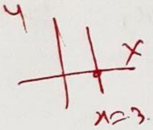
100, 100 → m = minimum.

400 → M = maximum.

minimum of $Z = 100$ at all points lying on line segment joining (0,50), (20,40)

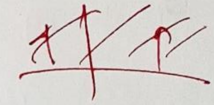
maximum of $Z = 400$ at (0,200).

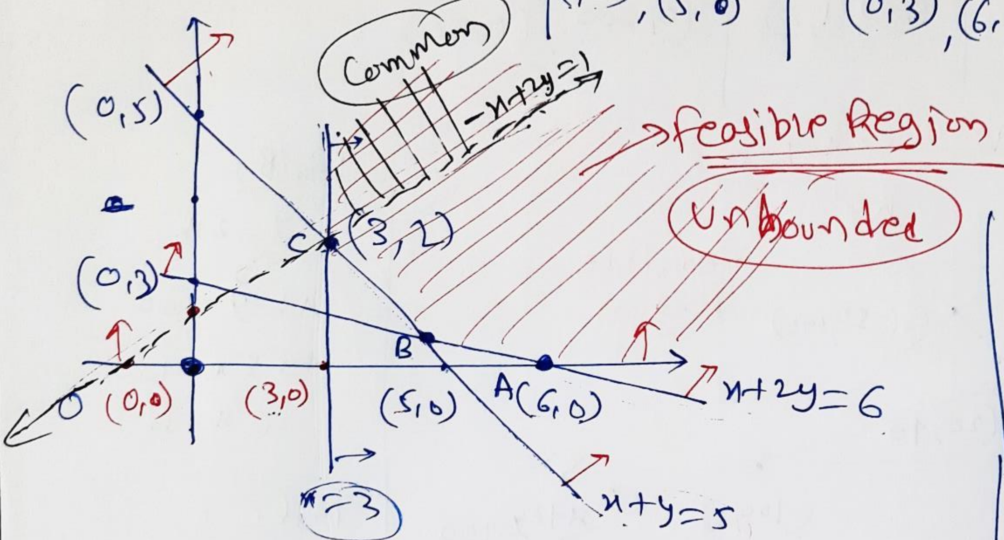
Q.9 Maximise $Z = -x + 2y$, subject to the constraints: $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.



$x = 3$

$x + y = 5$	$x + 2y = 6$
(0,5), (5,0)	(0,3), (6,0)





For B (4,1)

$x + 2y = 6$

$x + y = 5$

$y = 1$

For C

Table: ~~Corner~~ (maximise)

Corner Points	$Z = -x + 2y$
A (6,0)	-6
B (4,1)	-2
C (3,2)	1 ← M = Maximum,

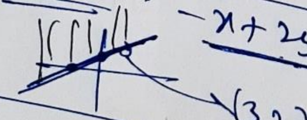
But feasible ↓ unbounded

Let $(-x + 2y) > 1$

$0 + 0 > 1$

$0 > 1$

Diagram, $-x + 2y = 1$, $(0, \frac{1}{2})$, $(-1, 0)$



Since $(-x + 2y > 1)$ & feasible region has some common portion.

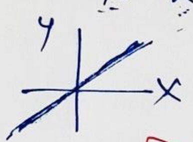
∴ No maximum value of Z.

Q.10 Maximise $Z = x+y$, subject to

$x - y \leq -1$, $-x+y \leq 0$, $x, y \geq 0$.

I-quad

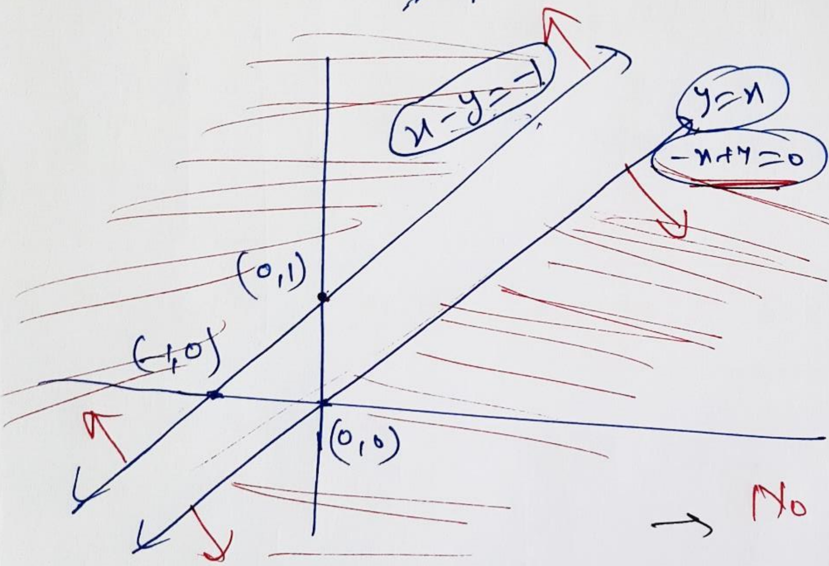
$x-y=-1$	$-x+y=0$
$(0,1)$, $(-1,0)$	$y=x$



$y = mx + c$

$y = x+1$ $\quad m_1 = 1$

$y = x$ $\rightarrow m_2 = 1$



$0 - 0 \leq -1$

$0 \leq -1$ ✗

$-x+y \leq 0$

$0+1 \leq 0$ ✗

→ No Common Region

→ No Feasible Region

→ No Feasible Solution

→ No maximum

Exercise 12.2

Q.1 let quantity of food 'P' = x kg
'Q' = y kg

	(₹/Kg) Cost	(units/Kg) A	B
P	60	3	5
Q	80	4	2

Vitamin 'A' $\geq$ 8 units.
Vitamin 'B' $\geq$ 11 units.

Inequality

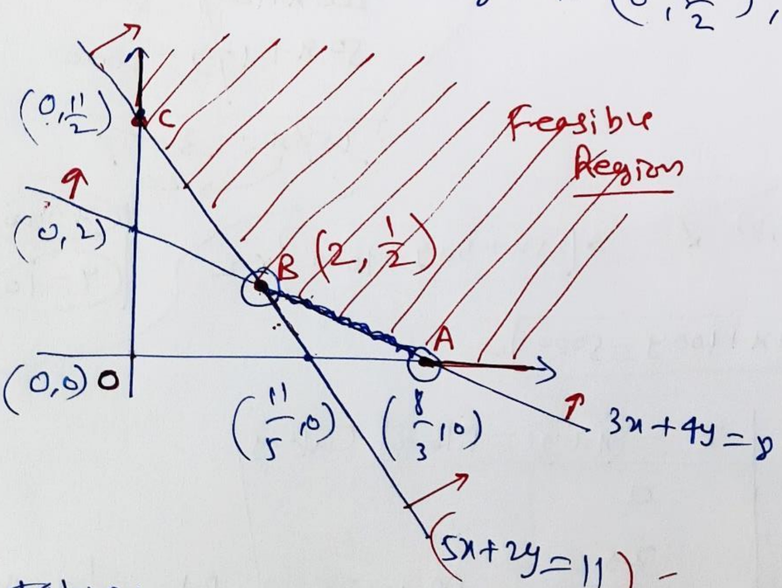
$$\begin{cases} 3x + 4y \geq 8 & (\text{Vit 'A' Constraint}) \\ 5x + 2y \geq 11 & (\text{Vit 'B' Constraint}) \\ x, y \geq 0 \end{cases}$$

Objective Function $\boxed{\text{Cost } (z) = 60x + 80y}$ (minimise)

$3x + 4y \geq 8 \longrightarrow 3x + 4y = 8 \quad (0, 2), \left(\frac{8}{3}, 0\right)$

$5x + 2y \geq 11 \longrightarrow 5x + 2y = 11 \quad \left(0, \frac{11}{2}\right), \left(\frac{11}{5}, 0\right)$

$x, y \geq 0$
I-quadrant



For (B)

$3x + 4y = 8$
$10x + 4y = 22$
$-7x = -14$
$x = 2$ $\quad y = \frac{1}{2}$

Table,

Corner Points	$Z = 60x + 80y$
A $(8/3, 0)$	160
B $(2, 1/2)$	160
C $(0, 11/2)$	440

$\left.\begin{matrix}160\\160\end{matrix}\right\}$ minimum value of cost

Minimum Cost = 160 at all points lying on line segment joining $(8/3, 0)$ & $(2, \frac{1}{2})$

