[redacted]

MATRICES

Class 12 MATHS

Basics of MATRICES

Tables

	Notebooks	Pens
Radha	15	6
Fauzia	10	2
Simran	13	5

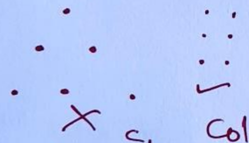
Matrix (आव्यूह) [ ]

$$\begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \end{array}\begin{bmatrix} 15 & 6 \\ 10 & 2 \\ 13 & 5 \end{bmatrix}$$

	R	F	S
N.B. →	15	10	13
Pens →	6	2	5

$$\begin{bmatrix} 15 & 10 & 13 \\ 6 & 2 & 5 \end{bmatrix}$$

★ An ordered rectangular array of numbers



$$\begin{array}{cc} & \begin{array}{ccc} \text{स्तंभ Columns} \\ c_1 & c_2 & c_3 \end{array} \\ \text{(पंक्ति) Row } (R_1) \rightarrow & \begin{bmatrix} 15 & 10 & 13 \\ 6 & 2 & 5 \end{bmatrix} \\ (R_2) \rightarrow & \end{array}$$

~~order~~

Elements (Entries) of Matrix.

Order of a matrix (size)/shape.

No. of Rows × No. of Columns

(by)

e.g. = 2×3

$$\begin{array}{cc} R_1 \rightarrow & \begin{bmatrix} 15 & 10 & 13 \\ 6 & 2 & 5 \end{bmatrix}_{2\times 3} \\ R_2 \rightarrow & \\ & \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ c_1 & c_2 & c_3 \end{array} \end{array}$$

$$\begin{array}{cc} \begin{bmatrix} 15 & 6 \\ 10 & 2 \\ 13 & 5 \end{bmatrix} & \begin{array}{c} \leftarrow R_1 \\ \leftarrow R_2 \\ \leftarrow R_3 \end{array} \\ \begin{array}{cc} \uparrow & \uparrow \\ c_1 & c_2 \end{array} & 3\times 2 \end{array}$$

General Representation of Matrices

Construct a matrix 'A' of order $m \times n$.

no. of rows = m

—— columns = n

$A = [a_{ij}]_{m \times n}$

i = Row

j = column

$$A = \begin{array}{ccccc|l} \boxed{a_{11}} & \boxed{a_{12}} & \boxed{a_{13}} & \cdots & \boxed{a_{1n}} & \leftarrow R_1 \\ \boxed{a_{21}} & \boxed{a_{22}} & \boxed{a_{23}} & \cdots & \boxed{a_{2n}} & \leftarrow R_2 \\ \boxed{a_{31}} & \boxed{a_{32}} & \boxed{a_{33}} & \cdots & \boxed{a_{3n}} & \leftarrow R_3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \boxed{a_{m1}} & \boxed{a_{m2}} & \boxed{a_{m3}} & \cdots & \boxed{a_{mn}} & \leftarrow R_m \ (m^{th} \text{ row}) \\ \hline \uparrow & \uparrow & \uparrow & & \uparrow & m \times n \\ C_1 & C_2 & C_3 & \cdots & C_n & \end{array}$$

e.g. Construct a matrix Ⓐ of order 2×3 whose each element $a_{ij} = i^2 - j$.

$$A = [a_{ij}]_{\underset{\text{Rows}}{\uparrow}2 \times \underset{\text{Columns}}{\uparrow}3} = \begin{array}{l|ccc|} & & & \\ R_1 \rightarrow & a_{11} & a_{12} & a_{13} \\ R_2 \rightarrow & a_{21} & a_{22} & a_{23} \\ & \uparrow & \uparrow & \uparrow \\ & C_1 & C_2 & C_3 \end{array}_{2 \times 3}$$

$a_{ij} = i^2 - j$

$a_{11} = 1^2 - 1 = 0$

$a_{12} = 1^2 - 2 = -1$

$a_{13} = 1^2 - 3 = -2$

$a_{21} = 2^2 - 1 = 3$

$a_{22} = 2^2 - 2 = 2$

$a_{23} = 2^2 - 3 = 1$

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 3 & 2 & 1 \end{bmatrix}_{2 \times 3}$$

Types of Matrices (आव्यूहों के प्रकार)

$\boxed{\text{order} = \overset{\text{row}}{m} \times \overset{\text{Colum}}{n}}$

① Column Matrix (स्तंभ आव्यूह)

No. of Columns = 1 $\quad A = [a_{ij}]_{m\times 1}$

$$A = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}_{3\times 1}$$

② Row Matrix (पंक्ति आव्यूह)

No. of rows = 1 $\quad A = [a_{ij}]_{1\times n}$

$$A = \begin{bmatrix} 2 & 3 & -5 & 4 \end{bmatrix}_{1\times 4}$$

③ Square Matrix (वर्ग आव्यूह)

No. of rows = no. of columns

$\underline{(m = n)}$

$A = [a_{ij}]_{m\times m}$

$A = [a_{ij}]_{n\times n}$

e.g. $$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & x & 7 \\ y & -3 & 0 \end{bmatrix}_{3\times 3}$$

Note. There are two diagonals in a sq. matrix.
(विकर्ण)

★ (Primary (main) (Principal)) ✓

Secondary ×

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & x & 7 \\ y & -3 & 0 \end{bmatrix}_{3\times 3}$$

④ Diagonal matrix (विकर्ण आव्यूह)

Square matrix whose non-diagonal elements = 0

e.g.

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} x & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 7 \end{bmatrix}_{3\times 3}$$

$$\boxed{A = [a_{ij}]_{m\times m}, \text{ where } a_{ij} = 0,\ \forall\ i \neq j}$$

⑤ Scalar Matrix (अदिश आव्यूह)

Diagonal matrix whose diagonal elements are same (equal)

$$A = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}_{3\times 3}$$

Scalar Matrix,

$$A = [a_{ij}]_{m\times m}$$

$$a_{ij} = \begin{cases} K, & i = j \\ 0, & i \neq j \end{cases}$$

⑥ Identity matrix (तत्समक /इकाई आव्यूह)

Scalar matrix whose diagonal elems = 1

$$I_1 = [1]_{1\times 1}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

⑦ Zero matrix. (शून्य आव्यूह) (O)

All elements = 0

zero = 0

e.g. $A = \begin{bmatrix} 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equality of matrices. Two matrices A = B are said to be equal

if (i) order same

(ii) each corresponding element should also be equal.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 7 & 0 \end{bmatrix}_{2\times 3}$$

A = B

$$B = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 7 & 0 \end{bmatrix}_{2}$$

$$B = \begin{bmatrix} 2 & -1 \\ 3 & 7 \\ 4 & 0 \end{bmatrix}_{3\times 2}$$

×

MATRICES   Exercise 3.1

$\boxed{Q2,\ Q4(i),\ Q6(iii),\ Q8,\ Q9,\ Q10}$

$\boxed{Q.2}$ no. of elements = 24 ✓

$$A = [a_{ij}]_{\underset{\text{order}}{\underset{\uparrow}{m\times n}}} \qquad \text{no. of elements} = \underline{mn}\ ✓$$

Possible orders = $\bigcirc\!\!\!\!\!\!24$

$1\times24 \qquad 4\times6 \qquad 12\times2$

$2\times12 \qquad 6\times4 \qquad 24\times1$

$3\times8 \qquad 8\times3$

no. of elements = ⑬

$\underline{1\times13} \qquad \underline{13\times1}$

$\boxed{Q.4,\ (i)}$ $A = [a_{ij}]_{2\times2} \longleftrightarrow \boxed{a_{ij} = \frac{(i+j)^2}{2}}$

2×2 → Rows → Columns

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$$

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2\ ✓$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}\ ✓$$

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}_{2\times2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

Q.6 (iii)

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}_{3\times1} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}_{3\times1}$$

By equality of matrices.

$x+y+z=9$ —①

$x+z=5$ —②

$y+z=7$ —③

eqⁿ① − eqⁿ②

$$\begin{array}{r} \cancel{x}+y+\cancel{z} = 9 \\ -\cancel{x} - \cancel{z} = -5 \\ \hline y = 4 \end{array}$$

eqⁿ① − eqⁿ③

$$\begin{array}{r} x+\cancel{y}+\cancel{z} = 9 \\ -\cancel{y} - \cancel{z} = -7 \\ \hline x = 2 \end{array}$$

eqⁿ③ : $y+z=7$

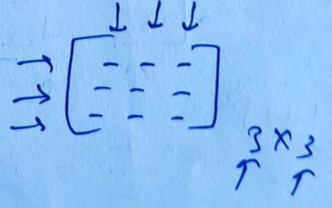
$4+z=7$

$\boxed{z=3}$

Q.8 $A=[a_{ij}]_{m\times n}$

Square matrix

no. of rows = no. of Columns



$m=n$

$\boxed{Q.9}$

$$\begin{bmatrix} 3x+7 & \boxed{5} \\ y+1 & \boxed{2-3x} \end{bmatrix}_{2\times2} = \begin{bmatrix} 0 & \boxed{y-2} \\ 8 & \boxed{4} \end{bmatrix}_{2\times2}$$

$\underline{3x+7=0} \Rightarrow \boxed{x=-\frac{7}{3}}$

$\underline{5=y-2} \Rightarrow y=7$ ✓

$\underline{y+1=8} \Rightarrow y=7$ ✓

$\underline{2-3x=4} \Rightarrow -2=3x \Rightarrow \boxed{x=-\frac{2}{3}}$

Not possible

$\boxed{Q.10}$

$$A=\begin{bmatrix} \boxed{-} & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times3}$$

No. of ways to fill it $= 2$

$\uparrow$

$\boxed{0 \text{ या } 1}$

$\underline{\text{Possible entries} = 0 \text{ या } 1}$

No. of ways to fill all places

$= 2\times2\times2\times2\times2\times2\times2\times2\times2 = 2^9$

$= 512$

P&C → multiplication rule of counting

necessary

Addition, Subtraction, Multiplication of Matrices

(+) (−) (×)

Addition & Subtraction of Matrices.

$A = [a_{ij}]_{m \times n}$ $B = [b_{ij}]_{m \times n}$

$\boxed{A+B=B+A}$

Commutative ✓

$$A \pm B = [a_{ij}]_{m\times n} \pm [b_{ij}]_{m\times n}$$

$$= [\underset{\uparrow}{a_{ij}} \pm \underset{\uparrow}{b_{ij}}]_{m\times n}$$

corresponding elements

e.g. $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} 7 & 3 \\ -2 & 5 \end{bmatrix}_{2\times 2}$

$C = \begin{bmatrix} 2 & 3 & 1 \\ 7 & -2 & 0 \end{bmatrix}_{2\times 3}$

$$A + B = \begin{bmatrix} \textcircled{2} & \boxed{-1} \\ 0 & 3 \end{bmatrix}_{2\times 2} + \begin{bmatrix} \textcircled{7} & \boxed{3} \\ -2 & 5 \end{bmatrix}_{2\times 2}$$

$$= \begin{bmatrix} 9 & 2 \\ -2 & 8 \end{bmatrix}_{2\times 2}$$ ✓

$A - B$

$= \begin{bmatrix} \textcircled{2} & -1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \textcircled{7} & 3 \\ -2 & 5 \end{bmatrix}$

$= \begin{bmatrix} -5 & -4 \\ 2 & -2 \end{bmatrix}$

$$A + C = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}_{2\times 2} + \begin{bmatrix} 2 & 3. & \textcircled{1} \\ 7 & -2 & \textcircled{0} \end{bmatrix}_{2\times 3}$$

Not Defined

Multiplication of a Scalar with a Matrix.

$(K \neq 0)$

$$A = [a_{ij}]_{m \times n}$$

$$kA = k[a_{ij}]_{m \times n} = [k\,a_{ij}]_{m \times n}$$

e.g. $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 7 & -5 \end{bmatrix}_{2 \times 3}$

$$kA = K \begin{bmatrix} 2 & 3 & 1 \\ 0 & 7 & -5 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 2K & 3K & K \\ 0 & 7K & -5K \end{bmatrix}_{2 \times 3}$$

$\boxed{K \neq 0}$

~~m×n~~ ~~m×n~~

$$-A = (-1)A = (-1).\begin{bmatrix} 2 & 3 & 1 \\ 0 & 7 & -5 \end{bmatrix}_{2\times 3} = \begin{bmatrix} -2 & -3 & -1 \\ 0 & -7 & 5 \end{bmatrix}_{2 \times 3}$$

MULTIPLICATION of Two Matrices

$\boxed{\text{कब Possible है ?}}$ $\boxed{\text{कैसे करते है ?}} \rightarrow ?$

$\Downarrow$

$$A = [a_{ij}]_{m \times \underline{n}} \qquad B = [b_{ij}]_{\underline{n} \times k}$$

$\boxed{AB \neq BA}$

Generally not Commutative

$$\underbrace{m \times \underline{n}}_{A} \quad \underbrace{\underline{n} \times K}_{B}$$

$(AB) \Longrightarrow$ AB का order $= \underline{m} \times \underline{K}$

$2 \times ③ \quad ③ \times 4.$

$\uparrow \quad \nearrow$

B.A $\Rightarrow$ BA का order $= 2 \times 4$

e.g.