Q.2 let no. of cakes of type$_I$ = $x \geq 0$

type$_{II}$ = $y \geq 0$

(5 kg = 5000 gm)

(1 Kg = 1000 gm)

	Floor	fat
Type - I	200 gm	25 gm
Type - II	100 gm	50 gm

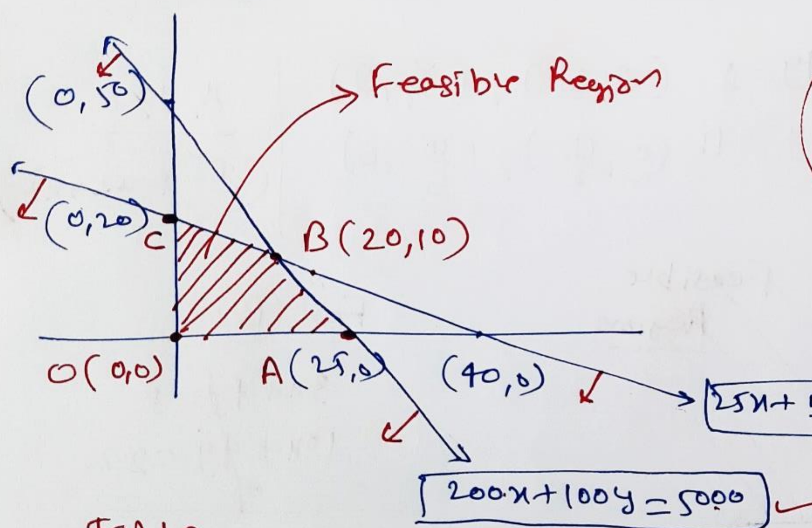
Constrains.

$x \geq 0, y \geq 0$ → I quad.

(flour) $200x + 100y \leq 5000$ → $200x + 100y = 5000$
(0,50), (25,0)

(Fat) $25x + 50y \leq 1000$ → $25x + 50y = 1000$
(0,20), (40,0)

Objective Function = No. of cakes (z) = x+y (maximum)



For (B)

$200x + 100y = 5000$

$50x + 100y = 2000$ ×2

$150x = 3000$ → 20

$x = 20$

$y = 10$

Table.

Corner Points	Z = x+y = No. of cakes
O (0,0)	0
A (25,0)	25
B (20,10)	30 = Maximum No. of cakes
C (0,20)	20

(Type$_I$ = 20 cakes
Type$_{II}$ = 10 Cakes)

Q.3

	Quantity	Machine time (hours)	Craftman's time (hours)	Profit
Tennis Racket	x	1.5	3	20 (₹)
Cricket Bat	y	3	1	10 (₹)
	$x+y$	maximum (42 hrs)	max. (24 hrs)	Total Profit $= 20x+10y$

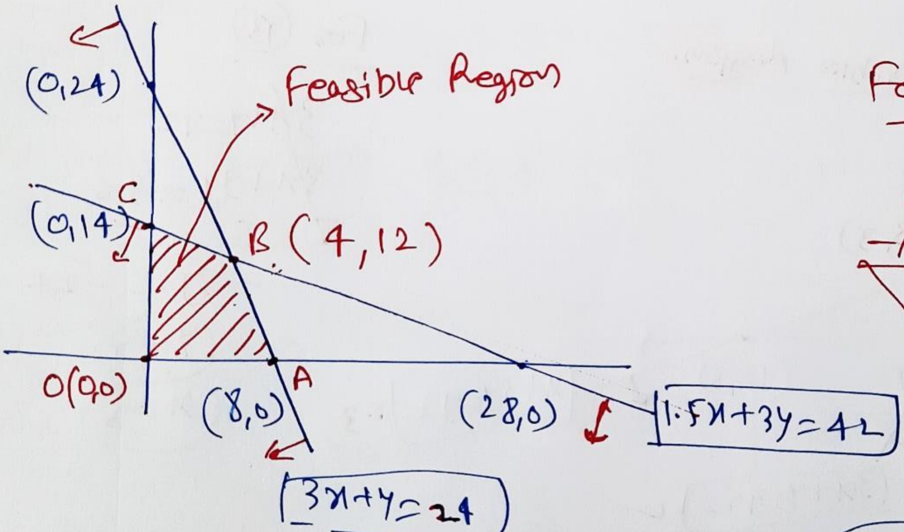
$x \geq 0$
$y \geq 0$

Inequalities.

Machine $1.5x + 3y \leq 42 \longrightarrow 1.5x + 3y = 42 \quad (0,14), (28,0)$

Craftman $3x + y \leq 24 \longrightarrow 3x + y = 24 \quad (0,24), (8,0)$

Objective Fn: Capacity $(z) = x+y$ ~~Mami~~ Maximise



× 2

For (B)

$3x + y = 24$
$3x + 6y = 84$
$- \quad - \quad -$
$-5y = -60$
$y = 12$
$x = 4$

Table.

Corner points	$Z = (x+y)$ Full Capacity
O (0,0)	0
A (8,0)	8
B (4,12)	16 = M = maximum.
C (0,14)	14

Rackets $= 4 = x$
Bats $= 12 = y$

Maximum Profit $= 20x + 10y$
$= 20 \times 4 + 10 \times 12 = 200$ ₹

Exercise 12.2 Class 12

Q.4

	Quantity	Machine 'A' time	Machine (B) time	(₹) Profit
Nuts	$x \geq 0$	1 hour	3 hours	17.50
Bolts	$y \geq 0$	3	1	7

max. 12 hrs   max. 12 hrs

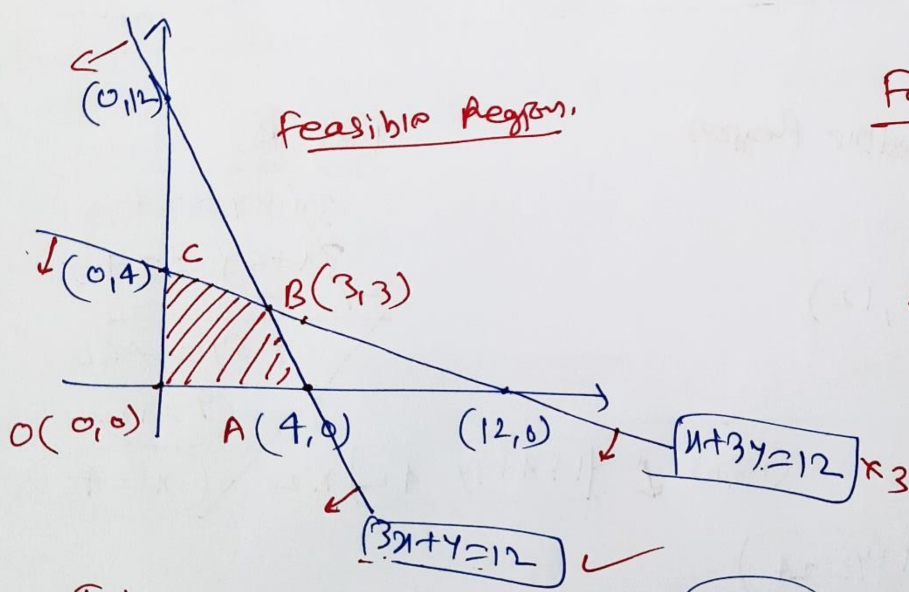
Inequality

(A) $x + 3y \leq 12$ → $x+3y=12$ (0,4), (12,0)

(B) $3x + y \leq 12$ → $3x+y=12$ (0,12), (4,0)

~~Objection~~

Objective function, Profit $(z) = 17.5x + 7y$ (maximise)



For (B)

$3x+y=12$

$3x+9y=36$

$-8y=-24$

$y=3$

$x=3$

Table

Corner Points	Profit $z = 17.5x+7y$ ↔ maximum
O (0,0)	0
A (4,0)	70
B (3,3)	$52.5+21=73.5$ ← maximum profit
C (0,4)	28

x = no. of packages of nuts = 3

y = ———————— Bolts = 3

Exercise (12.2)

Q.5

	Quantity	Profit (₹)	Automatic (time)	Hand operated (time)
Screw (A)	x	7	4 min.	6 min.
Screw (B)	y	10	6 min.	3 min.
			↓ 4 hrs (240 min.)	↓ 4 hrs. (240 min.)

$x, y \geq 0$

max.