$\boxed{\text{AB कैसे निकालते हैं?}}$

$A = [a_{ij}]_{2\times \underline{3}}$ $\quad B = [b_{ij}]_{\underline{3}\times 3}$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -3 & 4 & 5 \end{bmatrix}_{2\times\textcircled{3}} \qquad B = \begin{bmatrix} 1 & 0 & -2 \\ 7 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix}_{\textcircled{3}\times \underline{3}}$$

multiply करते वक्त → पहले वाली matrix $\boxed{\text{Row}}$

बाद वाली matrix $\boxed{\text{Column}}$

$$AB \downarrow \boxed{\underline{2}\times\underline{3}}$$

$$A.B = \begin{array}{c} R_1 \rightarrow \\ R_2 \rightarrow \end{array} \begin{bmatrix} \boxed{2 \quad -1 \quad 0} \\ -3 \quad 4 \quad 5 \end{bmatrix}_{2\times3} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 1 & 0 & -2 \\ 7 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix}_{3\times3} \end{array}$$

$$= \begin{bmatrix} \overbrace{(2\times1) + (-1\times7) + 0\times(-1)}^{\text{Due to } \textcircled{R_1} \,\&\, \textcircled{C_1}} & \overbrace{\underbrace{2\times0}+\underbrace{(-1)\times3}+\underbrace{(0\times-1)}}^{R_1 \,\&\, C_2} & (-4)+(-8)+0 \\ (-3)+(28)+(-5) & 0+12+15 & 6+32+0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -3 & -12 \\ 20 & 27 & 38 \end{bmatrix}_{\underline{2}\times\underline{3}}$$

Note: for multiplication of matrices.

$\boxed{AB \neq BA \text{ generally}}$

① $AB = BA$

$\downarrow\downarrow$

multiplication of ~~Dig~~ Diagonal matrices.

e.g. $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 15 \end{bmatrix} ✓$$

$$BA = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 15 \end{bmatrix} ✓$$

② $AB = \underset{\text{zero matrix}}{O} \implies \underbrace{A = 0 \text{ या } B = 0}$

जरूरी नहीं है !!!

e.g. $A = \begin{bmatrix} 0 & -2 \\ 0 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & -2 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \Theta$$

zero matrix

AB कैसे निकालते है?

e.g.

$A = [a_{ij}]_{2\times 3}$ $\qquad$ $B = [b_{ij}]_{3\times 3}$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -3 & 4 & 5 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 1 & 0 & -2 \\ 7 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix}_{3\times 3}$$

multiply करते वक्त → पहले वाली matrix (Row)

बाद वाली matrix (Column)

$$AB \downarrow 2\times 3$$

$$A.B = \begin{array}{c} R_1 \\ R_2 \end{array} \begin{bmatrix} 2 & -1 & 0 \\ -3 & 4 & 5 \end{bmatrix}_{2\times 3} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 1 & 0 & -2 \\ 7 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix}_{3\times 3} \end{array}$$

$$= \begin{bmatrix} \overbrace{(2\times 1) + (-1\times 7) + 0\times(-1)}^{\text{Due to } R_1 \,\&\, C_1} & \overbrace{2\times 0 + (-1)\times 3 + (0\times -1)}^{R_1 \,\&\, C_2} & (-4) + (-8) + 0 \\ (-3) + (28) + (-5) & 0 + 12 + 15 & 6 + 32 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -3 & -12 \\ 20 & 27 & 38 \end{bmatrix}_{2\times 3}$$

Exercise ③.② $\boxed{\text{Q6, Q7 (ii), Q13, Q17}}$ Class 12 - Maths

$\boxed{+, -, \times}$

$\boxed{\text{Q.6}}$ Simplify $\cos\theta \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cdot\cos\theta & \cos^2\theta \end{bmatrix}_{2\times2} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} (\cos^2\theta + \sin^2\theta) \xrightarrow{} 1 & \cos\cancel{\theta}\,\cancel{\sin\theta} - \sin\cancel{\theta}\cos\theta \\ 0 & (\cos^2\theta + \sin^2\theta) \xrightarrow{} 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2} = I = I_2 = I_{2\times2}$$

Identity / unit matrix.

$\boxed{\text{Q.7}}$ (ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ —① $\quad 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ —②

By Elimination

eqⁿ ① × 3 $\quad 6\cancel{X} + 9Y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$

eqⁿ ② × 2 $\quad 6\cancel{X} + 4Y = 2\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$

$\quad\;\; - \quad\;\; - \qquad\qquad\qquad\qquad\qquad -$

$$5Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix} \Rightarrow Y = \frac{1}{5}\begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$ ✓

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \underbrace{3\begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}}_{\boxed{3Y}}$$

$$\Rightarrow 2X = \begin{bmatrix} \boxed{2} & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \boxed{\frac{6}{5}} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\Rightarrow 2\boxed{X} = \underbrace{\begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}} \Rightarrow X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$ ✓

$\boxed{Q.13}$ If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$; show that $F(x).F(y) = F(x+y)$

To Prove $F(x).F(y) = F(x+y)$

$$\Rightarrow \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$LHS = F(X).F(y) \rightarrow . \downarrow$

$$= \underset{3\times\boxed{3}}{\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}} \cdot \underset{\boxed{3}\times 3}{\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} \underbrace{\cos x.\cos y - \sin x \sin y + 0} & \underbrace{-\cos x \sin y - \sin x \cos y + 0} & \underbrace{0+0+0} \\ \underbrace{\sin x \cos y + \cos x \sin y + 0} & \underbrace{-\sin x \sin y + \cos x \cos y + 0} & \underbrace{0} \\ \underbrace{0+0+0} & \underbrace{0+0+0} & \underbrace{0+0+1} \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y)$$

$= RHS.$

Q.17 If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\boxed{A^2 = kA - 2I}$. Find 'K'.

$$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

$$A^2 = kA - 2I$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow 1 . \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3K & -2K \\ 4K & -2K \end{bmatrix}$$

$$3 = 3K \Rightarrow \boxed{K = 1}$$

Exercise - 3.2 $\boxed{Q.18}$ ☆

$\boxed{\text{Class-12 Maths}}$

If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the Identity matrix of order 2, show that $\underbrace{(I+A)=(I-A)\cdot\begin{bmatrix}\cos\alpha & -\sin \\ \sin\alpha & \cos\end{bmatrix}}$

✓ $\sin\alpha = \dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ ✓

✓ $\cos\alpha = \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$ ✓

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin\underline{2}\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\cos\underline{2\theta} = \dfrac{1-\tan^2\theta}{1+\tan 2\theta}$

multiply $\quad I = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$

To Prove: $(I+A) = (I-A).\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}\textcircled{1} & 0 \\ 0 & 1\end{bmatrix} + \begin{bmatrix}\textcircled{0} & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0\end{bmatrix} = \left\{\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} - \begin{bmatrix}0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0\end{bmatrix}\right\}.$$

$$\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix} \times \begin{bmatrix} \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & -\dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \dfrac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$

$\rightarrow \cdot \downarrow$

$[\;].[\;]$

$$= \begin{bmatrix} \dfrac{1-\tan^2\frac{\alpha}{2}+2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \dfrac{-2\tan\frac{\alpha}{2}+\tan\frac{\alpha}{2}-\tan^3\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \dfrac{-\tan\frac{\alpha}{2}+\tan^3\frac{\alpha}{2}+2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \dfrac{2\tan^2\frac{\alpha}{2}+1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cancel{1+\tan^2\frac{\alpha}{2}}}{\cancel{1+\tan^2\frac{\alpha}{2}}} & -\dfrac{\tan\frac{\alpha}{2}\cancel{\left(1+\tan^2\frac{\alpha}{2}\right)}}{\cancel{\left(1+\tan^2\frac{\alpha}{2}\right)}} \\ \dfrac{\tan\frac{\alpha}{2}\cancel{\left(1+\tan^2\frac{\alpha}{2}\right)}}{\cancel{\left(1+\tan^2\frac{\alpha}{2}\right)}} & \dfrac{\cancel{1+\tan^2\frac{\alpha}{2}}}{\cancel{1+\tan^2\frac{\alpha}{2}}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} = \text{LHS}$$

$\boxed{\text{Class - 12}}$

Exercise ③.② $\boxed{\text{Q 19, Q 20, Q 21, Q 22}}$

$\boxed{\text{Q.19}}$

₹ 30,000

₹ x ₹ $(30,000 - x)$

(5%) (7%)

Matrix (Money) $M = \begin{bmatrix} x & 30000 - x \end{bmatrix}_{1\times 2}$

Matrix (Rate) $R = \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}_{2\times 1}$ $\leftarrow$ 5% $\leftarrow$ 7%

Matrix = Interest = $\underset{1\times 1}{T} = \overset{1\times ②}{M} . \overset{②\times 1}{R}$

$T = \begin{bmatrix} x & \xrightarrow{\quad} 30000 - x \end{bmatrix} . \begin{bmatrix} 5/100 \\ 7/100 \end{bmatrix}$

$T = \begin{bmatrix} x \times \frac{5}{100} + (30000 - x) \times \frac{7}{100} \end{bmatrix}_{1\times 1} =$

Part (i) $\begin{bmatrix} \frac{5x}{100} + \frac{210000 - 7x}{100} \end{bmatrix} = \begin{bmatrix} 1800 \end{bmatrix}$

$$\therefore \quad \frac{5x}{100} + \frac{210000 - 7x}{100} = 1800$$

$$\Rightarrow \quad \underline{5x} + 2,10,000 - \underline{7x} = 1,80,000$$

$$\Rightarrow \quad 30,000 = 2x$$

$$\Rightarrow \quad x = 15000$$

$$\begin{tikzcd} & x = 15000 \checkmark \\ 30000 \arrow[ur] \arrow[dr] & \\ & 30000 - x = 15000 \checkmark \end{tikzcd}$$

Part (ii)

$$\left[\frac{5x}{100} + \frac{210000 - 7x}{100}\right]_{1\times1} = \left[2000\right]_{1\times1}$$

$$\Rightarrow \quad 5x + 210000 - 7x = 200000$$

$$\Rightarrow \quad 10000 = 2x$$

$$\Rightarrow \quad x = 5000$$

$$\begin{tikzcd} & x = 5000 \quad \text{5\%} \\ 30000 \arrow[ur] \arrow[dr] & \\ & 30000 - x = 25000 \quad @ \ \text{7\%} \end{tikzcd}$$

Q.20 Matrix = Books = B = $\begin{array}{ccc} \swarrow C & \swarrow P & \swarrow E \\ \end{array}$ $\begin{bmatrix} 120 & 96 & 120 \end{bmatrix}_{1\times 3}$

Matrix = Price = P = $\begin{array}{cc} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} & \begin{array}{c} \leftarrow C \\ \leftarrow P \\ \leftarrow E \end{array} \end{array}$

All amount

$\downarrow$

$$\underset{\substack{\downarrow \\ \boxed{1\times 1}}}{A} = \underbrace{\begin{bmatrix} \overrightarrow{120 \quad 96 \quad 120} \end{bmatrix}_{1\times 3}}_{\text{no. of Books}} \cdot \underbrace{\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}_{3\times 1}}_{\text{Price.}}$$

$$\underset{\substack{\uparrow \\ \boxed{1\times 1}}}{A} = \begin{bmatrix} \boxed{(9600) + (5760) + (4800)} \end{bmatrix}_{1\times 1}$$

$$\underset{\uparrow}{A} = \begin{bmatrix} 20160 \end{bmatrix}_{1\times 1}$$

matrix for all amount. = ₹20,160

$$\begin{tikzcd} X \arrow[d] & Y \arrow[d] & Z \arrow[d] & W \arrow[d] & P \arrow[d] \\ (2\times n) & (3\times K) & (2\times P) & (n\times 3) & (P\times K) \end{tikzcd}$$

$$\boxed{\begin{tikzcd} A \pm B \arrow[d] \\ \text{order Same} \end{tikzcd}}$$

$\boxed{Q.21}$ $\boxed{n, K, P}$

$$\text{"}\overset{P\times K}{P}.\overset{n\times 3}{Y} + \overset{}{W}.Y\text{"}$$ will be defined

$$\underbrace{\underbrace{P\times \boxed{K}}\ \underbrace{\boxed{3}\times K}}_{K=3} \qquad \underbrace{\underbrace{n\times 3}\ .\ \underbrace{3\times K}}_{3}$$

order of $P.Y$ = order of $W.Y$

$$P\times K \qquad n\times 3$$

$\boxed{P=n}$, $\boxed{K=3}$ Option $\boxed{A}$

$\boxed{Q.22}$ If $n=P$, then order of "$7X-5Z$" = ?

$$'7X - 5Z' = \text{"Answer Matrix"}$$

$$\begin{tikzcd} 7X \arrow[d] & 5Z \arrow[d] \\ (2\times n) & (2\times P) \end{tikzcd}$$

$\boxed{2\times n}$ $\boxed{2\times n}$

या

$\boxed{2\times P}$ $\boxed{2\times P}$

$\boxed{2\times n}$ ✓

या

$\boxed{2\times P}$

Option $\boxed{B}$

MATRICES

Transpose, Symmetric Matrices, Skew Symmetric Matrices

Transpose (A^T / A')

let $A = [a_{ij}]_{m \times n}$ then it's ~~po~~ transpose

A' or $A^T = [a_{ji}]_{n \times m}$ (Interchange rows with columns)

e.g. $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}_{3 \times 2}$ $A^T = A' = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}_{2 \times 3}$

$B = \begin{bmatrix} 0 & 2 \\ -1 & 5 \end{bmatrix}_{2 \times 2}$ $B' = B^T = \begin{bmatrix} 0 & -1 \\ 2 & 5 \end{bmatrix}_{2 \times 2}$

Properties of Transpose.

(i) $(A^T)^T = A$ (ii) $(KA)^T = KA^T$ $\left(\begin{matrix} K \in R \\ K \neq 0 \end{matrix}\right)$

(iii) $(A \pm B)^T = A^T \pm B^T$ $(A+B+C)^T = A^T + B^T + C^T$

* (iv) $(AB)^T = B^T . A^T$ Reversal Law.