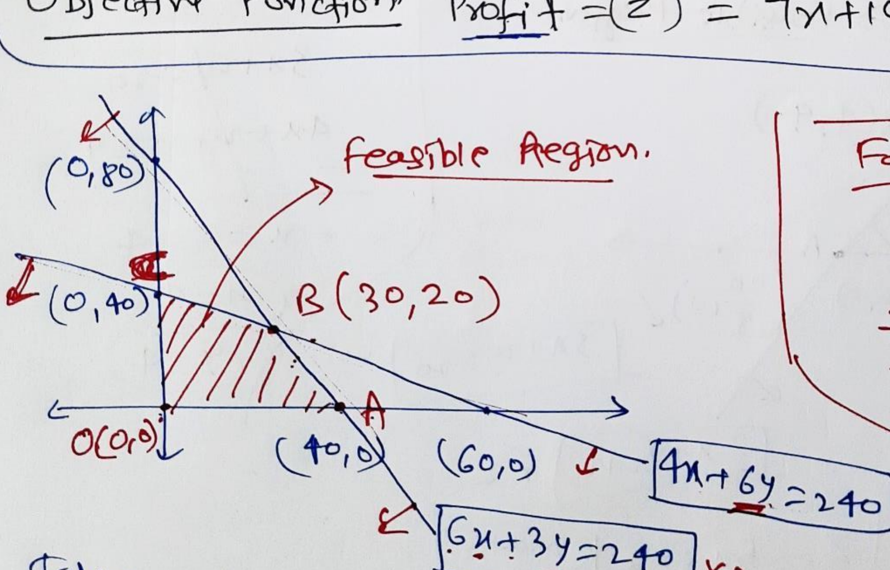
~~Date~~

Inequalities.

Automatic $4x+6y \leq 240 \longrightarrow 4x+6y=240$ $(0,40)$ $(60,0)$

Hand operated $6x+3y \leq 240 \longrightarrow 6x+3y=240$ $(0,80)$ $(40,0)$

Objective Function Profit $=(z) = 7x+10y$ (maximise)



For (B)

$4x+6y=240$

$12x+6y=480$

$-\quad-\quad-$

$-8x = -240$

$x = 30$

$y=20$

$\times 2$

210
200

Table.

Corner Points	$z=(7x+10y)$ ← Profit
O (0,0)	0
A (40,0)	280
B (30,20)	410 = maximum profit.
C (0,40)	400

$x=30$
$y=20$

Screw (A) → 30
Screw (B) → 20

Exercise 12.2 Class 12

Q.6

	Quantity	Profit (₹)	Grinding / Cutting machine (time)	Sprayer (time)
Lamps	x	5	2	3
shades	y	3	1	2
		max.?	Max. 12 hrs	max. 20 hrs

Inequalities.

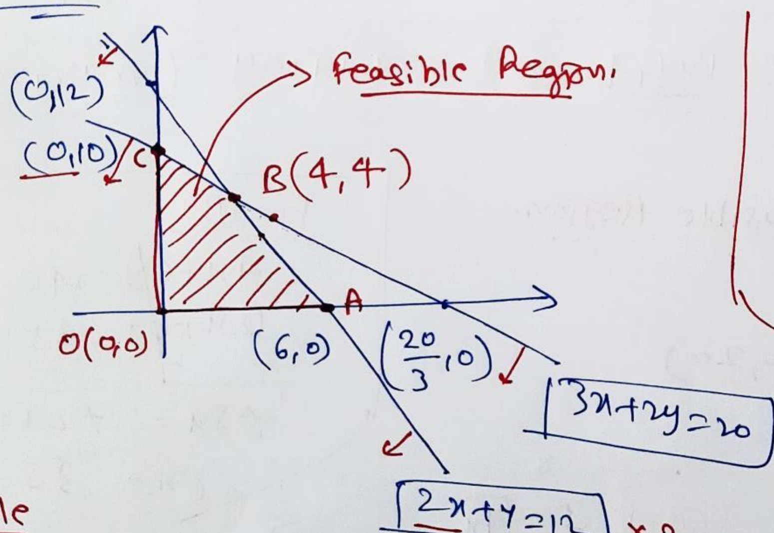
G/C → $2x+y \le 12 \to 2x+y=12$ $(0,12),(6,0)$

Sprayer → $3x+2y \le 20 \to 3x+2y=20$ $(0,10)$, $\left(\frac{20}{3},0\right)$

$x \ge 0, y \ge 0$ → I-quadrant.

$\boxed{\text{Objective Function. Profit } (Z) = 5x+3y \text{ (maximise)}}$

Diagram.



For B

$3x+2y=20$

$4x+2y=24$ ($\times 2$)

$-x=-4$

$x=4$

$y=4$

Table

Corner Points	$Z=5x+3y$	max.
O (0,0)	0	
A (6,0)	30	
B (4,4) ($x=4$, $y=4$)	32	← maximum profit
C (0,10)	30	

→ lamps = 4

→ shades = 4

Exercise 12.2 | Class 12

Q.7

	Quantity	Profit (₹)	Cutting Time	Assembling Time
Type (A)	x	5	5 min.	10 min.
Type (B)	y	6	8 min.	8 min.

max. available

3 hrs 20 min
$(3\times60+20)$ min.
$= 200$ min.

4 hrs
$= 240$ min.

Inequalities.

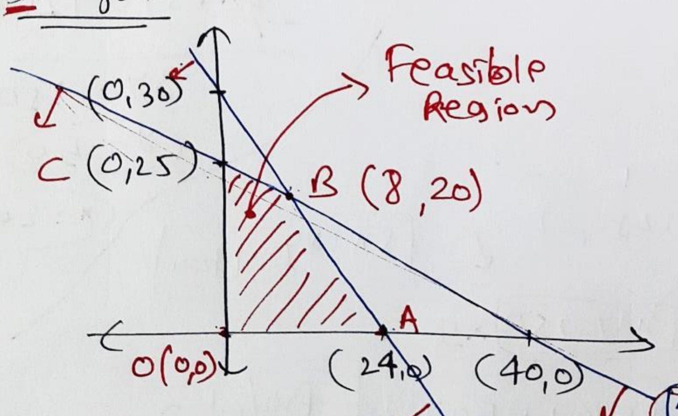
Cutting Time → $5x+8y \le 200 \to 5x+8y=200$ (0, 25) (40, 0)

Assembling Time → $10x+8y \le 240 \to 10x+8y=240$ (0, 30) (24, 0)

$x, y \ge 0$ → I quad.

Objective Function. Profit $(Z) = 5x+6y$ (maximise)

Diagram.



For (B)

$10x+8y=240$
$5x+8y=200$
$-\quad\cancel{-}\quad -$
$5x = 40$
$x=8$
$y=20$

Table.

Corner Points	$Z = 5x+6y$ (Profit) → max.
O (0,0)	0
A (24,0)	120
$\boxed{B (8,20)}$	$\boxed{160}$ ← max. profit.
C (0,25)	150

$x=8$ ← type (A)
$y=20$ ← type (B)

Exercise 12.2

Q. 8

	units	Profit	Cost
Desktop	x	4500	25000
Portable	y	5000	40000

max. ₹ 70,00,000

Inequalities.

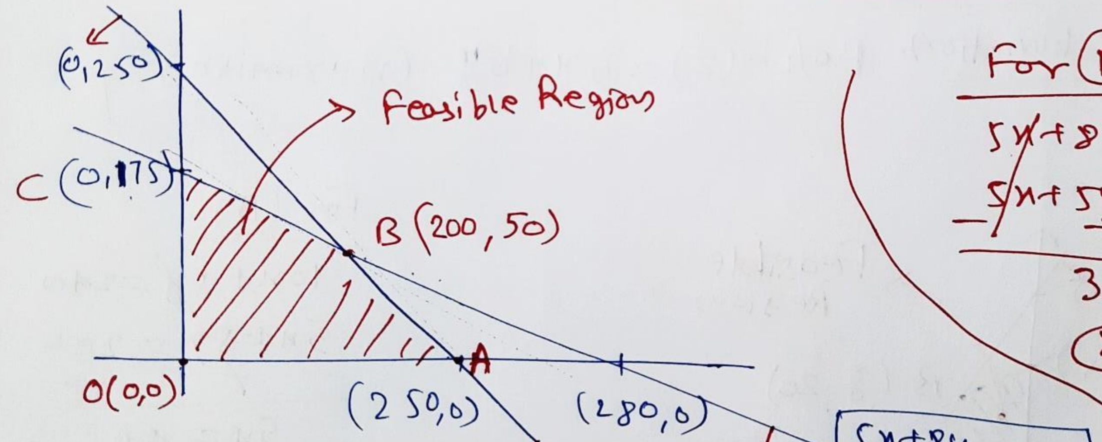
Demand $\rightarrow x+y \le 250 \rightarrow x+y=250$ (0,250), (250,0)

Investment $\rightarrow 25000x+40000y \le 7000000$

$\Rightarrow \boxed{5x+8y \le 1400} \rightarrow 5x+8y=1400$
(0,175)
(280,0)

$\boxed{x \ge 0,\ y \ge 0}$

Objective Function $\boxed{\text{Profit}(Z) = 4500x+5000y} \rightarrow$ maximise



For (B)

$5x+8y=1400$

$-\ \cancel{5x}+5y=1250$ ($x+y=250$) ×5

$3y=150$

$y=50$

$x=200$

Table.

Corner Points	Z = 4500x + 5000y ← Profit → max.
O (0,0)	0
A (250,0)	11,25,000
B (200,50)	11,50,000 → maximum Profit
C (0,175)	8,75,000

$x=200=$ Desktop

$y=50=$ Portable

Exercise 12.2 | Class 12

Q.9

	Units	Cost (₹/unit)	Vitamin A	Minerals
F_1	x	4	3	4
F_2	y	6	6	3
			at least 80	100 units at least

Inequalities.

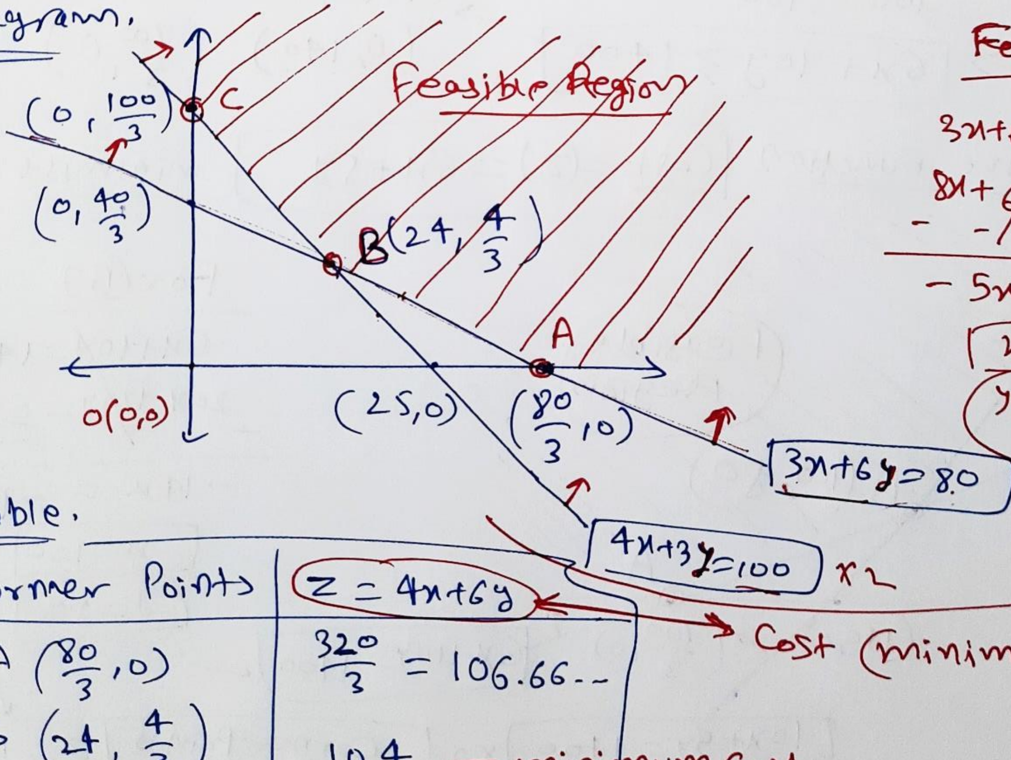
vit A → $3x+6y \geq 80 \rightarrow 3x+6y=80 \quad (0,\frac{40}{3}),(\frac{80}{3},0)$

mineral → $4x+3y \geq 100 \rightarrow 4x+3y=100 \quad (0,\frac{100}{3}),(25,0)$

$x\geq 0, y\geq 0$

Objective Function $\boxed{\text{Cost } (z) = 4x+6y}$ minimise

Diagram.



For B

$3x+6y=80$

$8x+6y=200$

$-5x=-120$

$\boxed{x=24}$

$y=\frac{4}{3}$

Table.

Corner Points	$z=4x+6y$
A $(\frac{80}{3},0)$	$\frac{320}{3}=106.66..$
B $(24,\frac{4}{3})$	104 = minimum cost
C $(0,\frac{100}{3})$	200

× 2 → Cost (minimise)

Exercise 12.2 Class 12

Q.10

	Quantity (Kg)	Cost (₹/Kg)	N_2	H_3PO_4
F_1	x	6	$10\% = x \times \frac{10}{100}$	$6\% = x \times \frac{6}{100}$
F_2	y	5	$5\% = y \times \frac{5}{100}$	$10\% = y \times \frac{10}{100}$

$x \geq 0$, $y \geq 0$

N_2: atleast 14 Kg, H_3PO_4: 14 Kg

Inequalities

$N_2 \rightarrow x \times \frac{10}{100} + y \times \frac{5}{100} \geq 14$

$\Rightarrow \boxed{10x + 5y \geq 1400} \rightarrow 10x + 5y = 1400$

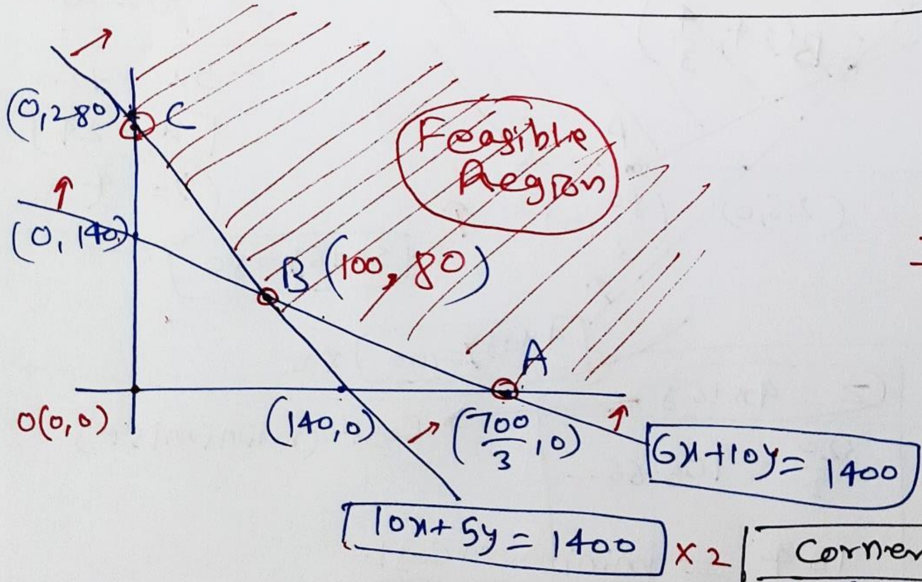
$(0, 280)$, $(140, 0)$

$H_3PO_4 \rightarrow \frac{6x}{100} + \frac{10y}{100} \geq 14$

$\Rightarrow \boxed{6x + 10y \geq 1400} \rightarrow 6x + 10y = 1400$

$(0, 140)$ $\left(\frac{700}{3}, 0\right)$

Objective function $\boxed{\text{Cost} = (Z) = 6x + 5y}$ minimise



For B

$6x + 10y = 1400$

$20x + 10y = 2800$ (×2)

$-14x = -1400$

$\boxed{x = 100}$

$\boxed{y = 80}$

Cost (↓)

Corner Points	$Z = 6x + 5y$
A $\left(\frac{700}{3}, 0\right)$	1400
B (100, 80)	1000
C (0, 280)	1400

Minimum Cost = 1000

$x = 100$ Kg ← F_1

$y = 80$ kg ← F_2

Exercise (12.2), Class (12)

Q.11

$2x+y \le 10$

$x+3y \le 15$

$x, y \ge 0$

$Z = px+qy$ $\quad p, q > 0$

$\Rightarrow$ maximum of Z occurs at Both (3,4) and (0,5) is —

(A) $p=q$ (B) $p=2q$

(C) $p=3q$ ~~(D)~~ $q=3p$.