$\boxed{(AB)^T \neq A^T . B^T}$ $\boxed{AB \neq BA}$ Generally

$(ABC)^T = C^T . B^T . A^T$ Generally

Symmetric Matrices

(Square matrix)

$$A^T = A$$

$$[a_{ji}]_{n\times n} = [a_{ij}]_{n\times n}$$

$$a_{ji} = a_{ij}$$

e.g. $A = \begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 4 \\ -7 & 4 & -1 \end{bmatrix}_{3\times 3}$

$$A^T = \begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 4 \\ -7 & 4 & -1 \end{bmatrix}_{3\times 3}$$

$\boxed{A^T = A}$ Sym. matrix.

Skew Symmetric Matrices

(Square matrix)

$$A^T = -A$$

$$[a_{ji}]_{n\times n} = -[a_{ij}]_{n\times n}$$

$$a_{ji} = -a_{ij}$$

For main Diagonal elements $(i=j)$

$$a_{ii} = -a_{ii}$$

$$\Rightarrow 2a_{ii} = 0$$

$\Rightarrow \boxed{a_{ii} = 0}$ ← Diagonal elements.

main Diagonal $(i=j)$

e.g. $B = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & -3 \\ -4 & 3 & 0 \end{bmatrix}_{3\times 3}$

$$B^T = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & 3 \\ 4 & -3 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & -3 \\ -4 & 3 & 0 \end{bmatrix}$$

$$B^T = -B$$

Theorem ① A is any square matrix

(i) $A + A^T$ is always symmetric.

(ii) $A - A^T$ is always skew symmetric.

Proof: let $X = A + A^T$

$$X^T = (A + A^T)^T = A^T + (A^T)^T = \underline{A^T + A} = X$$

$X \to$ sym.

$A + A^T \to$ sym.

Let $Y = A - A^T$

$$(Y)^T = (A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T)$$

$$\underline{Y^T = -Y}$$

$\therefore$ $Y \to$ skew sym.

$A - A^T \to$ skew sym.

Theorem ② 'A' $\to$ any sq. matrix.

$$A = (\text{sym.}) + (\text{skew sym.})$$

$$\boxed{A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)}$$

e.g. $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (zero matrix)

$$\boxed{\begin{aligned} O^T &= O \ ✓ \\ O^T &= -O \ ✓ \end{aligned}}$$

↓

Both → Sym.
→ Skew sym.

e.g. $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}$ → Sym ✗
→ Skew sym ✗

$A^T = \begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \\ 4 & 7 & 10 \end{bmatrix}$

Sym. part $= \frac{1}{2}(A + A^T)$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 8 & 12 \\ 8 & 12 & 16 \\ 12 & 16 & 20 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 8 \\ 6 & 8 & 10 \end{bmatrix} ✓$$

Skew sym. Part $= \frac{1}{2}(A - A^T)$

$$= \frac{1}{2}\left(\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix} - \begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \\ 4 & 7 & 10 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$A = \underbrace{\frac{1}{2}(A + A^T)}_{\text{Sym.}} + \underbrace{\frac{1}{2}(A - A^T)}_{\text{Skew}} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 8 \\ 6 & 8 & 10 \end{bmatrix} + \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

MATRICES Exercise 3.3 $\boxed{Q5, Q6, Q10, Q11, Q12}$

$\boxed{Q.5}$ (i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times1}$ $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}_{1\times3}$

$(AB)' = \cancel{A'B'}\ B'A'$

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{pmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{pmatrix}$$

$$LHS = (AB)' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$RHS = B'A' = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}' \cdot \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8. & 6 \\ 1 & -4 & 3 \end{bmatrix} = \cdot\ LHS\ \checkmark$$

$(AB)' = B'A'$

⑥ (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}_{2\times2}$

<u>To Prove</u>

$$\underset{2\times2}{\underset{\downarrow}{A'}} . \underset{2\times2}{\underset{\downarrow}{A}} = \underset{2\times2}{\underset{\downarrow}{I}}$$

I = Identity matrix (2×2)

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = RHS.$$

$$LHS = A'A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}' . \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} . \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\longrightarrow . \downarrow$$

$$= \begin{bmatrix} \overset{1}{\boxed{\cos^2\alpha + \sin^2\alpha}} & \overset{0}{\cancel{\cos\alpha}\sin\alpha - \sin\cancel{\alpha}\cos\alpha} \\ \underset{0}{\sin\alpha\cancel{\cos\alpha} - \cancel{\cos\alpha}\sin\alpha} & \underset{1}{\boxed{\sin^2\alpha + \cos^2\alpha}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2} = I = RHS.$$

$\boxed{Q.10}$ (iii) $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$A = \left(a\ \underbrace{\text{symmetric matrix}}\right) + \left(a \text{ skew sym. matrix}\right)$

$A = \frac{1}{2}A + \frac{1}{2}A$

★ $A = \underbrace{\frac{1}{2}(A+A')}_{\text{Sym.}} + \underbrace{\frac{1}{2}(A-A')}_{\text{Skew sym.}}$

$X^T = X \qquad\qquad X^T = -X$

$A' = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$

$RHS = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$

$= \frac{1}{2}\left\{\begin{bmatrix} \textcircled{3} & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} \textcircled{3} & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$

$+ \frac{1}{2}\left\{\begin{bmatrix} \textcircled{3} & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \overset{\cancel{+}}{-} \begin{bmatrix} \textcircled{3} & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right\}$

$A = \frac{1}{2}\left\{\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}\right\} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

⑪ $A \rightarrow A^T = A$ ⟵ Sym.

$B \rightarrow B^T = B$ ⟵

$(A \pm B)^T = A^T \pm B^T$

$(AB)^T = B^T A^T$

$X = AB - BA$

$X^T = (AB - BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T . B^T$

$= BA - AB$

$= -(AB - BA)$

$X^T = -(X)$ 'X' is skew sym.

$\Rightarrow$ 'AB-BA' is skew Sym.

Option (A)

$\boxed{Q.12}$ $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}_{2\times2}$

$A + A' = I$

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \boxed{\cos\alpha} \end{bmatrix}_{2\times2} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \boxed{\cos\alpha} \end{bmatrix}_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\alpha = ?$

Option - B

Comparison, $2\cos\alpha = 1$

$\cos\alpha = \frac{1}{2}$

$\alpha = 60° = \frac{\pi}{3}$

Elementary Operations (प्राथमिक संक्रियाएं)	Invertible Matrices (व्युत्क्रमणीय आव्यूह) ✓	Inverse of a matrix by Elementary Operations

Invertible Matrices.

let 'A' → order 'm' (Square matrix) (order m×m)

let matrix 'B' exists for 'A' such that

$$\underset{m\times m}{A}\,\underset{m\times m}{B} = BA = \underset{\text{Identity matrix }(m\times m)}{I}$$

Generally $AB \neq BA$

then 'A' is called Invertible Matrix.

A का inverse $= A^{-1} = B$

B का inverse matrix $= B^{-1} = A$

Note. Rectangular matrices $[\ \ ]_{2\times3}$ $[\ \ ]_{3\times1}$
↓
These are not invertible.