Corner Points

(0,0) (5,0) (3,4), (0,5)

I O D F C T

Table:

Corner Points	$Z = px+qy$
(0,0)	0
(5,0)	5p
(3,4)	3p+4q ← maximum
(0,5)	5q ← maximum

$3p+4q = 5q$

$\Rightarrow 3p = 5q-4q$

$\Rightarrow 3p = q$

Miscellaneous Exercise on Chapter 12

Q.1

	No. of Packets	Ca	Fe	Cholesterol	Vitamin (A)
Food (P)	x	12	4	6	6
Food (Q)	y	3	20	4	3
		min. 240	min. 460	max. 300	maximise?

$x, y \geq 0$ → I quad.

Inequalities

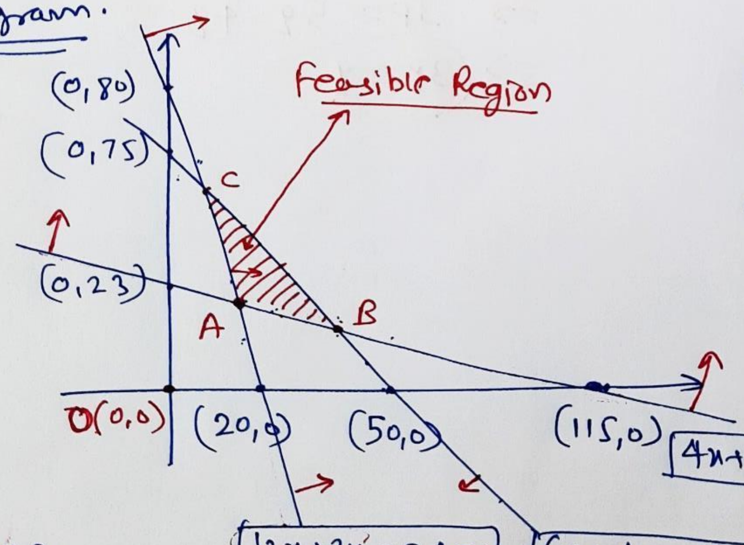
(Ca) $12x + 3y \geq 240 \longrightarrow 12x + 3y = 240$ (0, 80) (20, 0)

(Fe) $4x + 20y \geq 460 \longrightarrow 4x + 20y = 460$ (0, 23) (115, 0)

(Chol.) $6x + 4y \leq 300 \longrightarrow 6x + 4y = 300$ (0, 75) (50, 0)

Objective Function, (Vit. A) $= Z = 6x + 3y$ (maximise)

Diagram.



A (15, 20)
B (40, 15)
C (2, 72)

Table.

Corner Points	$Z = 6x + 3y$ (max.) (V. A)
A (15, 20)	90 + 60 = 150
B (40, 15)	240 + 45 = 285 = max. Vit. (A)
C (2, 72)	12 + 216 = 228

x = 40 = Food (P)
y = 15 = Food (Q)

Q.2

	No of Bags	Cost (₹)	N.E. A	B	C
Brand (P)	x	250	3	2.5	2
Brand (Q)	y	200	1.5	11.25	3
		minimise	↓	↓	↓
$x, y \geq 0$	min. requirement		18	45	24

→ Inequality.

Inequalities.

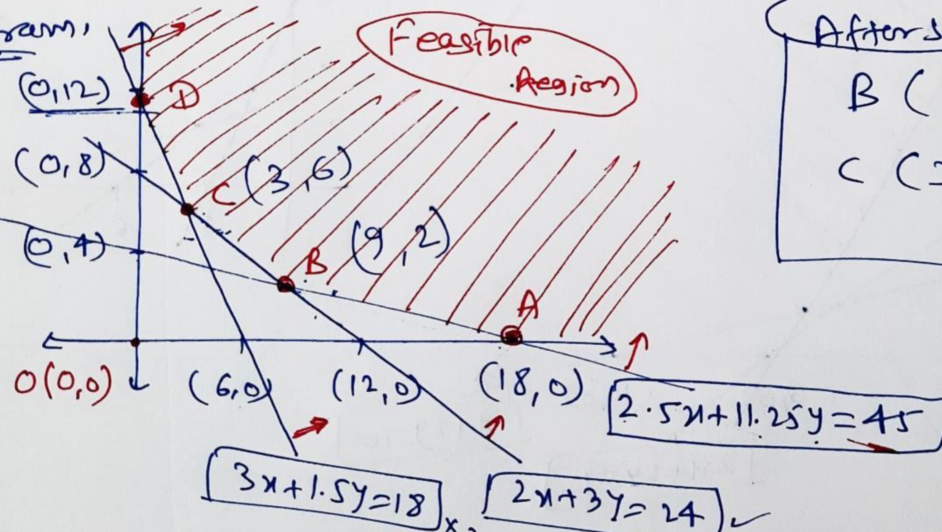
(A) → $3x + 1.5y \geq 18$ → $3x + 1.5y = 18$ (0,12) (6,0)

(B) → $2.5x + 11.25y \geq 45$ → $2.5x + 11.25y = 45$ (0,4) (18,0)

(C) → $2x + 3y \geq 24$ → $2x + 3y = 24$ (0,8) (12,0)

Objective function: $\boxed{\text{Cost } (z) = 250x + 200y}$ minimise

Diagram:



After solving

B (9, 2)

C (3, 6)

Table:

Corner Points	$Z = 250x + 200y$ → cost (min.)
A (18,0)	4500
B (9,2)	2250 + 400 = 2650
C (3,6)	750 + 1200 = 1950 = minimum cost
D (0,12)	0 + 2400 = 2400

$x = 3$ ← (P)

$y = 6$ ← (Q)

Miscellaneous Ex. Chapter (12) Class (12)

Q.3

Food	Quantity (Kg)	Cost (₹)	Vit (A)	Vit (B)	Vit (C)
X	x	16	1	2	3
Y	y	20	2	2	1
$x, y \geq 0$		minimise	10	12	8

minimum requirement

Inequalities.

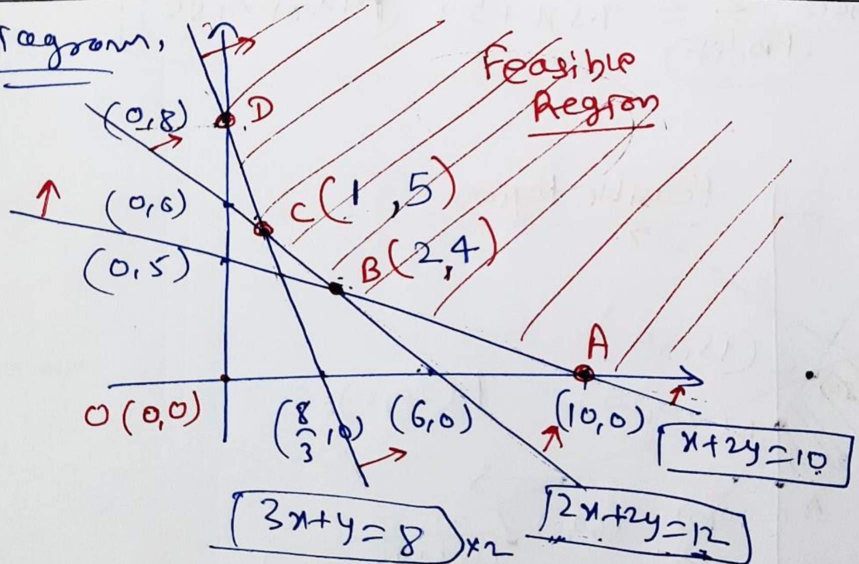
Vit (A) → $x+2y \geq 10$ → $x+2y=10$ → $(0,5), (10,0)$

Vit (B) → $2x+2y \geq 12$ → $2x+2y=12$ → $(0,6), (6,0)$

Vit (C) → $3x+y \geq 8$ → $3x+y=8$ → $(0,8), (\frac{8}{3},0)$

Objective Function, $\boxed{\text{Cost } (Z) = 16x+20y}$ minimise

Diagram,



B (2,4)

C (1,5)

Table

Corner Points	$Z = 16x+20y$ (Cost) minimum
A (10,0)	$160+0=160$
B (2,4)	$32+80=112$ ⇒ minimum cost
C (1,5)	$16+100=116$
D (0,8)	$0+160=160$

$X = 2$ Kg

$Y = 4$ Kg

112

miscellaneous Ex. Chapter 12 Class 12

Q.4

Types of Toys	No. of toys	Profit (₹)	minutes machines I	II	III
A	x	7.5	12	18	6
B	y	5	6	0	9
		maximum	6 hrs	6 hrs	6 hrs
		maximum availability	360 min.	360 min	360 min.

$x \geq 0$
$y \geq 0$ → I-quad

Inequality,

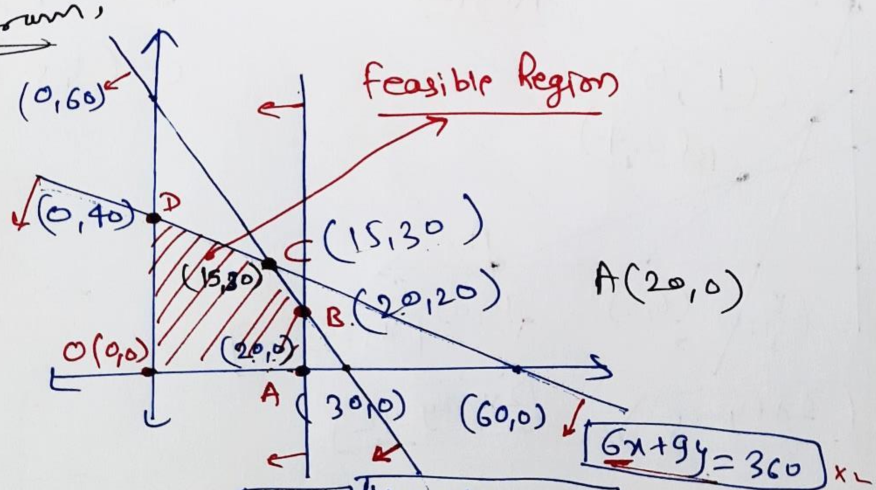
machine I $12x + 6y \leq 360 \rightarrow 12x + 6y = 360$ (0, 60), (30, 0)

machine II $18x + 0.y \leq 360 \rightarrow 18x = 360 \rightarrow \boxed{x = 20}$

machine III $6x + 9y \leq 360 \rightarrow 6x + 9y = 360$ (0, 40), (60, 0)

$$\boxed{\text{Objective function } Z = 7.5x + 5y \text{ (Maximise)}}$$
Profit

Diagram,



Table,

Corner Points	Z = 7.5x + 5y Profit (maximum)
O (0,0)	0
A (20,0)	150
B (20,20)	250
C (15,30)	262.5 → maximum Profit
D (0,40)	200

at C(15, 30)
x → Type (A), y → (B)

Miscellaneous Exercise on Chapter 12

Q.5

	No. of Tickets	Profit (₹ / ticket)
Executive Class	x	1000
Economy Class	y	600

No. of passengers that can be carried by Aeroplane = max. (200)

$\boxed{x+y \leq 200}$

Atleast 20 seats for Executive class ⇒ $\boxed{x \geq 20}$

Passengers in economy class = atleast 4 times of passengers in Executive class.

$\boxed{y \geq 4x}$ $x \geq 0$, $y \geq 0$

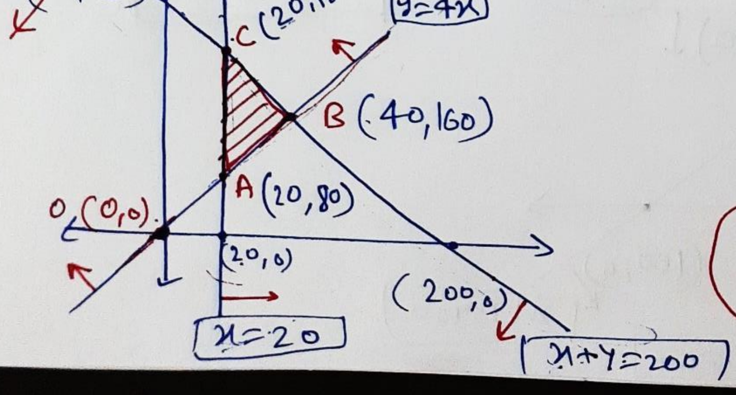
Objective Function $\boxed{\text{Profit } (Z) = 1000x + 600y}$ maximise

Diagram

$x+y \leq 200 \longrightarrow x+y = 200$ (0, 200), (200, 0)

$x \geq 20 \longrightarrow \boxed{x = 20}$

$y \geq 4x \longrightarrow \boxed{y = 4x}$ (0, 0) Slope m = 4 Positive



Table

Corner Points	$Z = 1000x + 600y$
A (20, 80)	68 000
B (40, 160)	136000
C (20, 180)	128 000

$x = 40$, $y = 160$ → max. Profit

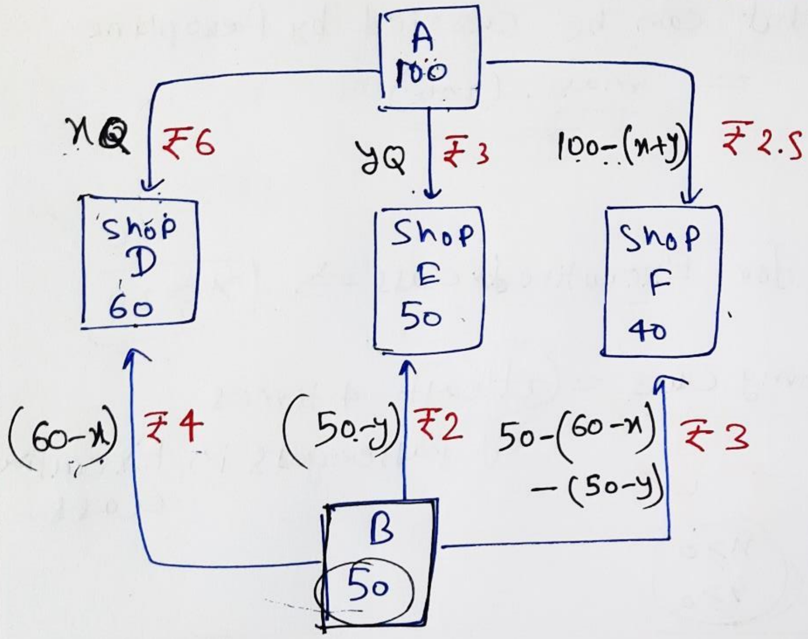
miscellaneous Ex. Chapter 12 Class 12

Q. 6 $\boxed{\text{Capacity of Godown A} = 100\,Q}$ $\boxed{\text{Capacity of Godown B} = 50\,Q}$

Demand of Shop D = 60 Q
Demand of Shop E = 50 Q
Demand of Shop F = 40 Q

Transportation Cost / Quintal (₹)

From | To	A	B
D	6	4
E	3	2
F	2.5	3



Inequalities:

- $\boxed{x \geq 0}$
- $\boxed{y \geq 0}$

supply ≥ 0

- $100-(x+y) \geq 0$

$\Rightarrow 100 \geq (x+y)$

$\Rightarrow \boxed{x+y \leq 100}$

- $60-x \geq 0$

$\Rightarrow \boxed{x \leq 60}$

- $50-y \geq 0$

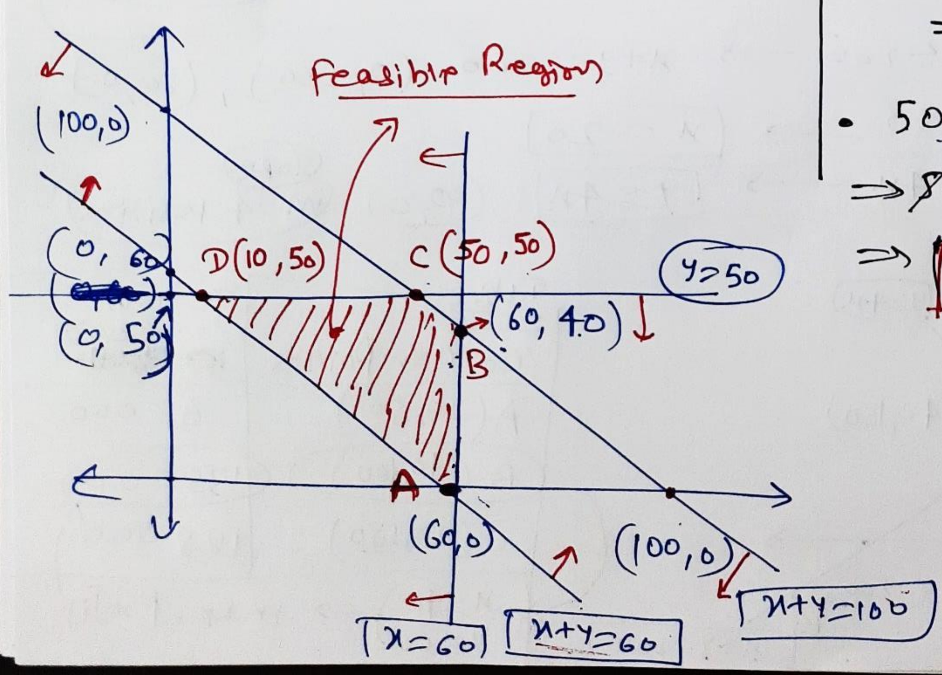
$\Rightarrow \boxed{y \leq 50}$

- $50-(60-x)-(50-y) \geq 0$

$\Rightarrow \cancel{50}-60+x-\cancel{50}+y \geq 0$

$\Rightarrow \boxed{x+y \geq 60}$

Diagram



Objective function (z) = [Transportation Cost]