Note: ① A के A^{-1} (B) unique होता है

② $(AB)^{-1} = B^{-1}.A^{-1}$ (Reversal Law)

~~$(AB)^{-1}$ [illegible] $A^{-1}.B^{-1}$~~ $(AB)^{T} = B^{T}.A^{T}$

e.g. $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}_{2\times 2}$ $\quad B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}_{2\times 2}$

$\boxed{AB = BA = I} \longrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

AB

$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4-3 & -\cancel{6}+\cancel{6} \\ \cancel{2}-\cancel{2} & -3+4 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$BA = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$AB = BA = I$.

∴ A का Inverse = B

B का inverse = A

Elementary Operations on a Matrix

Elementary Row operation ★

Elementary Column operation.

(I) $R_i \leftrightarrow R_j$ $\qquad C_i \leftrightarrow C_j$

$$\begin{array}{c} R_1 \leftarrow \\ R_2 \leftarrow \\ R_3 \leftarrow \end{array} \begin{bmatrix} 2 & 4 & 3 \\ 7 & 8 & 0 \\ -1 & -2 & -3 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 2 & 4 & 3 \\ -1 & -2 & -3 \\ 7 & 8 & 0 \end{bmatrix}$$

How to find inverse of 'A'

$A^{-1} = A$? let inverse of $A = B = A^{-1}$

$B = ?$

Given: 'A' square matrix

$$\boxed{AB = BA = I}$$

$A = A$

$$\begin{tikzcd}
A \arrow[d, squiggly, "\text{Elementary Row Operation.}"] \arrow[d, bend right=60, "\text{A से I लाना}"'] & = & I \cdot A \arrow[d, squiggly] \\
I & = & BA
\end{tikzcd}$$

$(\because IA = AI = A)$

$I \to$ multiplicative identity for matrices.

Identity matrix

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$

$2\times1 = 2$

$3\times1 = 3$

$$\boxed{A \times A^{-1} = I}$$

$A^{-1} = A$ का inverse $= B$

using Ele. Column op.

$$\begin{tikzcd}
A \arrow[d, squiggly] & = & A \cdot I \arrow[d, squiggly] \\
I & = & A\underset{\uparrow}{B}
\end{tikzcd}$$

$\boxed{A^{-1}}$

(II) $R_i \to kR_i$ $(K \neq 0)$ $\qquad$ $C_i \to kC_i$ $(K \neq 0)$

$$\begin{array}{c} \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}_{2\times 2} \\ \uparrow \quad \uparrow \\ C_1 \quad C_2 \end{array} \xrightarrow{C_2 \to -3C_2} \begin{bmatrix} 2 & -9 \\ 4 & -3 \end{bmatrix}$$

(iii) $R_i \to \underbrace{R_i \pm kR_j}$ $\qquad$ $C_i \to C_i \pm KC_j$

$$\begin{array}{c} R_1 \to \\ \\ R_3 \to \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow{R_3 \to \underbrace{R_3 - 7R_1}} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & -6 & 12 \end{bmatrix}$$

$\to 7 - 7\times(1) = 0$

$\to 8 - 7\times 2 = -6$

$\to 9 - 7\times 3 = 12$

How to find Inverse (A^{-1}) of a matrix (A) using Elementary transformations (operations)

Note:

Ele. Row Op. (only on Left matrix)

Equation (समीकरण) $\quad \underset{\uparrow}{A}.B = \underset{\uparrow}{C}.D.E$

$\underset{\uparrow}{X} = \underset{\uparrow}{A}B$

Ele. Column op. (only on Right matrix)

$A.\underset{\uparrow}{B} = C.D.\underset{\uparrow}{E}$

$\underset{\uparrow}{X} = A\underset{\uparrow}{B}$

e.g. Find inverse of $\begin{bmatrix}1&3\\2&7\end{bmatrix}_{2\times2}$.

(Using elementary row operations)

Ans. $A = \begin{bmatrix}1&3\\2&7\end{bmatrix}$

left

$\underset{\uparrow}{A} = \underset{\uparrow}{I}.A$

left

$I = \begin{bmatrix}1&0\\0&1\end{bmatrix}_{2\times2}$

$\Rightarrow R_2 \rightarrow \begin{bmatrix}1&3\\2&7\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}.A$

R_2

Final Target

$I = B.A$

$\begin{bmatrix}1&0\\0&1\end{bmatrix} = [\quad].A$

A^{-1}

Elementary Row Operations

$R_2 \rightarrow R_2 - 2R_1$

$\begin{bmatrix}\boxed{1}&\boxed{\ }\\\boxed{0}&\boxed{1.}\end{bmatrix}$

$2 \rightarrow 2 - 2\times1$
$= 0$

$7 \rightarrow 7 - 2\times3$
$= 7-6$
$= 1$

$0 \rightarrow 0 - 2\times1$
$= -2$

$1 \rightarrow 1 - 2\times0$
$= 1-0$
$= 1$

$R_i \leftrightarrow R_j$
$R_i \rightarrow kR_i$
$R_i \rightarrow R_i \pm kR_j$
$k \neq 0$

$\Rightarrow R_2 \begin{bmatrix}1&3\\0&1\end{bmatrix} = \begin{bmatrix}1&0\\-2&1\end{bmatrix}.A$

$R_1 \rightarrow R_1 - 3R_2$

$\Rightarrow \begin{bmatrix}1&0\\0&1\end{bmatrix} = \begin{bmatrix}7&-3\\-2&1\end{bmatrix}.A$

$\Rightarrow I = \begin{bmatrix}7&-3\\-2&1\end{bmatrix}.A$

A^{-1}

$\begin{bmatrix}1&3\\2&7\end{bmatrix}$ का inverse $= \begin{bmatrix}7&-3\\-2&1\end{bmatrix}$

Exercise 3.4 (Q.11) [Q12]

2×2 matrix का inverse

[Using Elementary Row Transformations]

$A = IA$

[Q.11]

$$A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}_{2\times 2}$$

I = Identity matrix

$= 2\times 2$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\underset{\uparrow}{A} = \underset{\uparrow}{I} \cdot A$$

Ele. Row op. left side

① $R_i \leftrightarrow R_j$

② $R_i \rightarrow kR_i \; (k \neq 0)$

③ $R_i \rightarrow R_i \pm kR_j \; (k \neq 0)$

$AB = BA = I$

B = A का inverse

$B = A^{-1}$

Target set

$I = BA$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = B \begin{bmatrix} & \\ & \end{bmatrix} \cdot A$$

$A = I \; A$

$A = IA$

$$\begin{array}{c} R_1 \rightarrow \\ R_2 \rightarrow \end{array} \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \leftrightarrow R_1$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot A$$

$\boxed{R_2} \to R_2 - 2R_1$

$2 - 2 \times 1$

$-6 - 2 \times (-2) = \underline{-6+4}$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \cdot A$$

$R_2 \to$

$$\boxed{R_2 \to \frac{R_2}{-2}}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

$$\Rightarrow I = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \cdot A$$

$$A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

$$\boxed{I = B \cdot A}$$

$$B = A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$\boxed{Q.12}$ $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

(Elementary <u>Row</u> operations)

$\textcircled{A} = \textcircled{I} A$

$$\Rightarrow \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} . A$$

Target

$I = BA$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [\ \ ] . A$

$\textcircled{R_1} \rightarrow R_1 + 3R_2$

$6 \Rightarrow 6 + 3(-2)$

$6 - 6 = 0$

$-3 \rightarrow -3 + 3(1)$

$= 0$

$1 \rightarrow 1 + 3(0)$

$= 1$

$0 \rightarrow 0 + 3(1)$

$= 3$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} . A$$

1. Complete Row/Column, 2×2 Elements

↳ $\textcircled{0}$

<u>Target</u> $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [\ \ ] . A$

$\therefore A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

is not invertible.