(minimise)

$$z = 6x + 3y + [100-(x+y)](2.5) + (60-x)\cdot 4 + (50-y)\cdot 2 + (x+y-60)\times 3$$

$$z = 6x + 3y + \boxed{250} - 2.5x - 2.5y + \boxed{240} - 4x + \boxed{100} - 2y + 3x + 3y - \boxed{180}$$

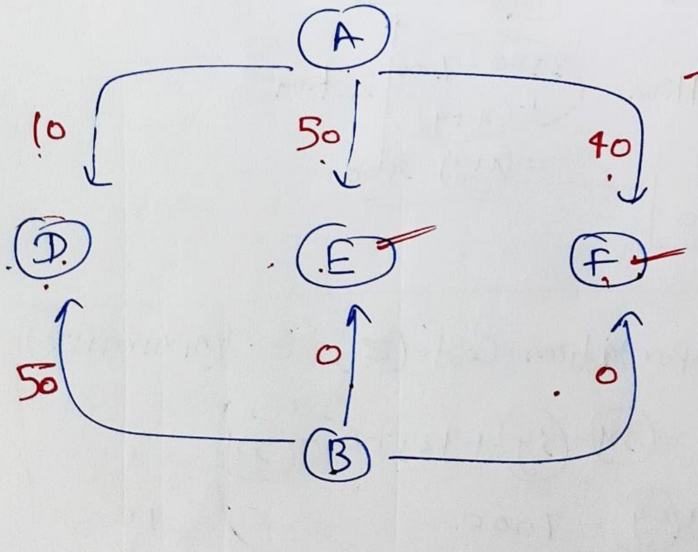
$$z = 2.5x + 1.5y + 410$$

Table

Corner Points	$z = 2.5x + 1.5y + 410$
A (60, 0)	150 + 410 = 560
B (60, 40)	150 + 60 + 410 = 620
C (50, 50)	125 + 75 + 410 = 610
D (10, 50)	25 + 75 + 410 = 510

x y

minimum cost



After putting

$x = 10$ & $y = 50$

Miscellaneous Ex. Chapter 12 class 12

Q.7 Capacity of Depot 'A' = 7000L | Capacity of Depot 'B' = 4000L

Requirement of Petrol Pump D = 4500 L

Requirement of Petrol Pump E = 3000 L

Requirement of Petrol Pump F = 3500 L

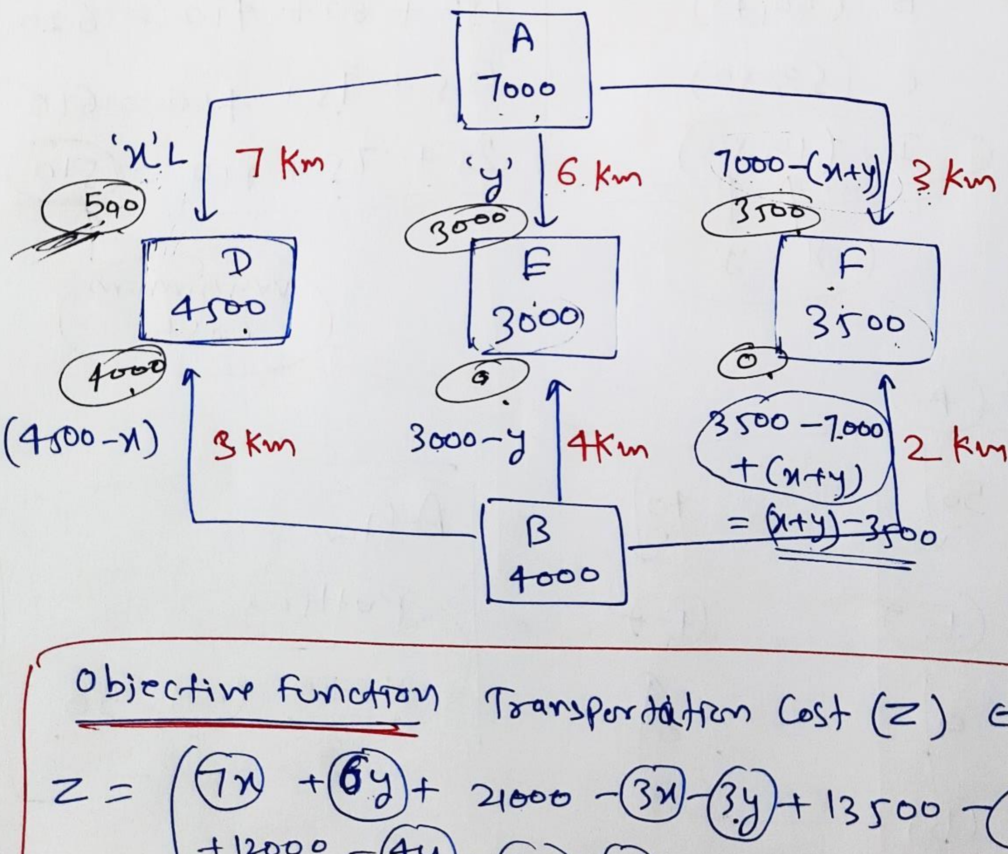
Distance (Km)

From/To	A	B
D	7	3
E	6	4
F	3	2

Transportation cost of 10 litres of oil = ₹1/km

Minimum Transportation Cost = ?

$\Rightarrow$ Transportation Cost of 1 litre of oil = ₹ $\frac{1}{10}$ /km



Objective function Transportation Cost (Z) ← (minimise)

$$Z = \left(7x + 6y + 21000 - 3x - 3y + 13500 - 3x + 12000 - 4y + 2x + 2y - 7000\right) \times \frac{1}{10}$$

$$= (3x + y + 39500)\frac{1}{10} = 0.3x + 0.1y + 3950$$

Inequalities. $\boxed{\text{Supply} \geq 0}$

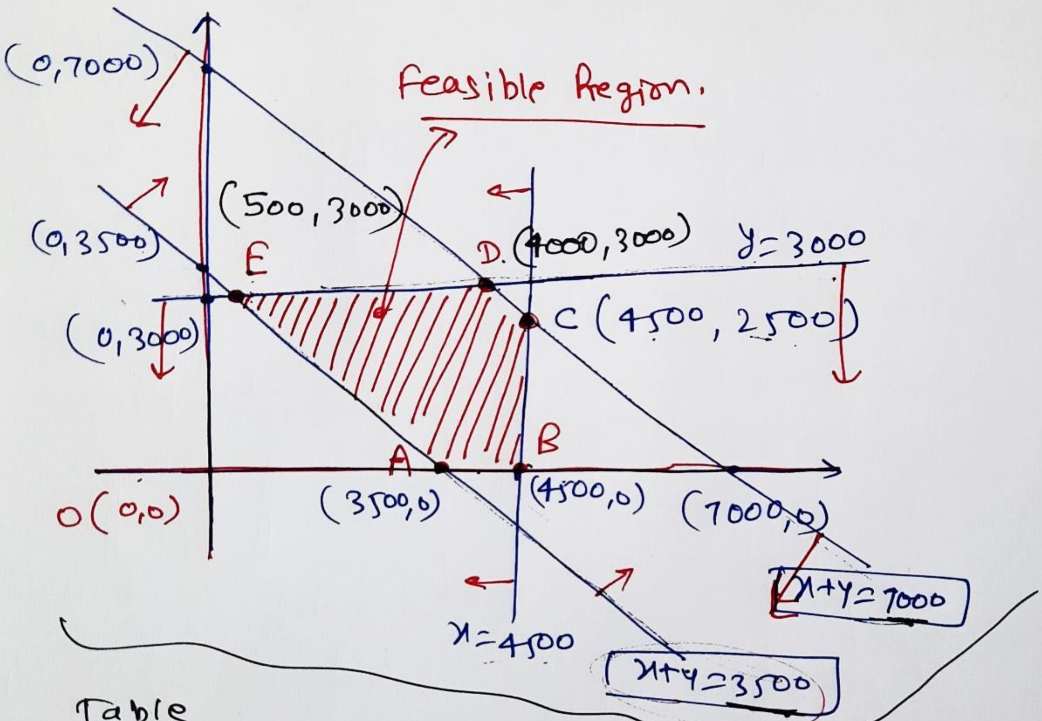
$\left.\begin{array}{l} x \geq 0 \\ y \geq 0 \end{array}\right\} \rightarrow$ I-quadrant

$\rightarrow 7000-(x+y) \geq 0 \longrightarrow x+y \leq 7000 \longrightarrow x+y=7000 \quad \left.\begin{array}{l}(0,7000) \\ (7000,0)\end{array}\right]$

$4500-x \geq 0 \longrightarrow x \leq 4500 \longrightarrow x=4500$

$3000-y \geq 0 \longrightarrow y \leq 3000 \longrightarrow y=3000$

$\rightarrow x+y-3500 \geq 0 \longrightarrow x+y \geq 3500 \longrightarrow x+y=3500 \quad \left.\begin{array}{l}(0,3500) \\ (3500,0)\end{array}\right]$



Table

Corner Points.	Cost (minimise) $Z = 0.3x + 0.1y + 3950$
A (3500,0)	$1050 + 0 + 3950$
B (4500,0)	$1350 + 0 + 3950$
C (4500,2500)	$1350 + 250 + 3950$
D (4000,3000)	$1200 + 300 + 3950$
E (500,3000)	$150 + 300 + 3950$

x y

= minimum cost

= 4400 ₹

Miscellaneous Exercise on Chapter (12)

Q.8 → minimum amount of N_2 added

Q.9 → maximum amount of N_2 added.

	No. of Bags	N_2	H_3PO_4	Potash	Chlorine
Brand (P)	x	3	1	3	1.5
Brand (Q)	y	3.5	2	1.5	2
$x, y \geq 0$		?	min.(240 kg)	min.(270 kg)	max (310) kg.

Inequalities.

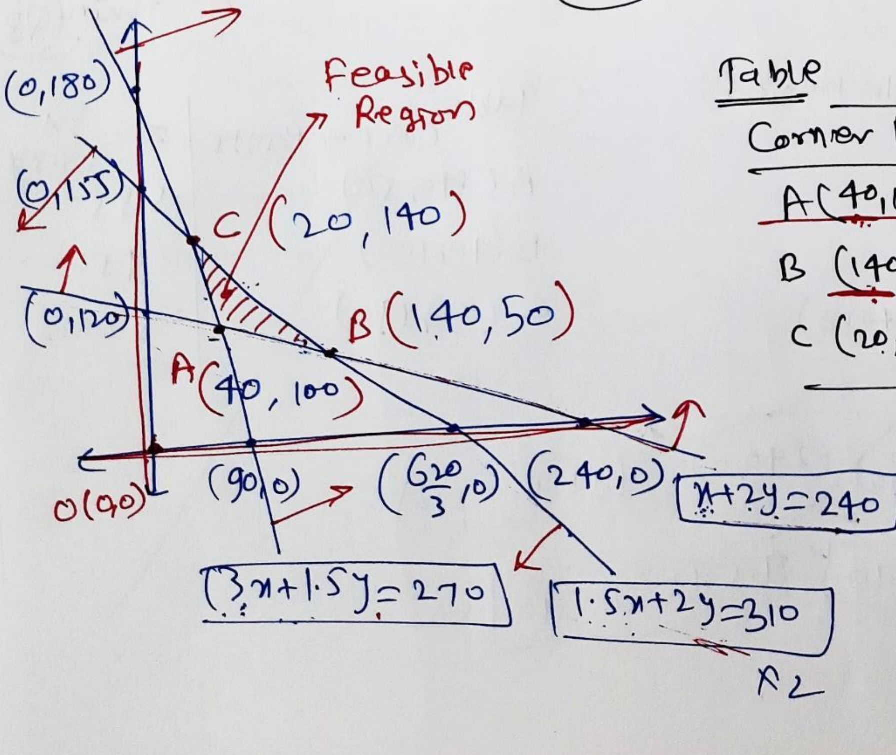
H_3PO_4 → $x + 2y \geq 240$ → $x + 2y = 240$ (0, 120), (240, 0)

Potash → $3x + 1.5y \geq 270$ → $3x + 1.5y = 270$ (0, 180), (90, 0)

Cl_2 → $1.5x + 2y \leq 310$ → $1.5x + 2y = 310$ (0, 155), $(\frac{620}{3}, 0)$

$\boxed{\text{Objective function Nitrogen } (Z) = 3x + 3.5y}$ → max. → Q.9; → min. → Q.8

N_2



Table

Corner Points	N_2: $Z = 3x + 3.5y$
A (40, 100)	470 = min.
B (140, 50)	595 max.
C (20, 140)	550

Q.9

Miscellaneous Exercise on Chapter (12) Class (12)

Q.10

	No. of Dolls	Profit (₹/Doll)
Doll 'A' →	x	12
Doll 'B' →	y	16

$x, y \geq 0$

Objective function Profit (Z)

$Z = 12x + 16y$ maximise

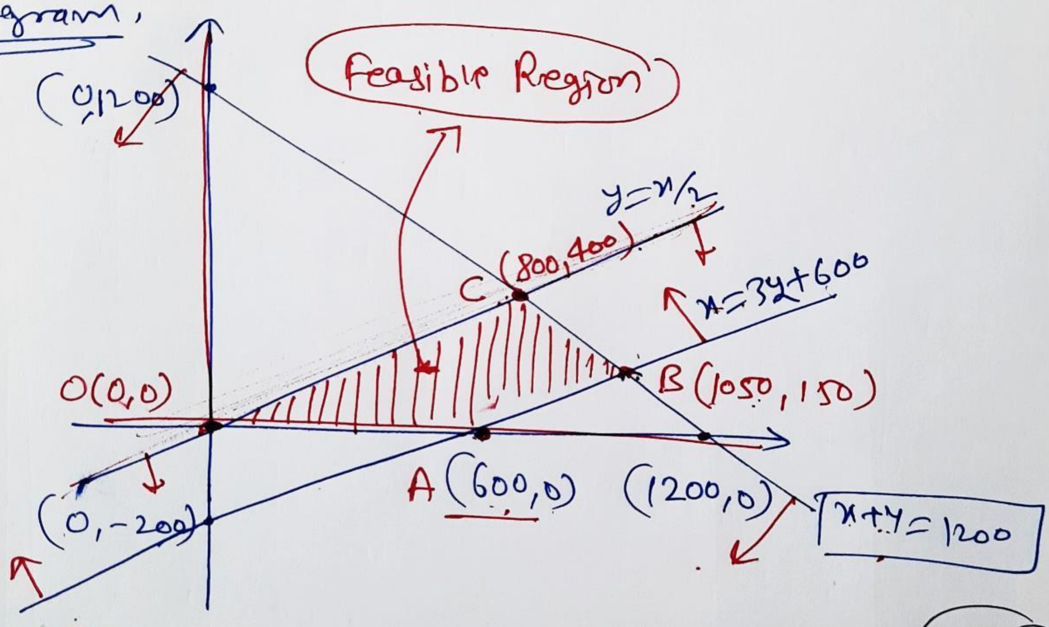
Inequalities

$x + y \leq 1200 \longrightarrow x + y = 1200 \quad (0, 1200), \ (1200, 0)$

$y \leq \frac{x}{2} \longrightarrow y = \frac{x}{2} \quad (0, 0)$ Slope = (+) $(1050, 525)$

$x \leq 3y + 600 \longrightarrow x = 3y + 600 \quad (0, -200), \ (600, 0)$

Diagram,



Table

Corner Points	$Z = 12x + 16y$
O (0, 0)	0
A (600, 0)	7200
B (1050, 150)	15000
C (800, 400)	16000

Profit maximise

16000 = maximum profit

C (800, 400): x → Doll (A), y → Doll (B)