3×3 Matrix का Inverse $(A^{-1}) = ?$ (How)

(using Elementary Row operations)

Exercise 3.4 (Q.17)

① $R_i \leftrightarrow R_j$

② $R_i \to kR_i$

③ $R_i \to R_i \pm kR_j$

Ele. Row. opⁿ.

left matrix.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}_{3\times3}$$

$$I = \overset{A^{-1}}{AB} = BA$$

$$A = I.A$$

$$I = BA$$

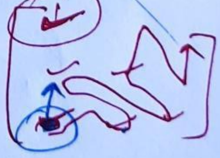
Target

I = identity M.

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

$R_1 \to \frac{R_1}{2}$



$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

$R_2 \to R_2 - 5R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

$R_3 \to R_3 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} \cdot A$$

R_3

$R_3 \to 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} \cdot A$$

$-\frac{5}{2} - \frac{5}{2}(5) = \frac{-30}{2}$

$1 + \cancel{\frac{5}{2}(2)} = 6$

$R_2 \to R_2 - \frac{5}{2} R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{R_3}{2}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \cdot A$$

$$I = \underbrace{\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}}_{A^{-1}} \cdot A$$

Miscellaneous Exercise on Chapter - 3

① $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

Prove: P(n): $\boxed{(aI + bA)^n = a^n.I + na^{n-1}b.A}$ $n \in N$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= O$ (ओ)

(Zero matrix)

$O.B = O$

$I.B = B$

$B.I = B$

I^2

$I.I = I$

PMI

Principle of Mathematical Induction

① $n=1$ के लिए prove.

② $n=K$ (माना)

$n=K+1$ के लिए prove.

Step-I: P(1) is true ??

$n=1$

$P(n): (aI+bA)^n = a^n I + na^{n-1}.b.A$

$P(1): \underline{aI + bA} = \underline{aI + bA}$

∴ P(1) is true.

Step-②: let (माना) P(K) is true

$\hookrightarrow (aI+bA)^K = a^K I + Ka^{K-1}.bA$ —①

Now we have to prove that P(K+1) is true.

$$\boxed{(aI+bA)^{K+1} = a^{K+1}.I + (K+1).a^K.bA}$$

$$LHS = (aI + bA)^{K+1}$$

$$= \boxed{(aI + bA)^K} \cdot (aI + bA)$$

$\downarrow$ by eq^n. ①

$$= \left(\underline{a^K I} + \underline{K a^{K-1} b A}\right) \cdot \left(\underline{aI} + \underline{bA}\right)$$

$$= a^{K+1} \cdot \overset{\to I}{\boxed{I^2}} + a^K b . \overset{\to A}{\boxed{I A}} + K a^K b . \overset{\to A}{\boxed{A I}}$$

$$+ \underbrace{K a^{K-1} b^2 . \underset{\downarrow\ 0}{\boxed{A^2}}}$$

$\uparrow$ (zero matrix)

$$A^2 = 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$5 . A^2 = 5.0 = 5.\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= a^{K+1} . I + \boxed{a^K b} . \boxed{A} + K \boxed{a^K b} . \boxed{A} + \underset{\uparrow\ \text{zero matrix}}{0}$$

$$= a^{K+1} . I + (1+K) a^K b A = \underline{\underline{RHS}}$$

$\therefore$ P(K+1) is also true.

$\therefore$ P(n): $(aI + bA)^n = a^n I + n a^{n-1} b.A$

is true. (by using PMI)

Miscellaneous Exercise on Chapter ③

Q4, Q5

Q.4 $\left.\begin{matrix} A^T = A \\ B^T = B \end{matrix}\right\} \leftarrow$ Sym. matrices

To Prove $(AB-BA) \longrightarrow$ Skew Sym.

$$(AB-BA)^T = -(AB-BA)$$

$$(XY)^T = Y^T.X^T$$

$$(AB-BA)^T = (AB)^T - (BA)^T$$

$$= B^T.A^T - A^T.B^T$$

$$= B.A - A.B$$

$$\boxed{(AB-BA)^T = -(AB-BA)}$$

∴ AB−BA is Skew Sym.

Q.5 $B'AB$

$$(X^T)^T = X$$

$$(B'.A.B)' = B'A'(B')'$$

$$\boxed{(B'AB)' = B'A'B} \text{ — ①}$$

Part (I) Let 'A' is Sym. $\Longrightarrow A^T = A = A'$

by eqⁿ ①: $(B'AB)' = B'AB$ ∴ Symmetric

Part (II) Let A is Skew Sym $\Longrightarrow A^T = -A = A'$

By eqⁿ ①: $(B'AB)' = B'(-A).B = -B'AB \longrightarrow$ Skew Sym.

$\boxed{Q.9}$

$$\underset{1\times3}{\begin{bmatrix} x & -5 & -1 \end{bmatrix}} \cdot \underset{3\times3}{\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}} \cdot \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = \underset{\text{Zero matrix}}{0}$$

$$\Rightarrow \underset{1\times3}{\begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}} = \underset{\text{zero matrix}}{0} \text{ (ओ)} \qquad \boxed{x} = ?$$

$\rightarrow \cdot \downarrow$

$$\Rightarrow \begin{bmatrix} \underline{(x-2) \cdot x - 40 + 2x - 8} \end{bmatrix}_{1\times1} = \begin{bmatrix} \underset{\uparrow \text{ Zero}}{\underline{0}} \end{bmatrix}_{1\times1}$$

By Comparison,

$$\Rightarrow x^2 - \cancel{2x} - 40 + \cancel{2x} - 8 = 0$$

$$\Rightarrow x^2 - 48 = 0 \qquad 48 = \boxed{16} \times 3$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm\sqrt{48}$$

$$\Rightarrow \boxed{x = \pm 4\sqrt{3}}$$

$\boxed{Q.10}$ $\xleftarrow{\text{Miscellaneous Exercise on chapter ③}}$

Market	ⓧ	ⓨ	ⓩ
I →	10000	2000	18000
II	6000	20000	8000

$$\text{Matrix} = \overset{2\times 3}{M} = \begin{array}{c|c} \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} & \begin{array}{l} \to \text{Ⓘ} \\ \to \text{II} \end{array} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ x & y & z \end{array} & \end{array}$$

market

$$\text{matrix} = \overset{3\times 1}{S} = \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix} \begin{array}{l} \leftarrow x \\ \leftarrow y \\ \leftarrow z \end{array}$$

Selling price (₹)

$$\text{Revenue} = \overset{2\times 1}{R} = \underset{2\times 3}{M} \times \underset{3\times 1}{S} = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \cdot \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

(matrix)

$$= \begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix}_{2\times 1}$$

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}_{2\times 1} \begin{array}{l} \to \text{market I's Revenue} \\ \to \text{market II's revenue} \end{array}$$

SP

Ⓘ

$$\text{matrix} = \underset{\uparrow}{C} = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}_{3\times 1}$$

Cost price (₹)
(CP)

$$\text{Total cost} = \underset{\underset{2\times 1}{\swarrow}}{T} = \underset{\underset{2\times 3}{\downarrow}}{M} \times \underset{\underset{3\times 1}{\downarrow}}{C}$$

$$= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$T = \begin{bmatrix} 20000 + 2000 + 9000 \\ 12000 + 20000 + 4000 \end{bmatrix}_{2\times 1}$$

$$\underset{\uparrow}{T} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}_{2\times 1} \begin{matrix} \rightarrow \text{I} \\ \rightarrow \text{II} \end{matrix} \qquad \underset{\uparrow}{R} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} \begin{matrix} \rightarrow \text{I} \\ \rightarrow \text{II} \end{matrix}$$

Total cost (CP)

Revenue (S.P.)

$$\text{Gross Profit} = G = R - T$$

matrix,

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

$$= \begin{bmatrix} 15000 \\ 17000 \end{bmatrix} \begin{matrix} \leftarrow \text{I} \\ \leftarrow \text{II} \end{matrix}$$

Miscellaneous Exercise on Chapter-3 $\boxed{\text{Q11, Q12}}$

$\boxed{\text{Q.11}}$

$$\underset{\substack{\downarrow\\ \text{Matrix}}}{X}\cdot\underbrace{\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}}_{A}{}_{2\times3} = \underbrace{\begin{bmatrix}-7&-8&-9\\2&4&6\end{bmatrix}}_{B}{}_{2\times3}$$

$$\begin{array}{ccccc} X & \cdot & A & = & B\\ \downarrow & & \downarrow & & \downarrow\\ \boxed{2}\times\boxed{2} & & 2\times3 & & 2\times3\end{array}$$

X का order $= 2\times2$

let $X = \begin{bmatrix}a & b\\ c & d\end{bmatrix}_{2\times2}$

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\cdot\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix} = \begin{bmatrix}-7&-8&-9\\2&4&6\end{bmatrix}$$

multiply → solve → Compare

↓

equalitions

a, b, c, d ← solve ←

✓

Q.12

$AB = BA$ (Given) **To Prove** Ⓘ $AB^n = B^n A$ $(n \in N)$

Statement P(n)

Ⓘ

Ⓘⓘ $(AB)^n = A^n B^n$

Let

$P(n): AB^n = B^n A$

Principle of mathematical Induction: (PMI)

Ⓘ $n = 1$ के लिए Prove.

Ⓘⓘ $n = K$ (माना) $P(K)$ ✓ ①

$n = K+1$ के लिए Prove.

Target

$P(K+1): AB^{K+1} = B^{K+1} . A$

1st step:

$n = 1$

$P(1): AB = BA$

True.

P(1) is true.

2nd step: Let P(K) is true.

$$P(K): \boxed{AB^K = B^K A} \text{ ———— ①}$$

Now we have to prove that P(K+1) is also true (i.e. $AB^{K+1} = B^{K+1} . A$)

$$LHS = AB^{K+1}$$

$$= (A . B^K) . B$$

By eqⁿ ①

$$= (B^K . A) B \quad \left(\because AB = BA \text{ Given}\right)$$

$$= B^K . \underline{BA}$$

$$= B^{K+1} . A = RHS.$$

$\therefore$ P(K+1) is true

$\therefore$ P(n): $AB^n = B^n A$ is true.

H.P

(II) To Prove $P(n): (AB)^n = A^n . B^n$ $\quad n \in N$

1st step.

$P(1)$ is true.

~~$P(1) = (AB)^n$~~ $=$ $\quad P(1): (AB)^1 = A^1 . B^1$

$\Rightarrow AB = AB$

$\therefore P(1)$ is true.

2nd step. Let $P(K)$ is true

$$P(K): (AB)^K = A^K . B^K \quad \text{——} ①$$

Now we have to prove that $P(K+1)$ is also true.

$$\boxed{P(K+1): (AB)^{K+1} = A^{K+1} . B^{K+1}} \star$$

$$LHS = (AB)^{K+1}$$

$$= (AB)^K . (AB)^1$$

$\downarrow$ By equation ①

$$= A^K . B^K . AB$$

$(\because AB^n = B^n A)$

$AB^K = B^K . A$

$$= A^K . AB^K . B$$

$$= A^{K+1} . B^{K+1}$$

$$= RHS$$

$\therefore P(K+1)$ is also true.

$\therefore P(n): (AB)^n = A^n . B^n$ is also true.

MISCELLANEOUS EXERCISE ON CHAPTER $\boxed{3}$

$\boxed{Q13, 14, 15}$

Q.13

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \qquad A^2 = I$$

I = identity matrix
↓
2×2

$$\Rightarrow A^2 = I$$

$$\Rightarrow A \cdot A = I$$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \cdot \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\rightarrow \cdot \downarrow$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \cancel{\alpha\beta} - \cancel{\alpha\beta} \\ \gamma\cancel{\alpha} - \alpha\cancel{\gamma} & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

comparison

$$\alpha^2 + \beta\gamma = 1 \Rightarrow \boxed{0 = 1 - \alpha^2 - \beta\gamma}$$

Q. 14

$A^T = A$ ←	Sym ✓	Sym ✗	Sym ✗	Sym ✓
$A^T = -A$ ←	Skew ✗	Skew ✓	Skew ✗	Skew ✓

↓

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{Zero matrix} = (0)$$

$\boxed{Q.15}$ $A \rightarrow$ square matrix

$\underbrace{A^2 = A}$ ✓

$$\boxed{\begin{aligned} A^3 &= A^2 \cdot A \\ &= A \cdot A \\ &= A^2 \\ &= A \end{aligned}}$$

$$(I+A)^3 - 7A = ?$$

?

$$\boxed{\begin{aligned} I\cdot A = A\cdot I = A \\ I^2 = I^3 = I^n = I \end{aligned}}$$

$$\boxed{(a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b}$$

$$(I+A)^3 - 7A$$

$$= I^3 + A^3 + 3IA^2 + 3I^2A - 7A$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$= I + A + 3\boxed{A^2} + 3A - 7A$$

$$\downarrow$$

$$= I + \cancel{\underbrace{A + 3A + 3A}_{\boxed{7A}}} \cancel{- 7A}$$

$$= I$$

option $\boxed{c}$